On waiting time for elective surgery admissions

Following:

Armstrong, 2000 A,B

Sobolev, Levy, and Kuramato, 2000

Sobolev and Kuramato, 2008

The waiting times for coronary artery bypass grafting (CABG) were such that 25%,50%,75% and 90% of the patients underwent surgery within 5,12,23 and 46 weeks.

(Sobolev & Kuramato, 2008)

Vladimir:

What do we do now?

Estragon:

Wait.

Waiting for Godot

Samuel Beckett

How long do people wait?

England, 1994

A Department of Health spokesperson insisted that "… *'half of all patients on waiting lists are already seen in five weeks'* …"

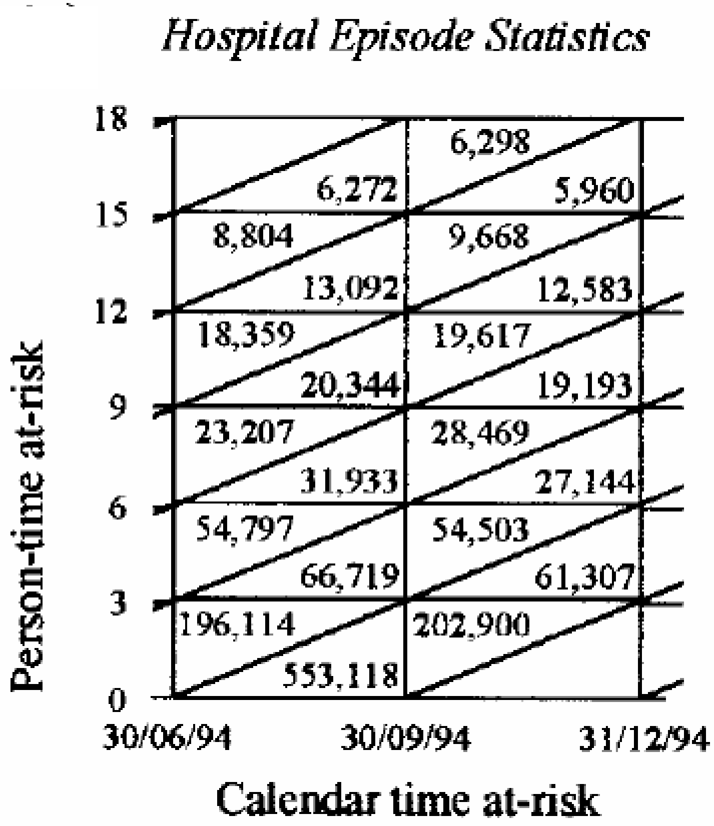
And of those *'already seen'* between 1 July and 31 December 1994, 1,506,883/2,017,685 = 75% had waited less than three months.

And of those 'still waiting' at 30 September 1994, 514,662/1,071,101 = 48% had been waiting less than three months.

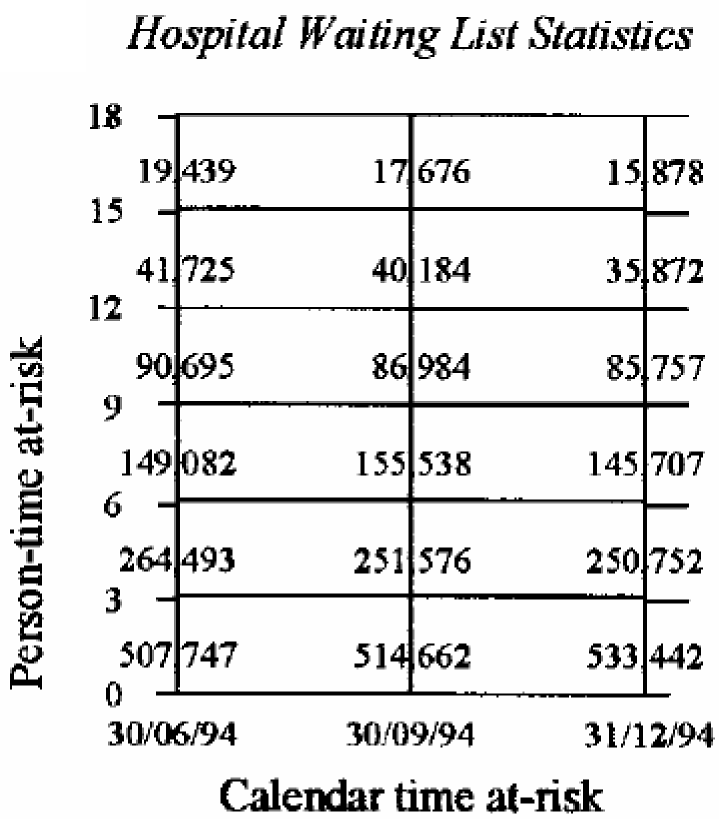
(Armstrong , 2000)

Databases:

1. List of time to admission



2. Waiting time census.



(Armstrong , 2000)

Part 1 – Introduction

Part 2 – Lists of time to admission

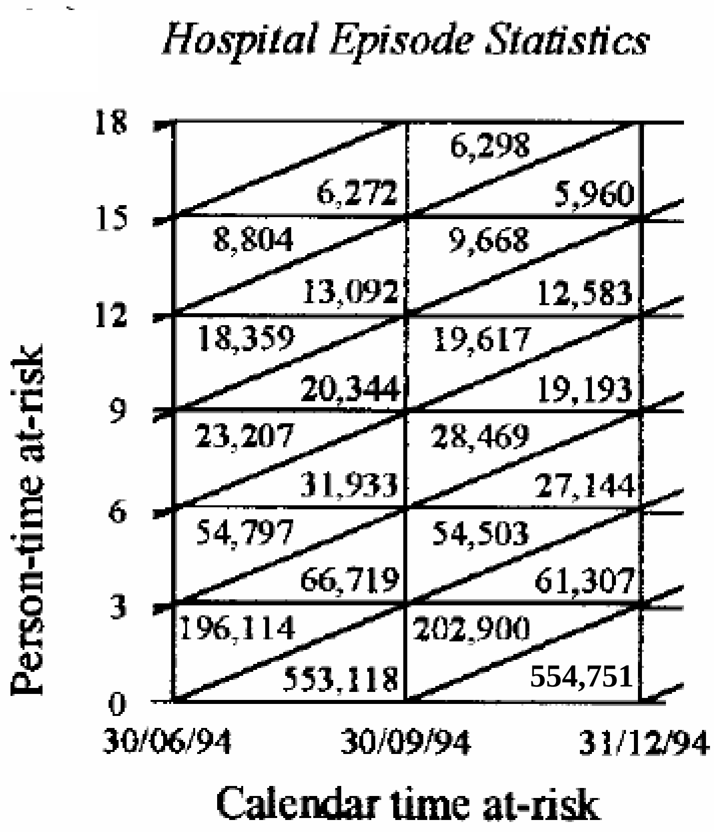
Part 3 – Waiting time census

Part 4 – Waiting time estimation

Part 5 – Censored observations

Part 6 – Competing risks

List of time to admission :



And of those *'already seen'* between 1 July and 31 December 1994, 1,506,883/2,017,685 = 75% had waited less than three months.

(Armstrong , 2000)

Lists of time to admission

And of those *'already seen'* between 1 July and 31 December 1994, 1,506,883/2,017,685 = 75% had waited less than three months.

Define:

A_t = Number admitted to surgery in period t

The probability for surgery in period t :

$$P(A \in [0,3]) \approx \frac{A_{0-3}}{A_{0-3} + A_{3-6} + A_{6-9} + A_{9-12} + A_{>12}}$$

Lists of time to admission

Patients are removed from the list if they reconsider the decision for surgery, if they accept surgery from another surgeon, if they decline admission, if they move out of the province, if they are deferred or suspended on medical grounds, if they are suspended for administrative reasons, if they die while awaiting surgery, if the physician decides to try a medical treatment instead of waiting for surgery, if their conditions preclude scheduling of surgery indefinitely, if their conditions improve and make the surgery unnecessary, if the operation is no longer possible, if the operation no longer offers the likelihood of improvement, or when surgery is done.

(Sobolev et al., 2000)

"We understand that 14% of patients in Australia may expect to be removed from the waiting list for some reason other than admission."

(Armstrong , 2000)

Lists of time to admission

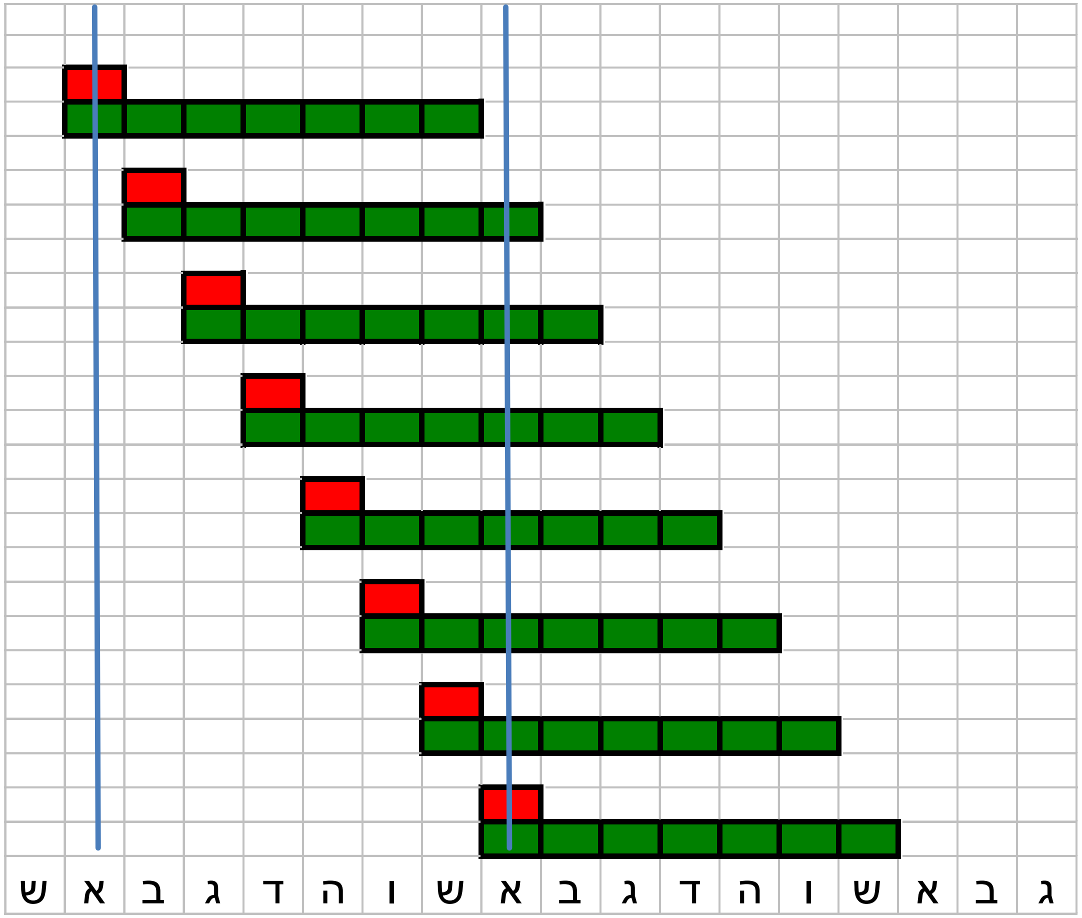
From the two types of patients:

7 of the one day waiting time

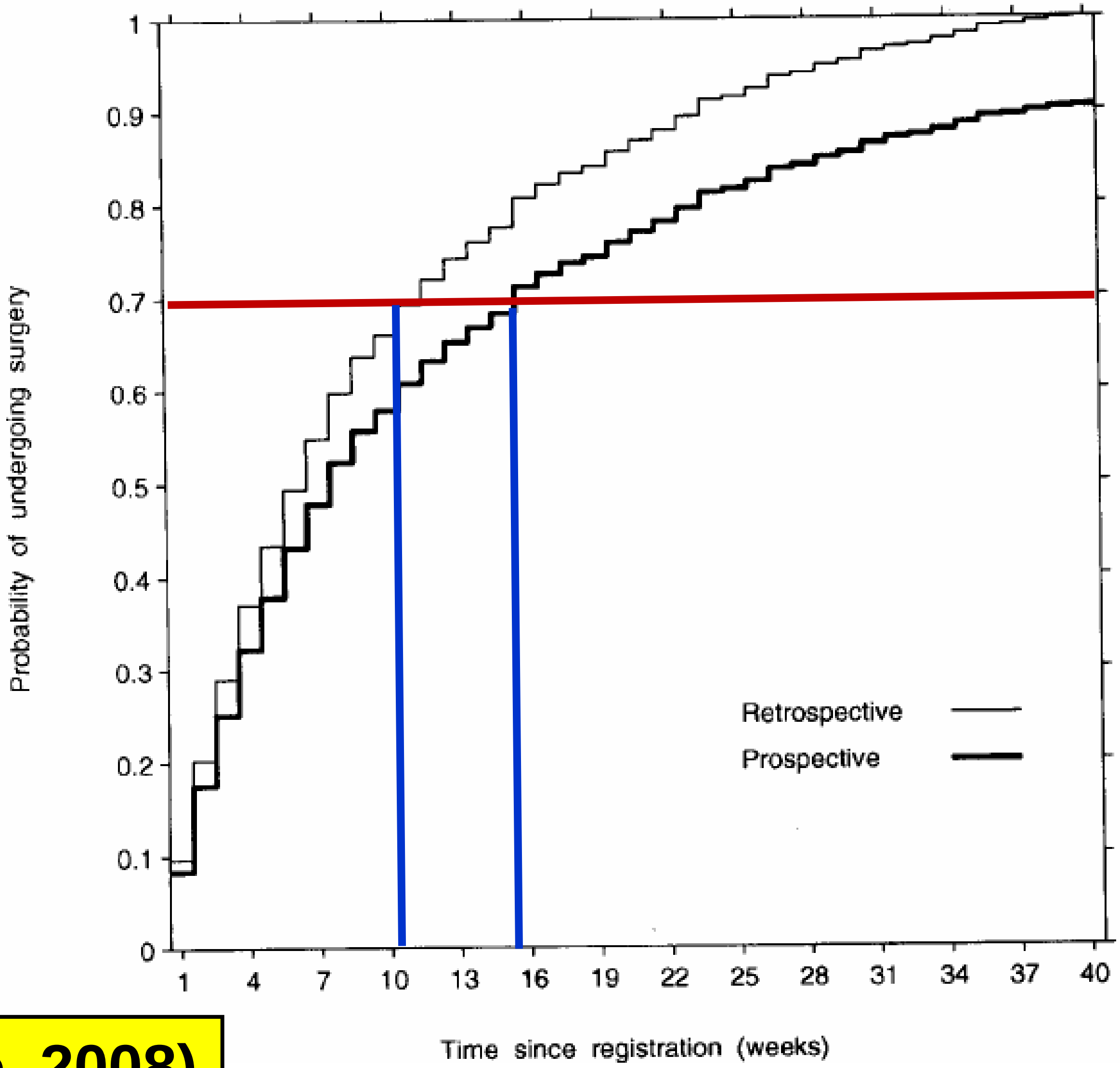
compared to

1 of the seven days waiting time

were counted.



Estimated probability of undergoing surgery as a function of waiting time. Data for a single group of patients awaiting vascular surgery.



(Sobolev & Kuramato, 2008)

Part 1 – Introduction

Part 2 – Lists of time to admission

Part 3 – Waiting time census

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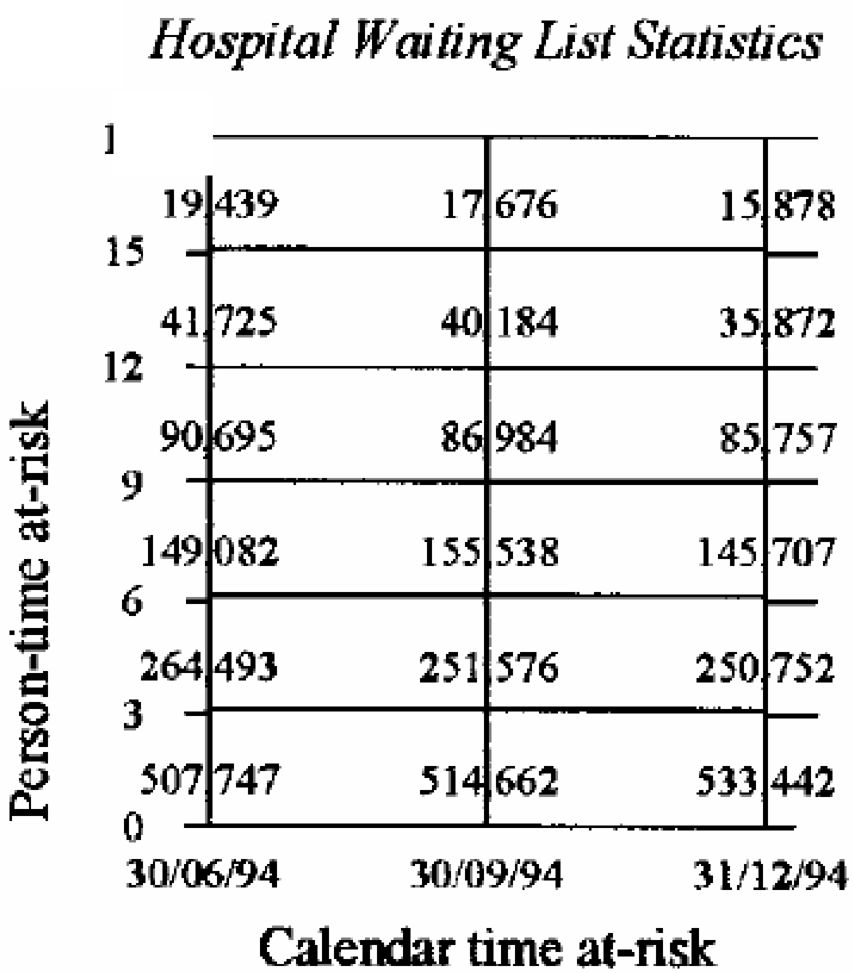
Part 6 – Competing risks

Waiting time census:



And of those ‘still waiting’ at 30 September 1994, 514,662/1,071,101 = 48% had been waiting less than three months.

(Armstrong , 2000)

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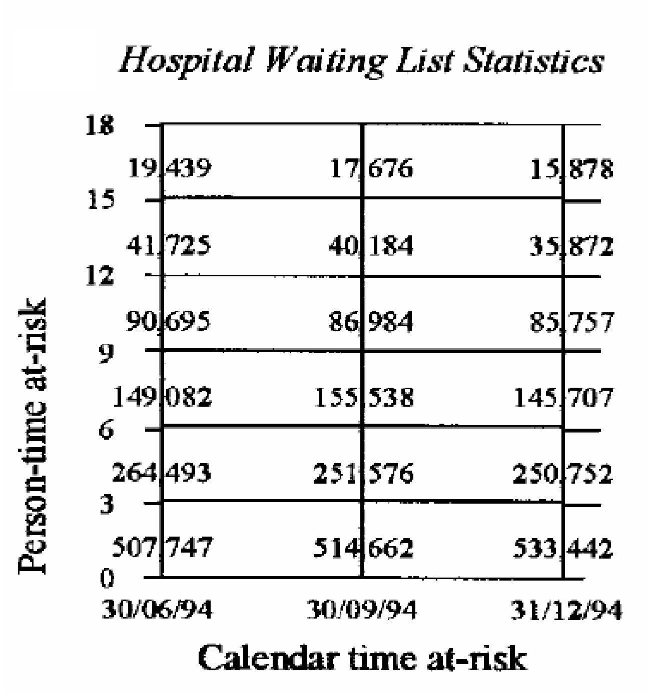


Define:

W_t = Number that waited period of t at time of census.

$$P(W \in [0,3]) \approx \frac{W_{0-3}}{W_{0-3} + W_{3-6} + W_{6-9} + W_{9-12} + W_{>12}}$$

$$P(W \in [0,3]) \approx \frac{514,662}{514,662 + 155,538 + 86,984 + 40,184 + 17,676}$$

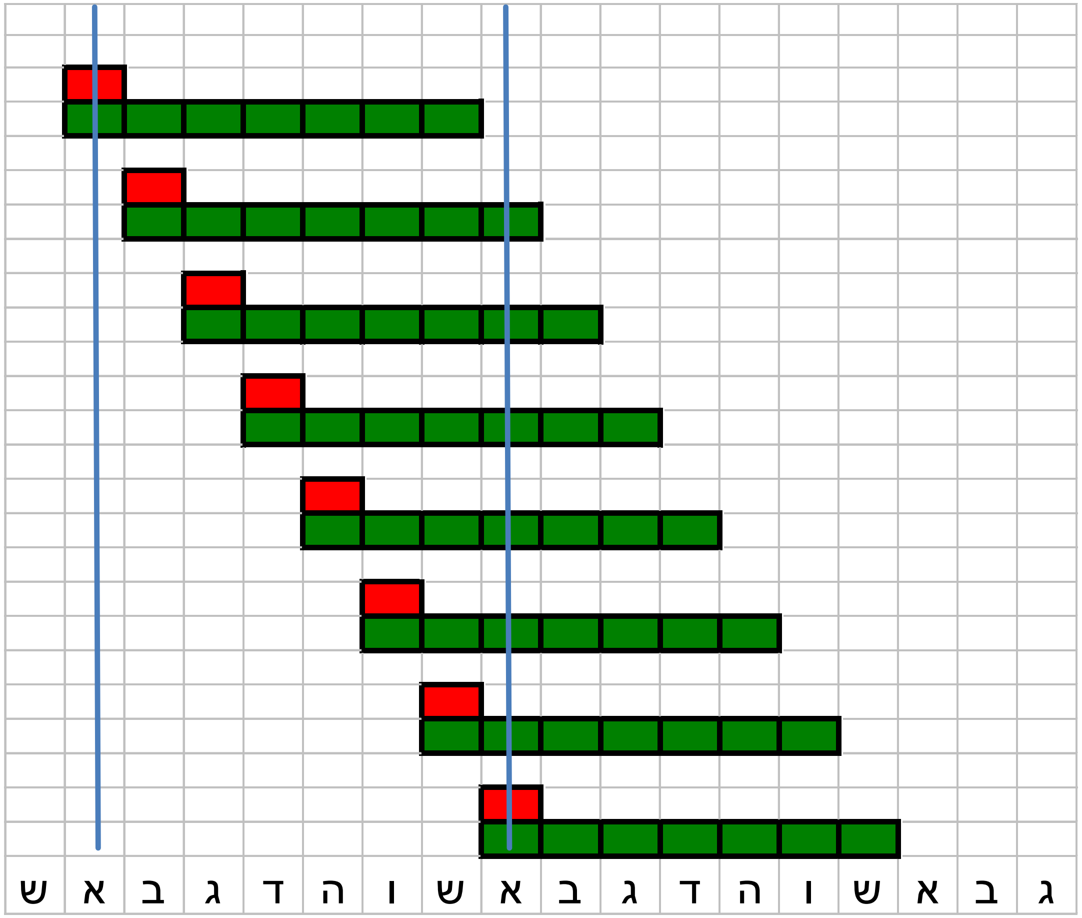
From the two types of patients:

1 of the one day waiting time

compared to

7 of the seven days waiting time

were counted.



Problems:

1. Short waiting periods do not appear in census.

2. Stationary assumption:
   Patients who enrolled in different periods are compared.

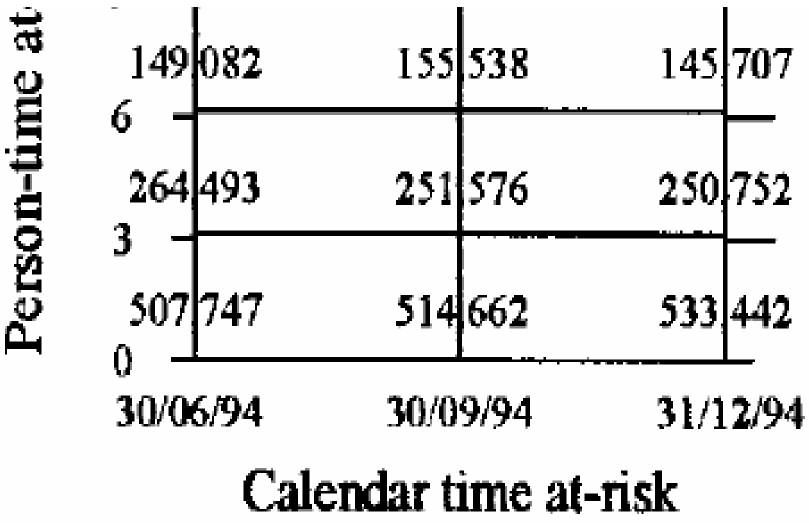
"But the hospital waiting list for England would not have attracted so much attention if it were really stationary..."

(Armstrong , 2000)

Problems:

3. It is not clear how long a patient waited if the patient appears in one census but not in the next.

More specifically, a patient was counted in the Sep. 0-3 category and does not appear in the Dec. 3-6 category.



How long did the patient wait?

Part 1 – Introduction

Part 2 – Lists of time to admission

Part 3 – Waiting time census

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Waiting time estimation:

Define:

S_t = Number at risk at the end of period t

A_t = Number admitted to surgery in period t

C_t = Number censored in period t

The probability for surgery in period t for patients still at risk at time t :

$$P(A = t \mid A \geq t) \approx \frac{A_t}{S_t + A_t}$$

Question: How to treat *censored* observations?

The probability for surgery in period t for patients still at risk at time t :

$$P(A = t | A \geq t) \approx \frac{A_t}{A_t + S_t}$$

By Bayes' rule

$$P(A = t | A \geq t) = \frac{P(A = t)}{P(A \geq t)}$$

hence

$$P(A \geq t + 1 | A \geq t) = \frac{P(A \geq t + 1)}{P(A \geq t)} \approx 1 - \frac{A_t}{A_t + S_t}$$

We have seen

$$P(A \geq t+1 | A \geq t) = \frac{P(A \geq t+1)}{P(A \geq t)} \approx 1 - \frac{A_t}{A_t + S_t}$$

The *survival function* can be estimated by

$$S(t) = P(A \geq t+1)$$

$$= \frac{P(A \geq 1)}{1} \cdot \frac{P(A \geq 2)}{P(A \geq 1)} \cdot \ldots \cdot \frac{P(A \geq t+1)}{P(A \geq t)}$$

$$\approx \prod_{j=1}^{t} \left(1 - \frac{A_j}{A_j + S_j} \right)$$

which is called the **Kaplan Meier** estimator

Censored observations:

Question 1:

How many censored observations are there?

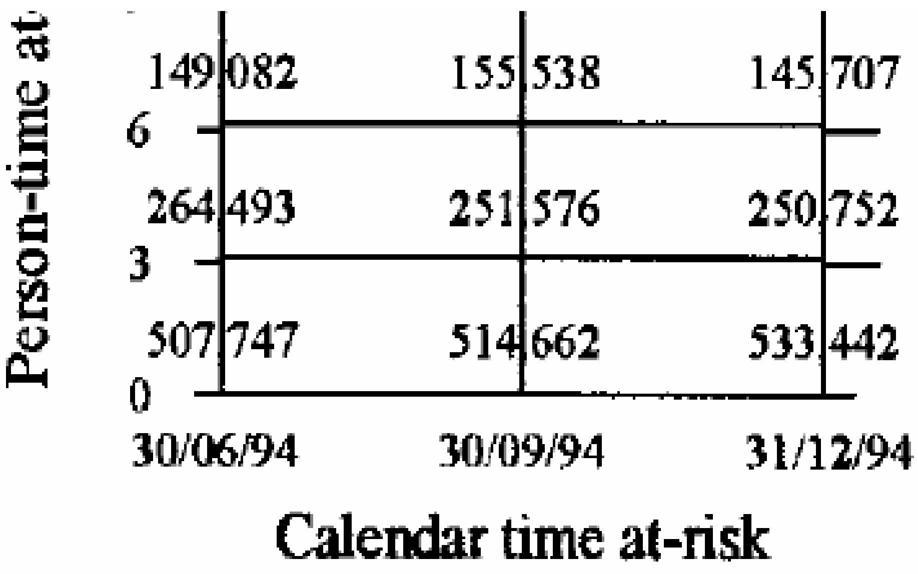
Note that censored observations are not counted either in the list of *time to admission* or in the *census*.

Question2:

How to treat censored observations?

Question 1:

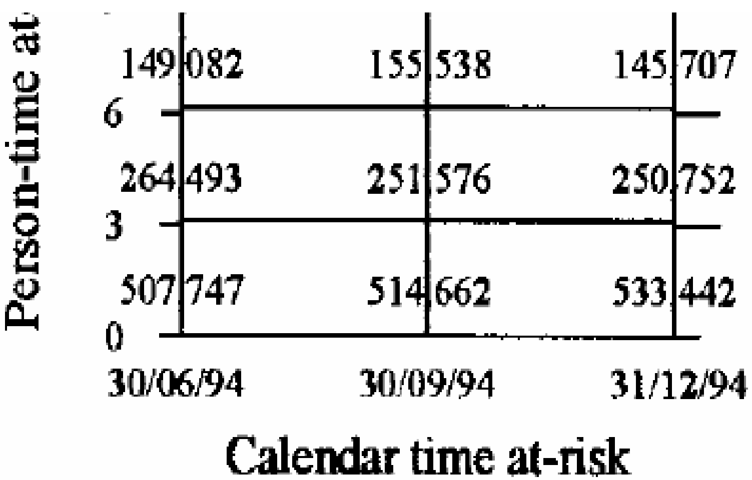
How many observations in the category of 0-3 months that enrolled between July to Sep. were censored between Oct. and Dec.?



Answer:

1. Calculate the difference between the number in category 0-3 in the Sep. census to those still waiting in the Dec. census.

Note: The difference accounted also for observations from 3-6 months category.

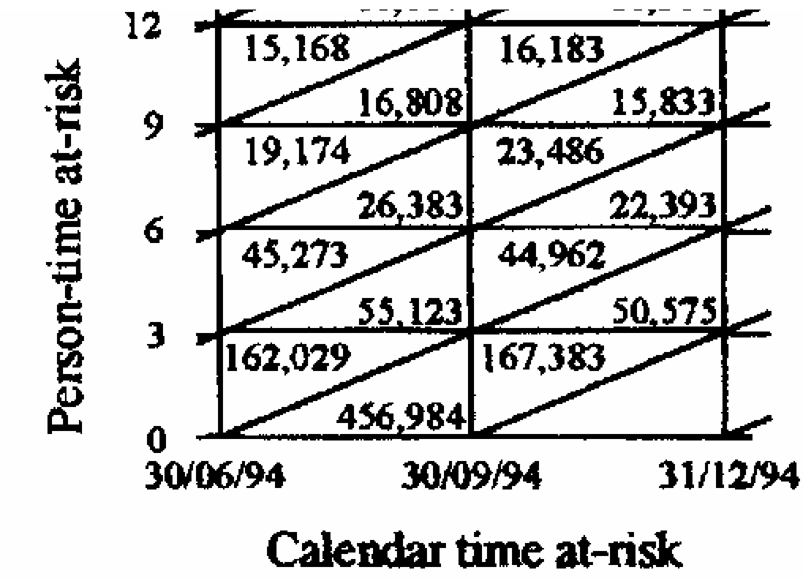


$$W_{0-3}^{\text{Sep}} - W_{3-6}^{\text{Dec}} = 514,662 - 250,752$$
$$= 263,910$$

2. Calculate how many of the patients that were enrolled between July and Sep. where admitted between Oct. and Dec.

$$\#\{A \text{ in Oct.-Dec.} \mid A \text{ ennrolled in July-Sep.}\} =$$

$$= 167,383 + 50,575 = 217,958$$



So far: Number of patients that enrolled between July and Sep. and were censored between Oct. and Dec. is:

enrolled in July-Sep. and not listed in Dec. census

minus

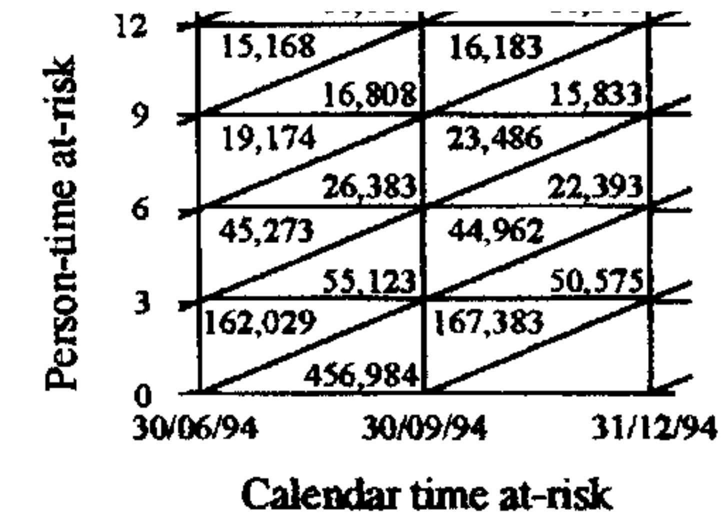
enrolled in July-Sep. and admitted Oct.-Dec.

$$\begin{array}{rr} & 263{,}910 \\ - & \\ & \underline{217{,}958} \\ = & 45{,}952 \end{array}$$

Question: How many of the censored observations were patients that waited 0-3 months?

3. Of the patients that enrolled between July and Sep. and were admitted between Oct. and Dec., calculate which percentage waited 0-3 months.

$$P(A \in [0,3] | A \text{ in } \ \text{Oct.-Dec.}) = \frac{167,383}{167,383 + 50,575} = 0.77$$



Censored observations

Conclude:

An estimate of the number of patients that:

enrolled between July and Sep.,

were censored between Oct. and Dec.

and waited 0-3 months:

$$45{,}952 \cdot 0.77 = 35{,}289$$

Number censored

% that waited 0-3 months

Censored observations:

Question2:

How to treat censored observations?

Answer:

First, note that in the **Kaplan-Meier** estimator, *censored observations* from periods $t+1$,… are indeed included.

$$\hat{S}(t) = \prod_{j=1}^{t} \left(1 - \frac{A_j}{A_j + S_j} \right)$$

Question2:

How to treat censored observations at period t?

Answer:

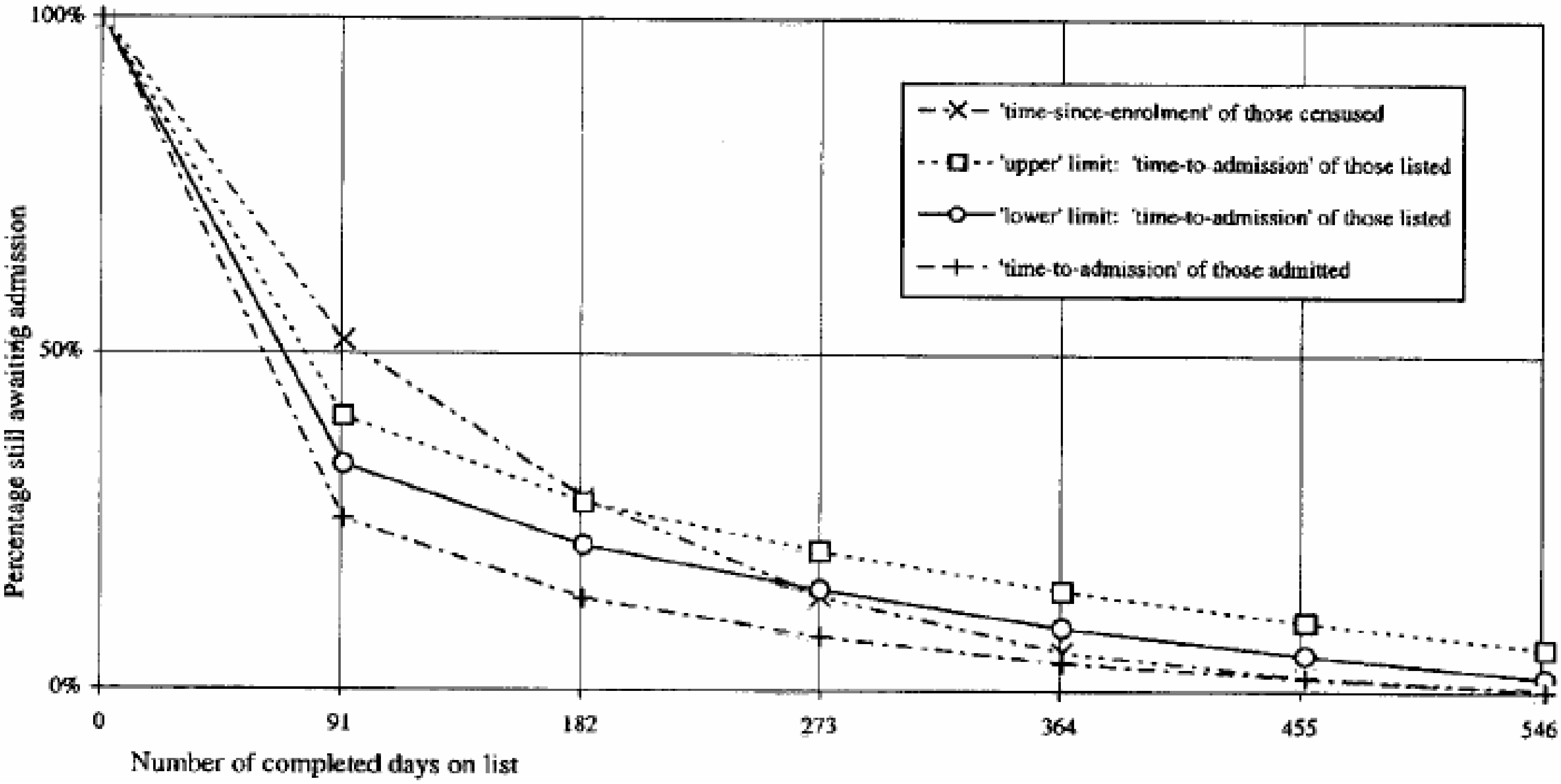
1. Assume that all were censored in the beginning of the period and need not be included (**Lower Bound**)

$$P(A = t \mid A \geq t) \approx \frac{A_t}{A_t + S_t}$$

2. Assume that all were censored at the end of the period and need to be included (**Upper bound**)

$$P(A = t \mid A \geq t) \approx \frac{A_t}{A_t + C_t + S_t}$$

Censored observations



(Armstrong , 2000)

Competing risks:

"A competing event is any event whose occurrence either **precludes the occurrence of another event** under examination or fundamentally alters the probability of occurrence of this other event."

(Gooley et al., 1999)

Examples:

1. **Medical:** Death is primary, surgery is competing event.
2. **Medical:** Surgery is primary, urgent surgery is competing event.
3. **Call centers:** Abandonment is primary, service is competing event.

Cumulative incidence function (CIF)

The probability of any event happening is partitioned to the probabilities of each **type** of event.

Define:

S_t = Number at risk at the end of period t

E_t = Number of primary events in period t

A_t = Number of competing events in period t

$$P(E = t \mid E \geq t) \approx \frac{E_t}{E_t + A_t + S_t}$$

Cumulative incidence function (CIF)

S_t = Number at risk at the end of period t

E_t = Number of primary event in period t

A_t = Number of competing event in period t

$$P(E = t \mid E \geq t) \approx \frac{E_t}{E_t + A_t + S_t}$$

Note:

$$P(E \geq t+1 \mid E \geq t) \neq 1 - \frac{E_t}{E_t + A_t + S_t}$$

=>Kaplan-Meier estimator does not work!

Cumulative incidence function (CIF)

Define the survival function as before (using **Kaplan-Meier**)

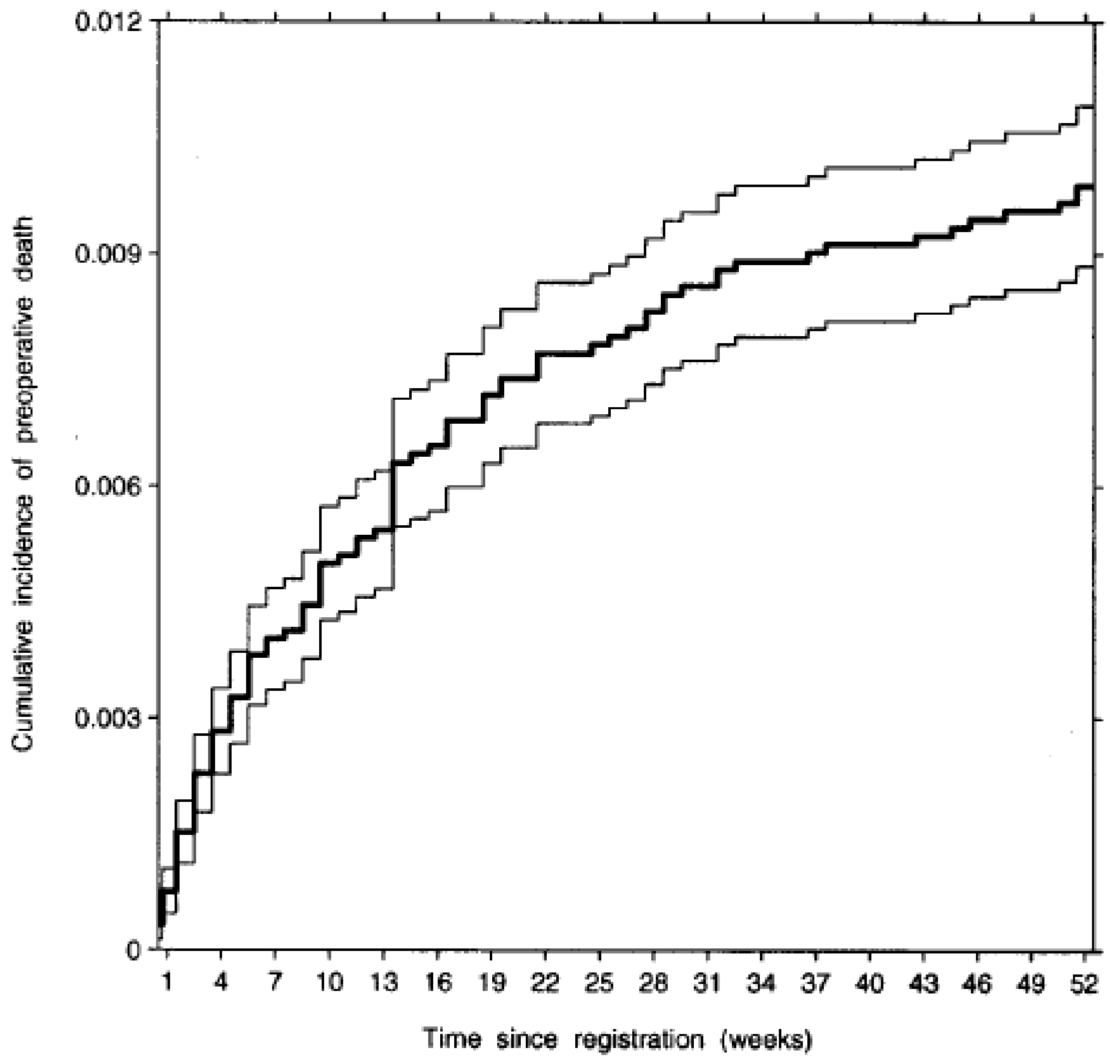
$$\hat{S}(t) = \prod_{j=1}^{t} \left(1 - \frac{A_j + E_j}{E_j + A_j + S_j} \right)$$

Define the **CIF** as

$$\widehat{F}_E(t) = \sum_{j=1}^{t} \frac{E_j}{E_j + A_j + S_j} \cdot \hat{S}(j-1)$$

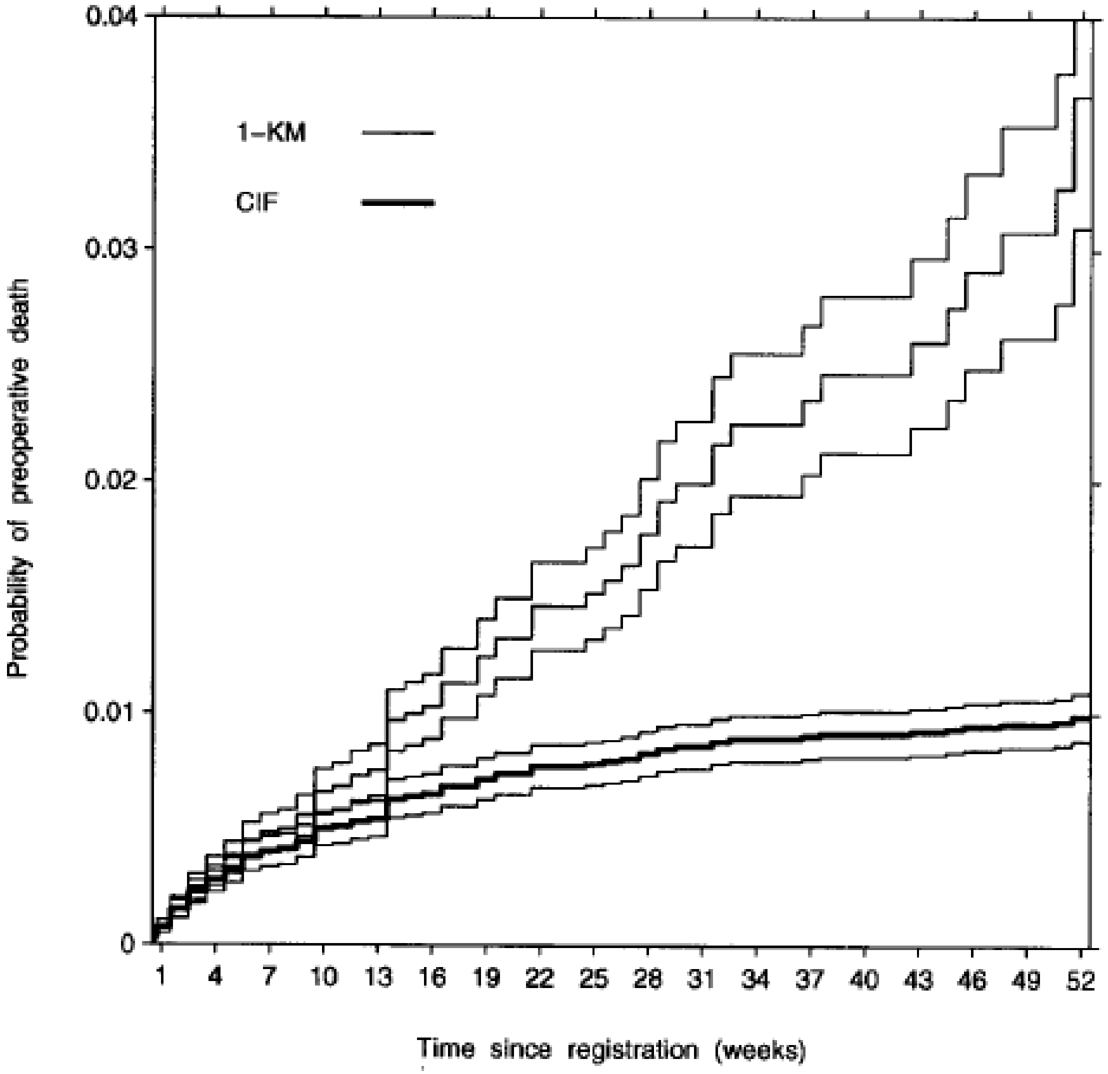
Death while waiting (CABG)

CIF of preoperative death during or before a certain week since registration for elective CABG



Death while waiting (CABG)

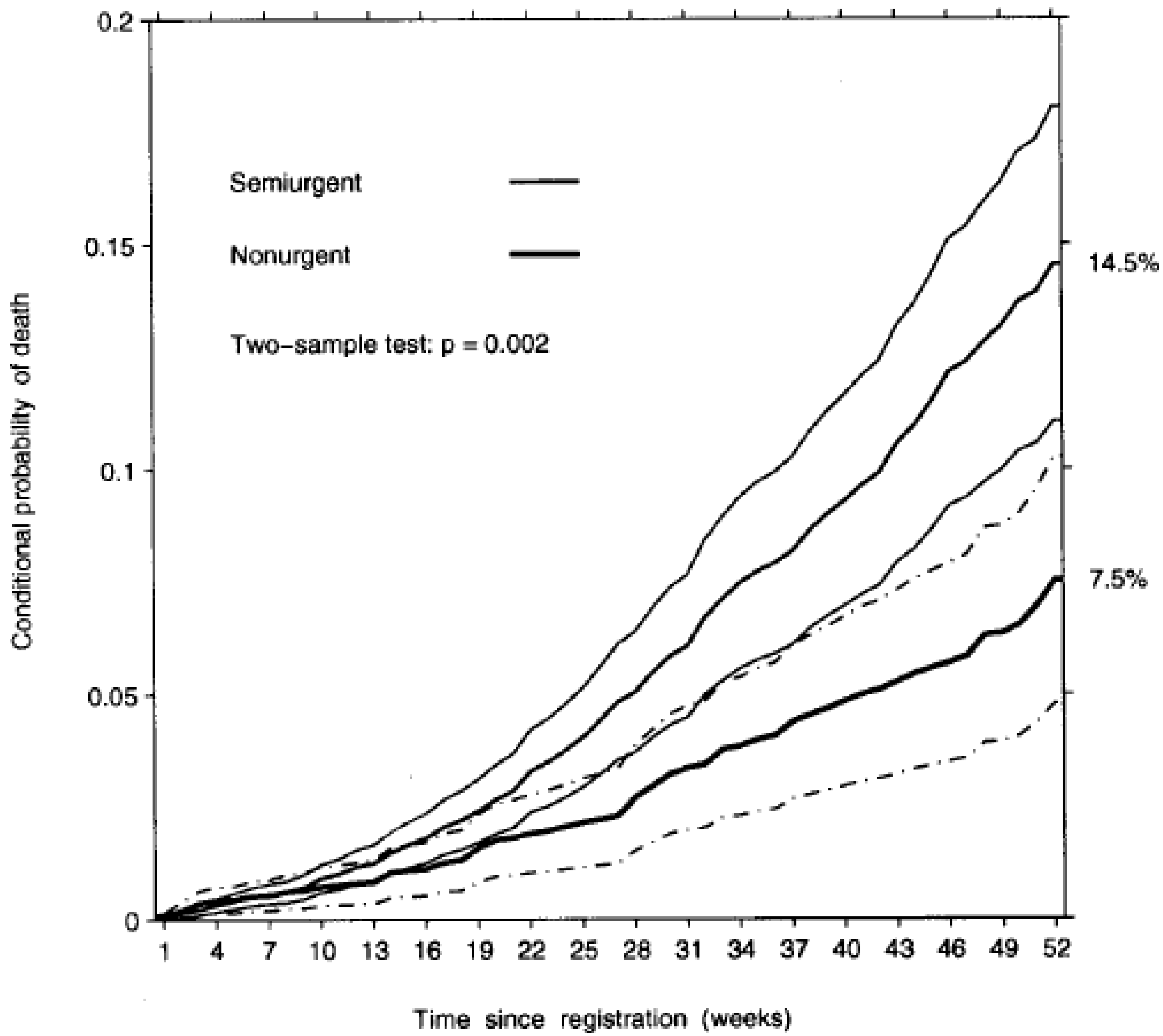
CIF compared to **Kaplan-Meier**



Death while waiting (CABG)

The **Conditional Probability Function (CPF)** of preoperative death from CABG.

$$\hat{CP}_E(t) = \frac{\hat{F}_E(t)}{1 - \hat{F}_A(t)}$$



References

1. Armstrong, P.W., “First steps in analysing NHS waiting times: avoiding the 'stationary and closed population' fallacy.” *Statist. Med.* 2000.
2. Armstrong, P.W., “Unrepresentative, invalid and misleading: are waiting times for elective admission wrongly calculated?”, *J Epidemiol Biostat.* 2000.
3. Sobolev, B., Levy, A., and Kuramoto, L., “Access to surgery and medical consequences of delays” In: *R. Hall ed. Patient Flow: Reducing Delay in Healthcare Delivery,* 2006.
4. Sobolev, B., Kuramoto, L., Analysis of Waiting-Time Data in Health Services Research. 1st edition. Hardcover, Springer, 2007;