

# **Empirically-Based Staffing in Call Centers**

## **Simple Models at the Service of Complex Realities**

Sergey Zeltyn

Technion, Haifa, Israel

IBM Thomas J. Watson Research Center, October 3, 2006

Joint work with Professor Avi Mandelbaum

## Outline

### Subject/Flow of the Talk:

Hierarchy of operational decisions in **call centers**, with emphasis on **staffing**.

### Main message I:

**Simple Models** can be excellent tools at the service of **Complex Realities**.

### Supported by:

**Erlang-A Model** and the **QED** operational **regime** applied to call center **staffing**.

### Main message II:

**Empirically-Based** Analysis is a Prerequisite for Research, Teaching and Practice of service operations.

### Supported by:

**DataMOCCA** – Data MModel for Call Center Analysis.

# Call Centers Industry

## U.S. Statistics

- Over 60% of annual business volume via the telephone
- 70,000 – 200,000 call centers
- 3 – 6.5 million employees (3% – 6% workforce)
- 20% annual growth rate
- \$100 – \$300 billion annual expenditures
- 1000's agents in a "single" call center.

## Quality/Efficiency Tradeoff

- 65 – 80% personnel costs
- Over 90% U.S. consumers form company image via call center experience.

## Objective:

Having the right number of appropriately skilled agents when needed.

## Staffing Problem

Determination of load-dependent **number of agents**.

**Prevailing method:**

**SIPP (Stationary Independent Period-by-Period)**,  
a constant number of agents over each period (15, 30 or 60 min).

**Agents** and **customers** are assumed **homogeneous**.  
In particular, every agent can potentially serve every customer.

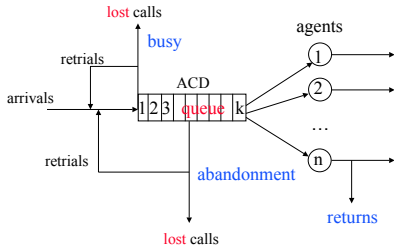
### Main Approaches to Staffing

**Constraint Satisfaction:** find minimal number of agents  $n^*$  that satisfies some performance goal(s) (e.g. less than 3% abandonment).  
Prevalent in practice, Service-Level Agreements (SLA) in outsourcing.

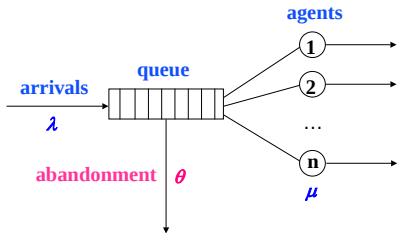
**Cost/Revenue Optimization:** find  $n^*$  that optimizes service revenues and costs of staffing, abandonment and waiting.

# M/M/n+M (Erlang-A, Palm): Main Staffing Model

## Call Center: Schematic Representation



## Erlang-A Queue



## Erlang-A Assumptions:

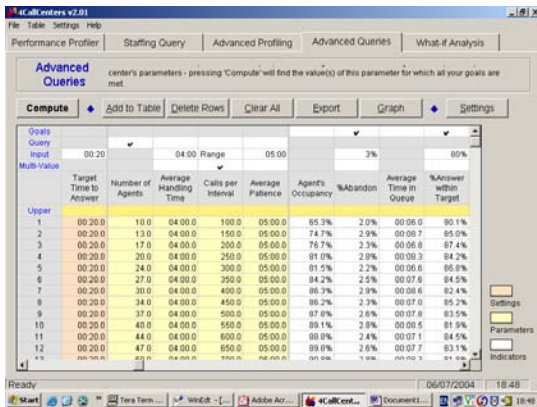
- $\lambda$  – **Poisson** arrival rate
- $\mu$  – **Exponential** service rate
- $n$  – number of service agents
- $\theta$  – **Exponential** individual abandonment rate
- **No busy** signals
- **First Come First Served.**

# Erlang-A Model: Calculations

Performance measures can be calculated relatively easily.

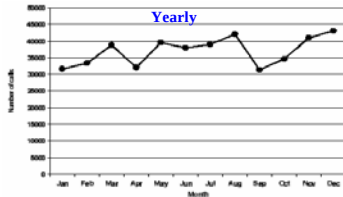
**4CallCenters** - A Personal Tool for Workforce Management.  
Based on the M.Sc. thesis of Ofer Garnett.

<http://iew3.technion.ac.il/serveng2006S/4CallCenters/Downloads.htm>

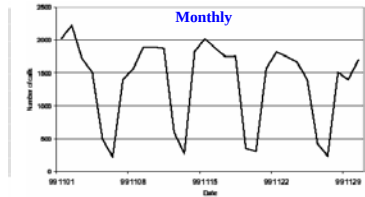


# Time Scale: Arrival to a Call Center in 1999

## Strategic



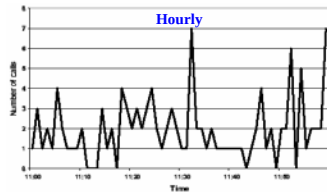
## Tactical



## Operational



## Stochastic



# Queueing Science: Arrival to a Call Center in 1976

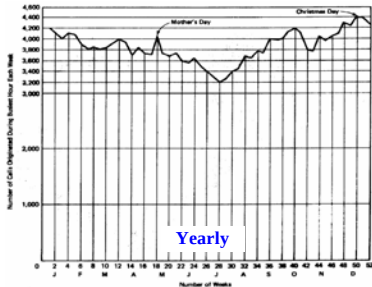


Figure 1 Typical distribution of calls during the busiest hour for each week during a year.

(E. S. Buffa, M. J. Cosgrove, and B. J. Luce,  
"An Integrated Work Shift Scheduling System")

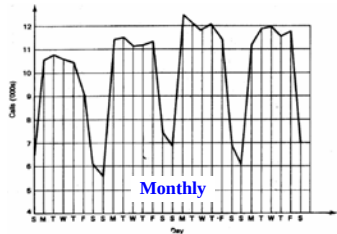


Figure 2 Daily call load for Long Beach, January 1972.

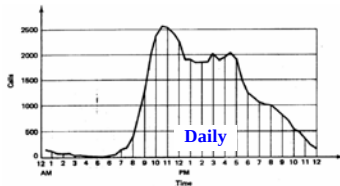


Figure 3 Typical half-hourly call distribution (Bundy D A).

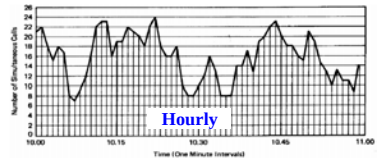


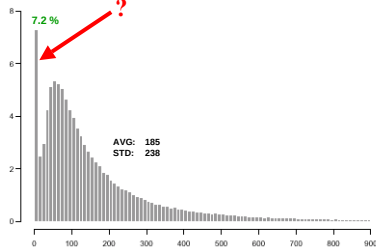
Figure 4 Typical intrahour distribution of calls, 10:00-11:00 A.M.

# Service Times: Distribution and Psychology

## Histogram of Service Times in an Israeli Call Center

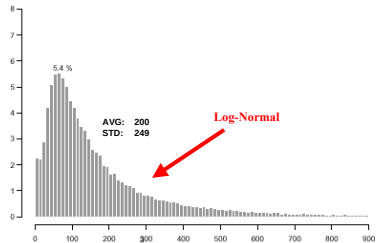
January-October

Jan - Oct:



November-December

Nov - Dec:



- **Lognormal** service times prevalent in call centers
- **7.2% Short-Services:** Agents' "Abandon" (improve bonus, rest)
- **Distributions**, not only Averages, must be measured.

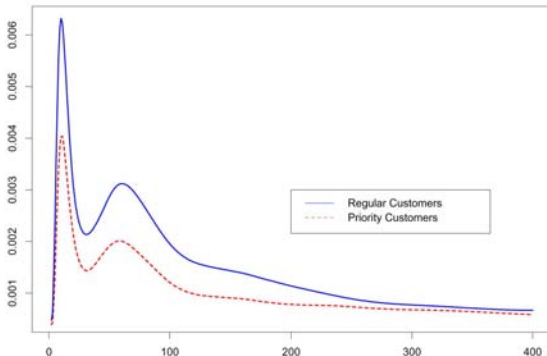
## Erlang-A: Modelling (Im)Patience

- **Patience Time:**  $\tau \sim \exp(\theta)$   
Time a customer is **willing to wait** for service.
- **Offered Wait:**  $V$   
Time a customer is **required to wait** for service.  
(= Waiting time of a customer with infinite patience.)
- If  $\tau \leq V$  then customer **abandons**, else **served**.
- **Actual Wait**  $W_q = \min(\tau, V)$ .

Patience data is **censored**: 2% abandoning implies 98% censored!

# Measuring Patience

## Hazard Rates of Patience in an Israeli Bank: Regular over VIP Customers



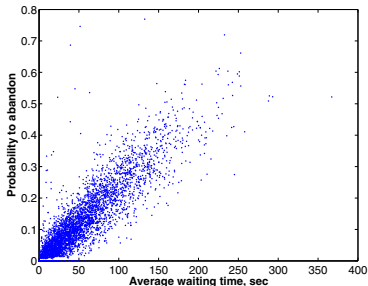
- **VIP** customers are **more patient** (needy).
- Why the peaks in abandonment? **Announcements!**
- **Call-by-call data** required to obtain this graph (+Uncensoring).

## Estimating Patience: $P\{Ab\} \propto E[W_q]$ Relation

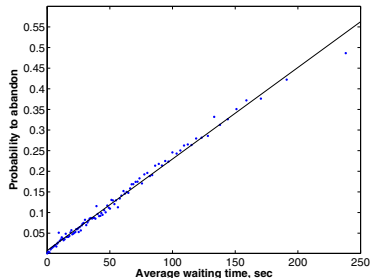
In queues with  $\exp(\theta)$  patience:  $P\{Ab\} = \theta \cdot E[W_q]$ .

### Israeli Bank: Yearly Data

Hourly Data



Aggregated

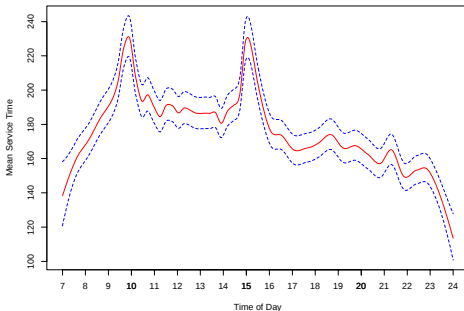


Graphs are based on 4158 hour intervals.

Estimate of mean patience:  $250/0.55 \approx 450$  seconds.

## Building block: Interrelation

### Average Service Time over the Day – Israeli Bank



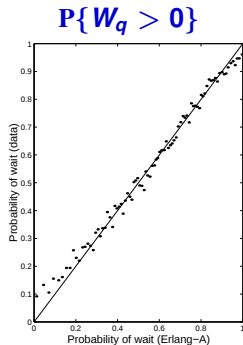
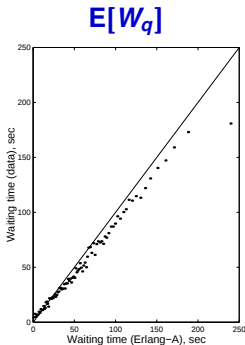
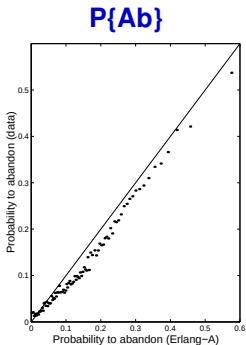
Prevalent: **Longest services** at **peak-loads** (10:00, 15:00). **Why?**

#### Explanations:

- Prevalent: Service protocol different (longer) at congestion
- Operational: The **needy** abandon less during peak loads; hence the **VIP** remain on line, with their **longer** service times.

# Erlang-A: Fitting a Simple Model to a Complex Reality

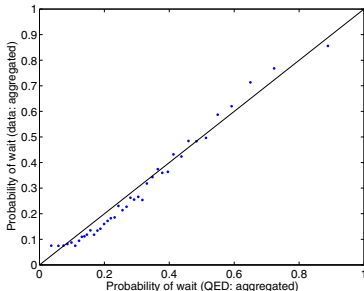
- Small Israeli bank (10 agents)
- Patience estimated via  $P\{Ab\} / E[W_q]$
- Graphs: **hourly performance vs. Erlang-A predictions**, over 1 year, aggregating groups with 40 similar hours.



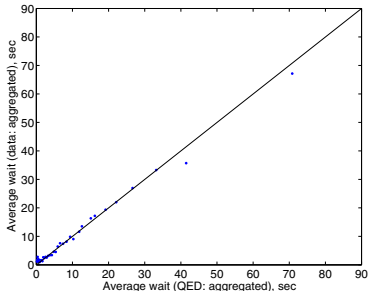
# Erlang-A: Fitting a Simple Model to a Complex Reality II

## Large U.S. Bank

Retail.  $P\{W_q > 0\}$



Telesales.  $E[W_q]$



Partial success, in some cases Erlang-A does not work well (Networking, SBR).

Large Israeli call center – underway.

## Erlang-A:

### Fitting a Simple Model to a Complex Reality III

We have learned:

- Arrival process can be approximated by **Poisson**
- Service times are **not exponential** (typically close to lognormal)
- Patience times are **not exponential** (various patterns are observed).

**Question:** why Erlang-A works with non-exponential patience and service times?

**Answer:** via study of **operational regimes** in call centers.

# Operational Regimes: Motivating Example

## Health Insurance. ACD Report.

| Time         | Calls         | Answered      | Abandoned%  | ASA       | AHT        | Occ%          | # of agents  |
|--------------|---------------|---------------|-------------|-----------|------------|---------------|--------------|
| <b>Total</b> | <b>20,577</b> | <b>19,860</b> | <b>3.5%</b> | <b>30</b> | <b>307</b> | <b>95.1%</b>  |              |
| 8:00         | 332           | 308           | 7.2%        | 27        | 302        | 87.1%         | 59.3         |
| 8:30         | 653           | 615           | 5.8%        | 58        | 293        | 96.1%         | 104.1        |
| 9:00         | 866           | 796           | 8.1%        | 63        | 308        | 97.1%         | 140.4        |
| 9:30         | 1,152         | 1,138         | 1.2%        | 28        | 303        | 90.8%         | 211.1        |
| 10:00        | 1,330         | 1,286         | 3.3%        | 22        | 307        | 98.4%         | 223.1        |
| 10:30        | 1,364         | 1,338         | 1.9%        | 33        | 296        | 99.0%         | 222.5        |
| 11:00        | 1,380         | 1,280         | 7.2%        | 34        | 306        | 98.2%         | 222.0        |
| 11:30        | 1,272         | 1,247         | 2.0%        | 44        | 298        | 94.6%         | 218.0        |
| 12:00        | 1,179         | 1,177         | 0.2%        | 1         | 306        | 91.6%         | 218.3        |
| 12:30        | 1,174         | 1,160         | 1.2%        | 10        | 302        | 95.5%         | 203.8        |
| 13:00        | 1,018         | 999           | 1.9%        | 9         | 314        | 95.4%         | 182.9        |
| <b>13:30</b> | <b>1,061</b>  | <b>961</b>    | <b>9.4%</b> | <b>67</b> | <b>306</b> | <b>100.0%</b> | <b>163.4</b> |
| 14:00        | 1,173         | 1,082         | 7.8%        | 78        | 313        | 99.5%         | 188.9        |
| <b>14:30</b> | <b>1,212</b>  | <b>1,179</b>  | <b>2.7%</b> | <b>23</b> | <b>304</b> | <b>96.6%</b>  | <b>206.1</b> |
| 15:00        | 1,137         | 1,122         | 1.3%        | 15        | 320        | 96.9%         | 205.8        |
| 15:30        | 1,169         | 1,137         | 2.7%        | 17        | 311        | 97.1%         | 202.2        |
| 16:00        | 1,107         | 1,059         | 4.3%        | 46        | 315        | 99.2%         | 187.1        |
| 16:30        | 914           | 892           | 2.4%        | 22        | 307        | 95.2%         | 160.0        |
| <b>17:00</b> | <b>615</b>    | <b>615</b>    | <b>0.0%</b> | <b>2</b>  | <b>328</b> | <b>83.0%</b>  | <b>135.0</b> |
| 17:30        | 420           | 420           | 0.0%        | 0         | 328        | 73.8%         | 103.5        |
| 18:00        | 49            | 49            | 0.0%        | 14        | 180        | 84.2%         | 5.8          |

## Efficiency-Driven (ED) Regime

| Time  | Calls | Answered | Abandoned% | ASA | AHT | Occ%   | # of agents |
|-------|-------|----------|------------|-----|-----|--------|-------------|
| 13:30 | 1,061 | 961      | 9.4%       | 67  | 306 | 100.0% | 163.4       |

- 100% occupancy
- High  $P\{Ab\}$
- Considerable waiting time
- $P\{W_q > 0\} \approx 1$ .

**Offered load:**

$$R_{ED} \triangleq \frac{\lambda}{\mu} = 1061 : \frac{1800}{306} = 180.37.$$

**Characterization:**

$$n = R_{ED} \cdot (1 - \gamma), \quad \gamma > 0.$$

Service grade

$$\gamma = 1 - \frac{n}{R_{ED}} = 1 - \frac{163.4}{180.37} = 0.094 \approx P\{Ab\}.$$

ED regime captured by **Fluid-Model**.

## Quality-Driven (QD) Regime

| Time  | Calls | Answered | Abandoned% | ASA | AHT | Occ%  | # of agents |
|-------|-------|----------|------------|-----|-----|-------|-------------|
| 17:00 | 615   | 615      | 0.0%       | 2   | 328 | 83.0% | 135.0       |

- Occupancy far below 100%
- Negligible  $P\{Ab\}$
- Very short waiting time
- $P\{W_q > 0\} \approx 0$ .

### Offered load:

$$R_{QD} = \frac{\lambda}{\mu} = 615 : \frac{1800}{328} = 112.07.$$

### Characterization:

$$n = R_{QD} \cdot (1 + \gamma), \quad \gamma > 0.$$

### Service grade

$$\gamma = \frac{n}{R_{QD}} - 1 = \frac{135}{112.07} - 1 = 0.205.$$

## Quality and Efficiency-Driven (QED) Regime

| Time  | Calls | Answered | Abandoned% | ASA | AHT | Occ%  | # of agents |
|-------|-------|----------|------------|-----|-----|-------|-------------|
| 14:30 | 1,212 | 1,179    | 2.7%       | 23  | 304 | 96.6% | 206.1       |

- High occupancy, but not 100%
- Small  $P\{Ab\}$  and waiting
- $P\{W_q > 0\} \approx \alpha$ ,  $0 < \alpha < 1$ .

**Offered load:**  $R_{QED} = \frac{\lambda}{\mu} = 1212 : \frac{1800}{304} = 204.69$ .

**Characterization:**  $n = R_{QED} + \beta \sqrt{R_{QED}}$ ,  $-\infty < \beta < \infty$ .

Service grade

$$\beta = \frac{n - R_{QED}}{\sqrt{R_{QED}}} = \frac{206.1 - 204.69}{\sqrt{204.69}} = 0.10.$$

**Square-Root Staffing Rule:** Described by Erlang in 1924!

Awaited the seminal formulation of **Halfin-Whitt** in 1981.

## Operational Regimes: Performance.

Assume that **offered load**  $R$  is not small ( $\lambda \rightarrow \infty$ ).

**ED regime:**  $n \approx R - \delta R$ ,  $0.1 \leq \delta \leq 0.25$ .

- Essentially **all** customers are delayed
- %Abandoned  $\approx \delta$  (10-25%)
- Average wait  $\approx 30$  seconds - 2 minutes.

**QD regime:**  $n \approx R + \gamma R$ ,  $0.1 \leq \gamma \leq 0.25$ .

Essentially **no** delays.

**QED regime:**  $n \approx R + \beta\sqrt{R}$ ,  $-1 \leq \beta \leq 1$ .

- %Delayed between 25% and 75%
- %Abandoned is 1-5%
- Average wait is one-order less than average service time (seconds vs. minutes).

## Erlang-A Queue: QED Approximations

Assume that **offered load**  $R$  is not small ( $\lambda \rightarrow \infty$ ).

Let  $\hat{\beta} = \beta \sqrt{\frac{\mu}{\theta}}$ ,  $h(\cdot) = \frac{\phi(\cdot)}{1 - \Phi(\cdot)}$  = hazard rate of  $\mathcal{N}(0, 1)$ .

- **Delay probability:**

$$\mathbb{P}\{W_q > 0\} \sim \left[ 1 + \sqrt{\frac{\theta}{\mu}} \cdot \frac{h(\hat{\beta})}{h(-\beta)} \right]^{-1},$$

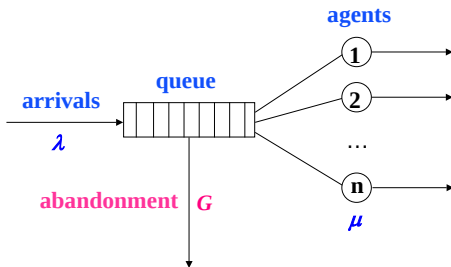
- **Probability to abandon:**

$$\mathbb{P}\{\text{Ab} | W_q > 0\} = \frac{1}{\sqrt{n}} \cdot \sqrt{\frac{\theta}{\mu}} \cdot [h(\hat{\beta}) - \hat{\beta}] + o\left(\frac{1}{\sqrt{n}}\right).$$

- **Linear relation between  $\mathbb{P}\{\text{Ab}\}$  and  $\mathbb{E}[W_q]$ :**

$$\frac{\mathbb{P}\{\text{Ab}\}}{\mathbb{E}[W_q]} = \theta.$$

## Generally Distributed Patience: M/M/n+G Model



Back to puzzle of “**Why Erlang-A works?**”

Assume patience times are **generally distributed**.

Density of patience time:  $g = \{g(x), x \geq 0\}$ , where  $g(0) \triangleq g_0 > 0$ .

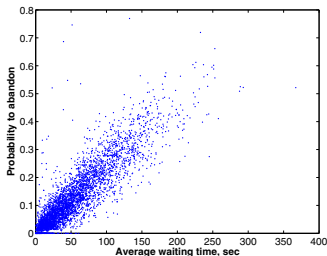
**QED regime:**  $n \approx R + \beta\sqrt{R}$ .

**QED approximations:** use Erlang-A, with  $g_0$  replacing  $\theta$ .

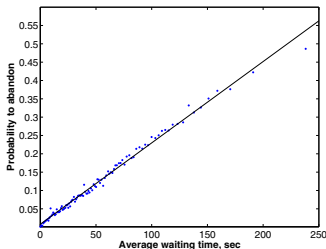
# Generally Distributed Patience: Fitting Erlang-A

## Israeli Bank: Yearly Data

Hourly Data



Aggregated



## Theory

**Erlang-A:**  $P\{Ab\} = \theta \cdot E[W_q]$ .

**M/M/n+G:**  $P\{Ab\} \approx g_0 \cdot E[W_q]$ .

## Recipe:

In both cases, use Erlang-A, with  $\hat{\theta} = \widehat{P\{Ab\}} / \widehat{E[W_q]}$  (slope above).

## Generally Distributed Service Times

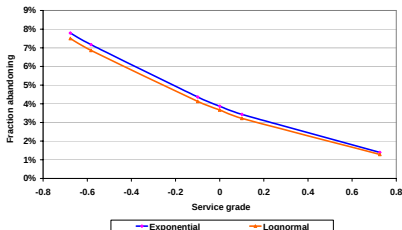
Established:  $M/M/n+M \approx M/M/n+G$  ( $\theta = g_0$ ).

**Next:**  $M/M/n+G \approx M/G/n+G$  (same mean service).

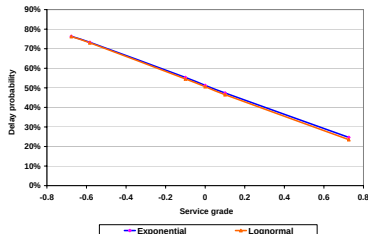
**Numerical Experiments:** Whitt (2004), Rosenshmidt (2006) demonstrate a **good fit** for typical call-center parameters.

Lognormal (CV=1) vs. Exponential Service Times, QED regime;  
100 agents, mean patience = mean service

Fraction Abandoning



Delay Probability



## Simple Model for Complex Realities: More on Applications of the QED Regime

- **Simple performance approximations** that are **robust** also in the ED and QD regimes: Mandelbaum, Zeltyn.
- Optimal staffing for **cost/revenue optimization problems** (staffing, abandonment and waiting costs): Borst, Mandelbaum, Reiman, Zeltyn.
- **General service times**: Puhalskii, Reiman; Jelencović, Mandelbaum, Momčilović.
- Generalizations to **time-varying queues**: Jennings, Feldman, Mandelbaum, Massey, Whitt.
- Generalizations to system with non-homogeneous customers and/or servers (**Skills-Based Routing**): Armony, Gurvich, Mandelbaum; Atar, Shaikhet.
- **Load-balancing**: Dai, Tezcan.

## The QED Regime and Stochastic-Ignorant Staffing: The Right Answer for the Wrong Reasons

If  $\beta = 0$ , QED staffing prescribes:

$$n = R,$$

$R$  = offered load (minutes of work that arrive per minute).

In word: **Assign number of agents that equals offered load**, which is common practice.

**No abandonment:** queue “explodes”.

**With abandonment**,  $n = 400$ , reasonable (im)patience:

- %Delayed  $\approx 50\%$
- %Abandoned  $\approx 2\%$
- $E[W_q] \approx 2\% \cdot E[S]$ , few seconds.

Very good service level.

# Call Centers: Hierarchical Operational View

Forecasting Customers (Statistics), Agents (HRM)

**Staffing: Queueing Theory** (Erlang-A based)

Service Level, Costs

# FTE's (Seats)  
per unit of time

**Shifts:** IP, Combinatorial Optimization; LP

Union constraints, Costs

Shift structure

**Rostering:** Heuristics, AI (Complex)

Individual constraints

Agents Assignments

**Skills-based Routing:** Stochastic Control (of Q's)

# DataMOCCA = Data MOdel for Call Center Analysis

**Project Goal:** Designing and Implementing a (universal) data-base/data-repository and interface for storing, retrieving, analyzing and displaying **Call-by-Call-Data**.

## System Components:

- Clean **Databases**: operational-data of individual calls, agents and operations.
- Friendly yet powerful **Online Interface**: enables convenient fast access to (mostly) operational and (some) administrative data (but no marketing/business data).

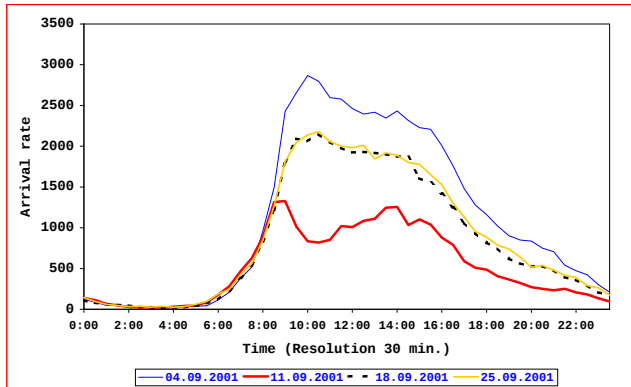
## Current Databases:

- Medium-sized U.S. Bank (**2.5 years; 220M calls, 40M via agents; 800 agents at peaks**) – Completed.
- Israeli Cell-Phone Company (**2 years; 110M calls, 25M via agents; 400 agents at peaks**) – Ongoing.
- Large Israeli Bank – Pilot.

DataMOCCA will now be used to illustrate the hierarchy of operational decisions, from forecasting to SBR.

## Arrivals to Service: Predictable vs. Random

### Arrival Rates on Tuesdays in a September – U.S. Bank



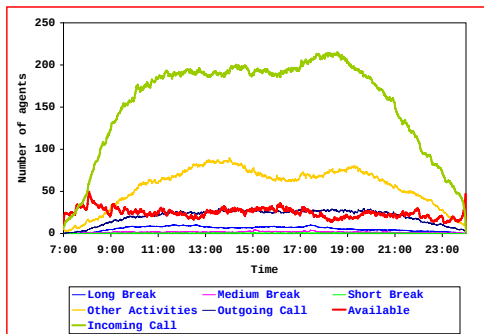
- **Tuesday**, September 4th: **Heavy**, following Labor Day
- **Tuesdays**, September 18 & 25: **Normal**
- **Tuesday, September 11th, 2001.**

# Agent Status

**Erlang-A Model**  $\Rightarrow$  optimal **Staffing Level**  $n$ .

$n$  = overall number-of-agents that come to work? **No!**

## Israeli Bank, Agent Status: Monthly Averages



Staffing Level (FTE) = Busy with “Incoming Calls” + “Available” for service.

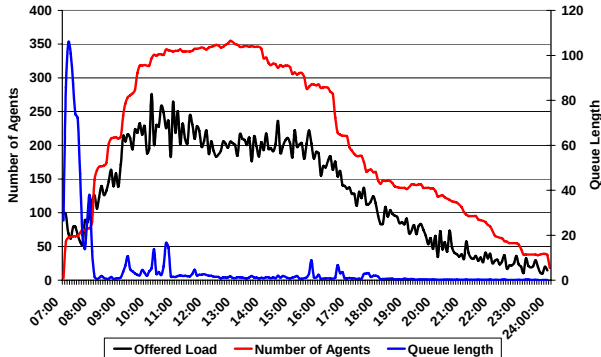
# Shifts Scheduling

**Integer Programming** given interval-based Staffing Levels.

Example of a shift scheduling problem:

should we bring agents early, given a predictable arrival peak?

## U.S. Bank: Queue-length and Staffing on May 3, 2002

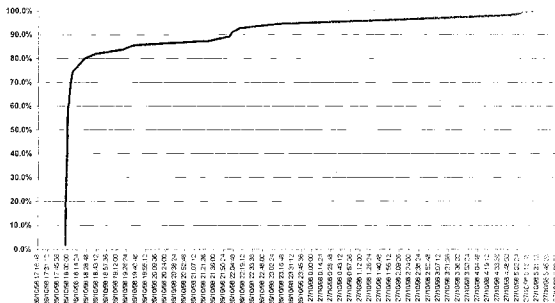


# Rostering

Assigning **individual agents** to **schedules**.

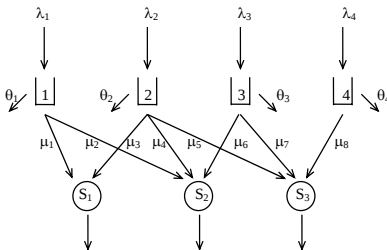
Typically, heuristics used to accommodate individual constraints.

## Israeli Technical Support Call Center: Online Shift Bidding



# Introduction to Skills Based Routing

## General Setup



## Major Control Decisions

- **Customer Routing:** If an agent turns idle and there are queued customers, which customer (if any) should be routed to this agent.
- **Agent Scheduling:** If a customer arrives and there are idle agents, which agent (if any) should serve this customer.
- **Load Balancing:** Routing of customers to distributed call centers (eg. nation-wide).

# Customers Relationship/Revenue Management (CRM)

## NationsBank CRM relationship groups:

- RG1: high-value customers
- RG2: marginally profitable customers (with potential)
- RG3: unprofitable customer.

## NationsBank's Design of the Service Encounter

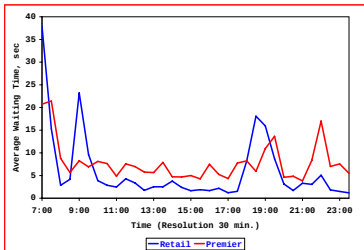
### Examples of Specifications: Assignable Grade Of Service

|                      | RG1                    | RG2                    | RG3                    |
|----------------------|------------------------|------------------------|------------------------|
| VRU Target           | 70% of calls           | 85% of calls           | 90% of calls           |
| Abandonment rate     | < 1%                   | < 5%                   | < 9%                   |
| Speed of Answer      | 100% in 2 rings        | 80% in 20 seconds      | 50% in 20 seconds      |
| Average Talk Time    | no limit               | 4 min. average         | 2 min. average         |
| Rep. Training        | universal              | product experts        | basic product          |
| Rep. Personalization | request rep / callback | FCFS                   | FCFS                   |
| Trans. Confirmation  | call / fax             | call / mail            | mail                   |
| Problem Resolution   | during call            | within 2 business days | within 8 business days |

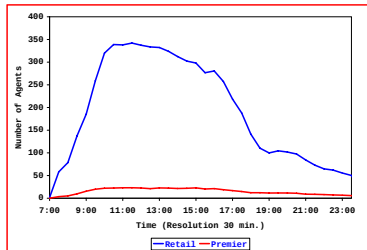
# Priorities and Economies-of-Scale

## U.S. Bank. Regular vs. VIP Customers. December 2002

Average Wait



Staffing Level



**Premier** customers do not get a better service level.

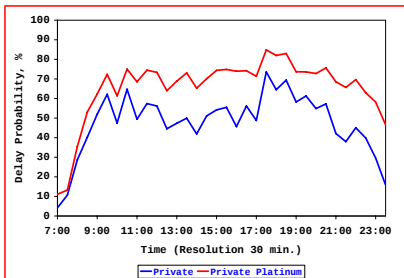
Number of agents assigned to Premier is small and they do not get enough help from regular agents.

**Challenge:** enable **better service level** for Premier and still serve most of them by a **small dedicated group of agents**.

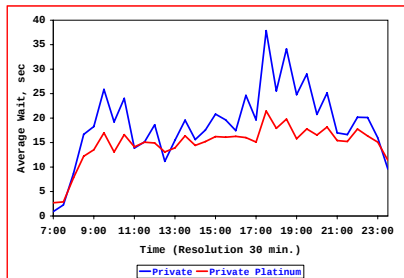
# Priorities and Routing Protocols I

Israeli Bank. Regular vs. VIP Customers. October 2004

Delay Probability



Average Wait

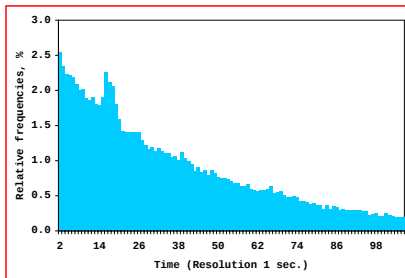


More **Platinum** customers have to wait, **but** their average wait is shorter. **How to explain?**

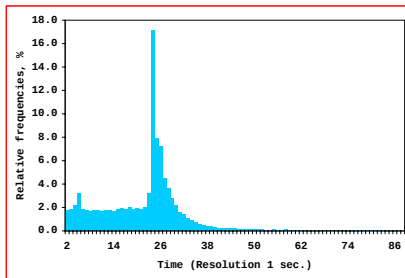
# Priorities and Routing Protocols II

## Histograms of Waiting Times. October 2004

Private



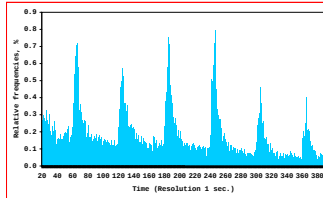
Private Platinum



After **25 seconds** of wait, Platinum are routed to Regular agents getting **high priority**. Hence, almost **no long waiting times** for Platinum.

# Dynamic Priority-Upgrade

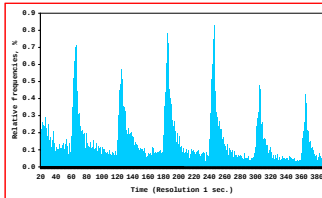
## Large Israeli Bank: Histogram of Waiting Times



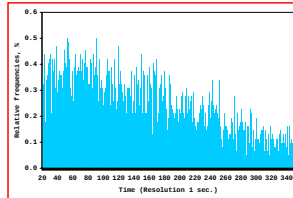
Peaks every 60 seconds. **Why?**

- System: **Priority-Upgrade** (unrevealed) every 60 seconds
- Human: **Voice-Announcement** every 60 seconds.

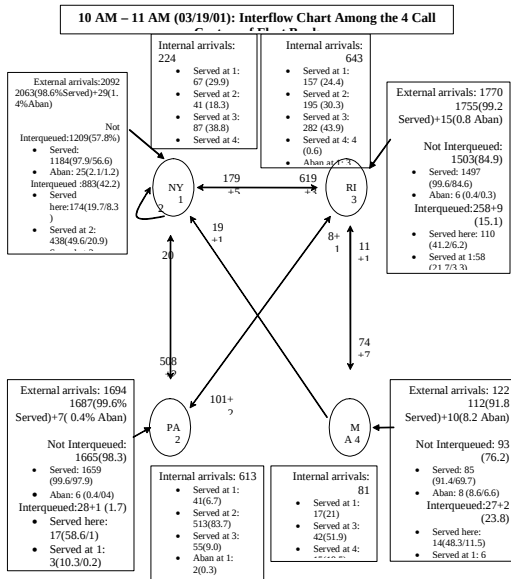
### Served Customers



### Abandoning Customers



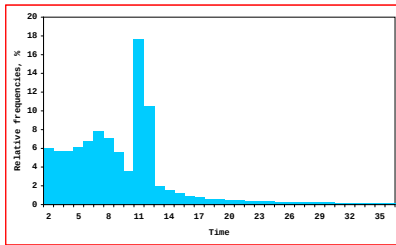
# Network Balancing via Interqueue in a U.S. Bank



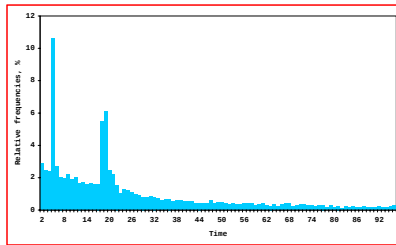
# Network Balancing Protocols and Performance Level

## U.S. Bank: Histograms of Waiting Times

Retail



Business



Why do we observe a peak for **Retail** service (**10 seconds**)?  
After 10 seconds of wait **Retail** customers were sent into the **interqueue**.

**Business** customers – peak at **5 seconds**, for the same reason.  
Second peak – unclear, maybe priority-upgrade.

## Main Research and Practical Challenges

- **Skills-Based Routing:** Convergence of Practice and Theory
- **Uncertainty:** in Reality, Model Parameters, Forecasting
- **Time-Varying Queues:** Time-Stable Performance
- **General Service-Times:** Theory
- **Economic Models:** Operations (Dimensioning), Marketing
- **Human Dimensions:**  
Measure, Model, Experiment, Validate, Refine, etc.

All of the above in a **Network** of distributed call centers.

See our **Service Engineering** site for downloads:

<http://iew3.technion.ac.il/serveng>