

Performance Analysis of Dynamic Stochastic

PERT/CPM Networks

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YONIT BARRON

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content

Abstract.	1
1. Dynamic Stochastic PERT Networks.	4
1.1 Literature Survey.	4
1.2 Model Overview.	27
1.2.1 Model Description.	27
1.2.2 System description.	29
1.2.3 Examples.	31
1.3 Redefinition's.	33
1.3.1 Critical Path.	33
1.3.2 Critical Task.	36
1.3.3 Time profile.	38
1.4 Simulation Example.	43
1.4.1 Model Description.	43
1.4.2 Homogeneous System.	44
1.4.2.1 Deterministic Service Times.	44
1.4.2.2 Stochastic Service Times.	44
First Method.	47
Second Method.	52
1.4.2.3 Stochastic Arrival.	57
1.4.3 NonHomogeneous System.	60
2. Parallel Serial System.	63
2.1 System's Description.	63
2.1.1 The Model.	63
2.1.2 Lindley's Equations.	64
2.1.3 Expansion of Lindley's Equations.	65
2.2 Analytic Results.	66
2.2.1 Definition of "Random Associated Variables".	66
2.2.1.1 Characteristics.	67

content

2.2.1.2 Proofs.	67
2.2.1.3 Results and Conclusions.	69
2.2.2 Deterministic Arrival.	72
2.3 Bounds.	73
2.3.1 Poisson Arrival.	73
Large and Fixed n .	75
2.3.2 Deterministic Arrival.	79
2.4 Approximations with the use of Simulations and Bounds.	80
2.4.1 Small m, n .	80
2.4.2 Large n .	85
2.4.2.1 Influence of First Synchronization.	85
2.4.2.2 Influence of Last Synchronization.	88
2.4.3 Large m .	91
2.4.4 Large m, n .	93
2.4.5 Deterministic Arrival.	94
2.5 Summary, Results and Main Conclusions.	97
3. Summary and Extensions.	99
4. Bibliography.	101
5. Appendix.	105
5.1 Tables.	105

List of Tables:

Number	Name	Page
1.3.1a	The Path's Criticality for the Dynamic Model.	35
1.3.1b	The Path's Criticality for the Stochastic Model.	35
1.3.2a	Tasks' Criticality.	37
1.3.2b	Queue-Length Average.	38
5.1.1-5.1.7	Compare-Tables.	104

List of Figures:

Number	Name	Page
1.1a	Project Network with Synchronization Queues.	4
1.1b	Dynamic Stochastic Model.	17
1.2.1a	Stochastic Network.	27
1.2.2a	Dynamic Stochastic Network.	29
1.2.3a	PERT Network.	31
1.2.3b	PERT Network.	32
1.2.3c	Processing Network Representation.	32
1.3.3a	Serial Network.	39
1.3.3b	Parallel Network.	40
1.3.3c	Gantt Network for the Dynamic Model.	42
1.4.1a	PERT Network.	43
1.4.1b	Dynamic Model.	44
2.1.1a	PERT Network.	63

List of Graphs:

Number	Name	Page
1.1a	Static Stochastic Model, Task's Times Exponentially Distributed.	9
1.1b	Dynamic Stochastic Model.	17
1.4.2.1a	Throughput Time in a Homogeneous System, as Function of Utilization.	44
1.4.2.2a	Throughput Time as Function of Utilization, CV fixed.	45
1.4.2.2b	Throughput Time as Function of CV, Utilization fixed.	45
1.4.2.2c	Percent of Waiting Time, as Function of Utilization & CV, According to the First Method.	47
1.4.2.2d	Percent of Synchronization Time, as Function of Utilization & CV, According to the First Method.	48
1.4.2.2e	Profile at Different CV, as Function of Utilization, according to the First Method.	50
1.4.2.2f	Percent of Waiting Time, as Function of Utilization & CV, According to the Second Method.	52
1.4.2.2g	Percent of Synchronization Time, as Function of Utilization & CV, According to the Second Method.	53
1.4.2.2h	Profile at Different CV, as Function of Utilization, according to the Second Method.	55
1.4.2.3a	Average Throughput Time, as Function of CV's.	57
1.4.2.3b	Percent of Waiting Time, as Function of CV's, According to the Second Method.	58
1.4.2.3c	Percent of Synchronization Time, as Function of CV's, According to the Second Method.	59
1.4.3a	Average Throughput Time in Nonhomogeneous System..	62
2.4.1a	Lower Bounds for Small m,n.	81
2.4.1b	Expected Throughput Time and Bounds for Small m,n.	82
2.4.1c	e as Function of n.	82
2.4.1d	e as Function of m.	82

Number	Name	Page
2.4.1e	e as Function of ρ .	83
2.4.1f	Throughput Time Density, 2*2 Network.	84
2.4.1g	Throughput Time Density, 8*2 Network.	84
2.4.2a	Throughput Time and Bounds.	85
2.4.2b	a as Function of n.	86
2.4.2c	a as Function of m.	86
2.4.2d	a as Function of ρ .	86
2.4.2e	b as Function of n.	88
2.4.2f	b as Function of m.	88
2.4.2g	b as Function of ρ .	88
2.4.2h	Expected Throughput Time (From Simulations).	90
2.4.3a	Throughput Time and Bounds.	91
2.4.3b	The Normalized Difference, as Function of Utilization.	92
2.4.4a	Throughput Time.	93
2.4.5a	Throughput Time in System, as Function of n.	94
2.4.5b	Throughput Time in System, as Function of m,	94

ABSTRACT.

In the traditional approaches to management, planning and control of projects it is generally assumed that the projects go through a system separately and independently of each other. Handling of projects in accordance with these approaches is generally deterministic, with the results of analysis of a single project being a "sure" forecast of the date of conclusion. In reality, however, a large number of projects characterized by uncertainty are handled simultaneously, and in this way a dynamic system is created.

In this system, projects compete to receive service from a group of limited resources, and this results in the forming of queues of activities belonging to different projects. The study separates the queues into two types:

- Queues waiting for resources, because resources are limited.
- Synchronization queues formed because of constraints of priorities and independence between the activities.

In this study we attempt to examine the influence of the various parameters (for instance: efficiency, variability factors etc.) on the time profile of a project in the system.

The work contains two main chapters:

The first chapter begins with a survey of the literature, in which we gradually build the dynamic model. The basic model is the deterministic model. With the addition of distributions to the service times, we receive a static stochastic model. Finally, with the addition of arrival of projects for the system, we receive the dynamic model.

For the purposes of the study we redefined traditional concepts, such as: critical path, critical task etc.

In addition, we divide the time profile of a project in the system into three types:

Processing time, waiting time and synchronization time.

We proposed two methods for defining these times:

Method 1: Relieving the system from constraints of resources and synchronization, while measuring the benefit in performances. This method has four stages that can be executed in an arbitrary order.

Method 2: Treatment of each task separately. For each task these times are well defined. We then define a performance index that equals to the sum of times spent by the tasks at the stations.

At the end of the chapter we illustrate applications of these methods and examination of other parameters in a small, but sufficiently interesting system. In this way we supplied a framework for empirical investigation based on simulations of queue networks.

The main conclusion derived from the example is that the waiting time for resources constitutes most of the overall time, in both methods. On the other hand, the synchronization time is low. Therefore, if we wish to reduce the throughput time in the system, we will invest in reducing the waiting time (for instance by increasing resources). Naturally, lowering of efficiency or reduction of the coefficient of variation (arrival and service) will improve the system.

Another conclusion is that the arrival variability has almost no influence on the synchronization time between the tasks.

On the other hand, the arrival variability and the service variability both have an influence on the waiting time.

An analytical analysis of a general system is difficult and most times impossible. Accordingly, in the second chapter we restrict attention to a simple and symmetric system. The system included $n \times m$ resources, each devoted to an activity. The project is split into n groups of m tasks each, and the n groups operate in parallel.

The task duration is exponentially distributed, and the arrival of the projects to the system constitutes a renewal process.

We built three bounds for the expected throughput time of the project in the system:

- Upper bound received by a system with n independent arrival of projects.
- Lower bound received by a system without arrival of projects, a static stochastic system.
- Lower bound, received by a system with one group ($n=1$).

Through the use of these bounds and with results of simulations, we received analytical approximations for the mean throughput time of a project, for a border cases of the parameters m, n :

- Fixed m , $n \rightarrow \infty$
- Fixed n , $m \rightarrow \infty$
- $m \rightarrow \infty$, $n \rightarrow \infty$

In each case we matched an approximate analytical formula for a Poisson phase and a deterministic phase. We also examined the influence of the various parameters on the expectancy, for instance: efficiency of the resources, size of the group (m), number of groups (n), and coefficient of variation.

III

We distinguished between the two types of synchronization existing in the system:

- Initial synchronization, formed as a result of joint arrival of projects to the system.
- Final synchronization, formed in light of the constraint whereby the project finishes its processing on conclusion of processing of all its tasks.

As a conclusion, we recommend using the following approximation for the mean throughput time of a project in a system with a Poisson arrival (for each $m > 10$, and for each n):

$$\begin{aligned} \bar{T}(n, m) &= \frac{m}{\mu - \lambda} \cdot \left(1 + \frac{\ln n}{m} \cdot \frac{\ln m - \ln \rho}{2} \right) \\ \frac{\bar{T}(n, m)}{1/\mu} &= \frac{m}{1 - \rho} \cdot \left(1 + \frac{\ln n}{m} \cdot \frac{\ln m - \ln \rho}{2} \right) \end{aligned}$$

This approximation usually gives an error of up to 5% of the mean as calculated by the simulation.

Form the approximation it can be seen that the mean throughput time of a project grows at a linear rate as a function of m and at a logarithmic rate as a function of n . If we attribute the number of tasks in group (m) to waiting queues, and the number of groups (n) to synchronization queues, then limited resources have a linear influence, while the synchronization gap has a logarithmic influence. Accordingly, it will be advisable to invest more extensively in limited resources in order to improve the system.