Data-Based Service Networks:

A Research Framework for

Asymptotic Inference, Analysis & Control of Service Systems

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http://ie.technion.ac.il/serveng

Kellogg Operations Workshop, September 2012

- **Overheads available at my Technion website**

Research Partners

- **Students**:
  Aldor*, Baron*, Carmeli*, Cohen*, Feldman*, Garnett*, Gurvich*, Khudiakov*, Maman*, Marmor*, Reich*, Rosenshmidt*, Shaikhet*, Senderovic, Tseytlin*, Yom-Tov*, Yuviler, Zaied*, Zeltyn*, Zychlinski*, Zohar*, Zviran*, . . .
- **Theory**:
  Armony, Atar, Cohen, Gurvich, Huang, Jelenkovic, Kaspi, Massey, Momcilovic, Reiman, Shimkin, Stolyar, Trofimov, Wasserkrug, Whitt, Zeltyn, . . .
- **Empirical/Statistical Analysis**:
  Brown, Gans, Shen, Zhao; Zeltyn; Ritov, Goldberg; Gurvich, Huang, Liberman; Armony, Marmor, Tseytlin, Yom-Tov; Nardi, Plonsky; Gorfine, Ghebali; Pang, . . .
- **Industry**:
  Mizrahi Bank (A. Cohen, U. Yonissi), Rambam Hospital (R. Beyar, S. Israelit, S. Tzafrir), IBM Research (OCR Project), Hapoalim Bank (G. Maklef, T. Shlasky), Pelephone Cellular, . . .
- **Technion SEE Center / Laboratory**:
  Feigin; Trofimov, Nadjharov, Gavako, Kutsy; Liberman, Koren, Plonsky, Senderovic; Research Assistants, . . .

Contents

- Service Networks: Call Centers, Hospitals, Websites, $\cdots$
- Redefine the paradigm of modeling/asymptotics via Data
- **ServNets: QNets, SimNets; FNets, DNets**
- **Ultimate Goal: Data-based creation and validation of ServNets, automatically in real-time**

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- Why be Optimistic? Pilot at the Technion SEELab
  - Lacking but Feasible: Dynamics, Durations, Protocols
  - Simple Models at the Service of Complex Realities
    - State-Space Collapse (Queues, Waiting Times)
    - Congestion Laws (LN, Little, ImPatience, Staffing)
    - Universal Approximations: Simplifying the Asymptotic Landscape
    - Stabilizing Time-Varying Performance (Offered-Load)
- Successes: Palm/Erlang-R (ED Feedback = FNet), Palm/Erlang-A (CC Abandonment = DNet)

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- Elsewhere: Process Mining (Petri Nets, BPM), Networks (Social, Biological, Complex, . . .), Simulation-based, . . .
- Scenic Route: Open Problems, New Directions, Uncharted Territories

Applying Queueing Asymptotics

There are by now numerous insightful asymptotic queueing models at our disposal, and many arise from, and create, deep beautiful theory:

Has it helped one approximate or simulate a service system more efficiently, estimate its parameter more accurately, teach it to our students more effectively, perhaps even manage the system better?

I am of the opinion that the answers to such questions have been too often negative, that positive answers must have theory and applications nurture each other, which is good, and my approach to make this good happen is by marrying theory with data.

Prevalent Asymptotic Approximations

System (Data)

Phenomenology

Accuracy

Models $\longleftrightarrow$ **Approximations**

QNets **SimNets**

Data–Based Prevalent Asymptotic Approximations Models

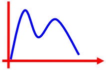
System (Data)

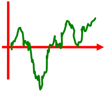
Phenomenology

Accuracy

Models ⟷ Approximations

QNets SimNets ⟷ FNet DNet Models





Data–Based Prevalent Asymptotic Approximations Models

System (Data)

Phenomenology

Value

Accuracy

Models ⟷ Approximations

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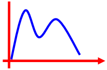

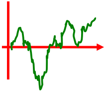

Data–Based Prevalent Asymptotic Approximations Models

System (Data)

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Models $\longleftrightarrow$ Approximations

QNets

SimNets $\longleftrightarrow$ FNet DNet Models

System = Coin Tossing, **Model** = Binomial ; de Moivre 1738

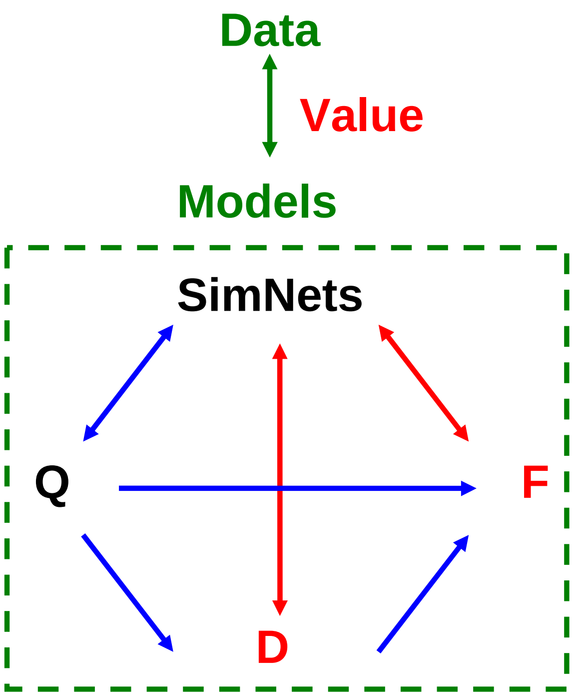
Approx. / Models: SLLN **(FNets)**, CLT **(DNets)** ; Laplace 1810

Value: Exceeds Value of originating stylized model

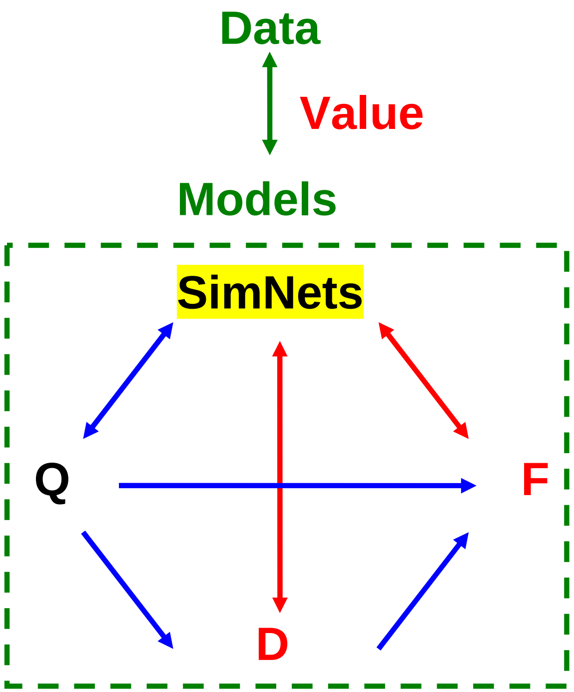
Normal, Brownian Motion ; Bachalier 1900

Poisson ; Poisson 1838

Data–Based Framework: (Almost) All Models Born Equal



Data–Based (Asymptotic) Framework: Simulation Mining

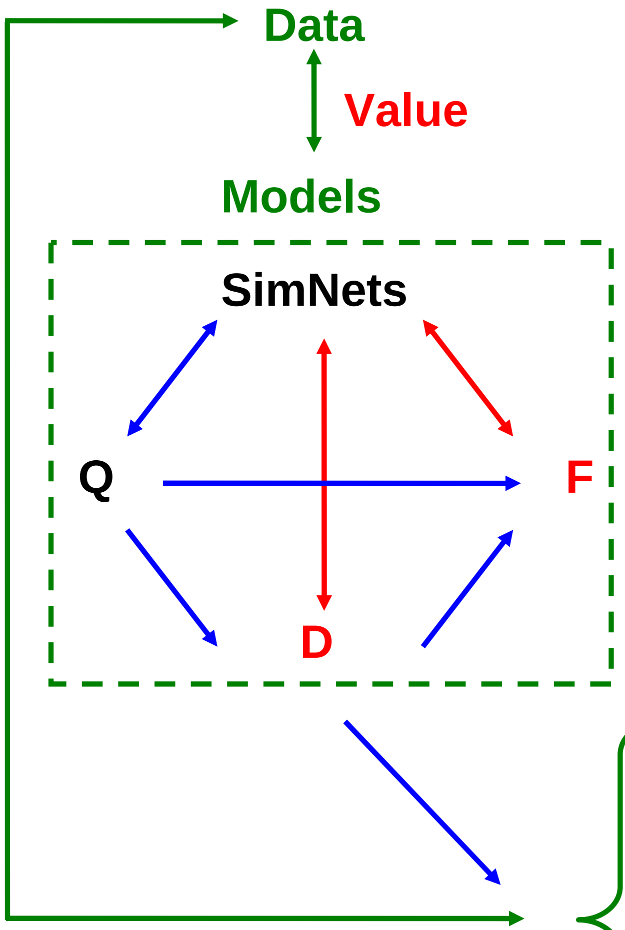


Data-Based
Real-Time

- Creation
- Application

- "Implies" other ServNets
- Virtual Realities
- Validation: Parsimony
- Theory: Reproducibility
- Application: Trust
- Data: Proprietary

Ultimately: Automatic "Discovery, Conformance, Enhancement"



Substitution Principle ⟷ **Congestion Laws**

- Inference: Patience, Predictors
- Performance: G/G/N, QED
- Control: Gcμ , SBR, FJ
- Design: $\sqrt{\ }$ - Staffing, Pooling
- Predictions

- State-Space Collapse
- Multiple Scales (Regimes, UA)
- Snapshots
- Pathwise Little, ASTA, …

Service Science

Scope of the Service Industry

Guangzhou Railway Station, Southern China

Call Centers, Then Hospitals, Now Internet

Call Centers - U.S. Stat.

- \$200 – \$300 billion annual expenditures
- 100,000 – 200,000 call centers
- **“Window"** into the company, for better or worse
- Over 3 million agents = **2% – 4% workforce**

Healthcare - similar, plus unique challenges:

- Cost-figures far more staggering
- Risks much higher
- ED (initial focus) = **hospital-window**
- Over 3 million nurses

Internet - . . .

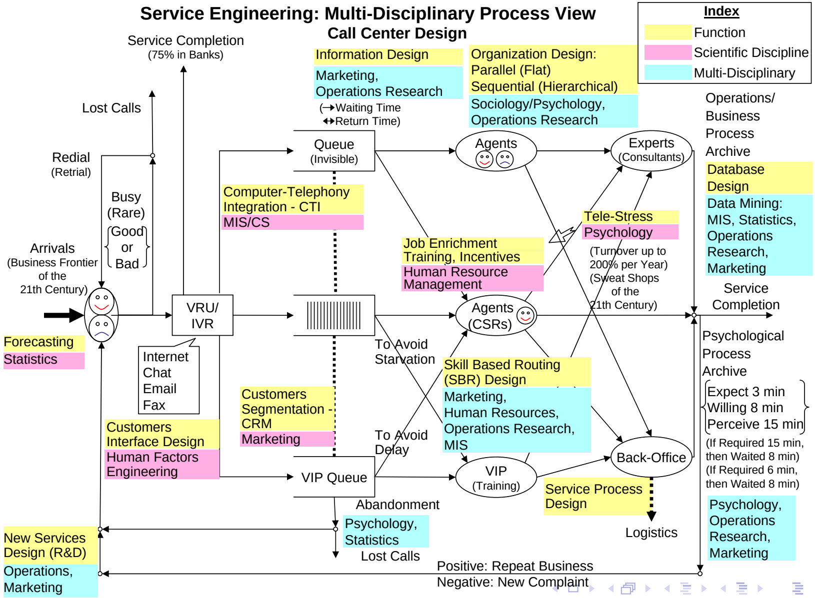
Call-Center Environment: Service Network

Operational Focus

Operational Measures:

- Surrogates for overall performance: Financial, Psychological; Clinical
- Easiest to quantify, measure, track online, react upon / Research

Call-Center Network: Gallery of Models



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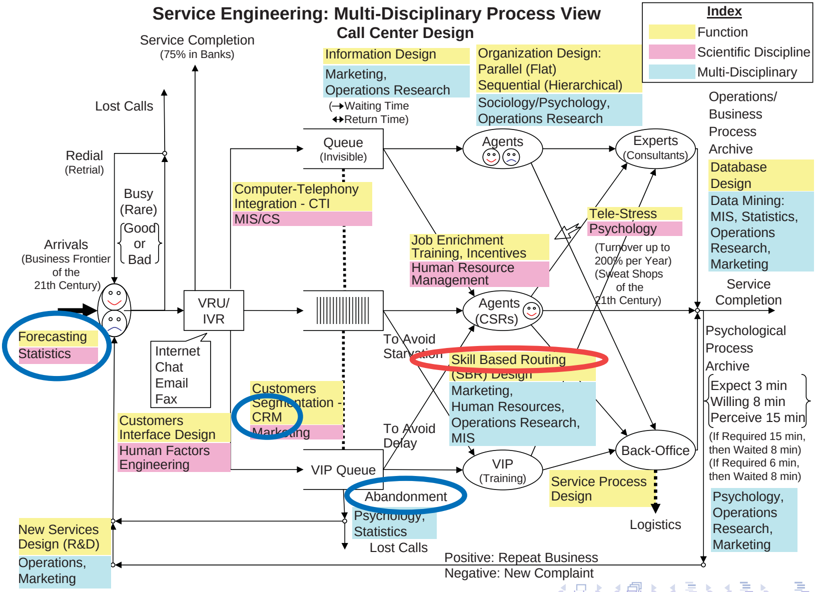


Skills-Based Routing in Call Centers

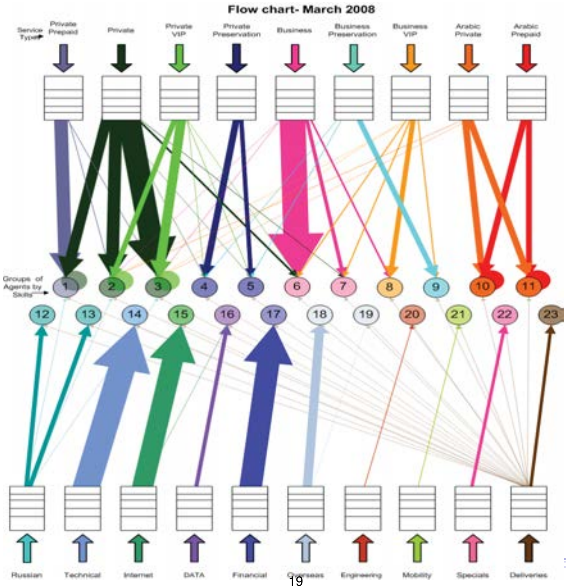
EDA and OR, with **I. Gurvich and P. Liberman**

Mktg. ⇒

OR ⇒

HRM ⇒

MIS ⇒



ER / ED Environment: Service Network

Acute (Internal, Trauma)

Walking

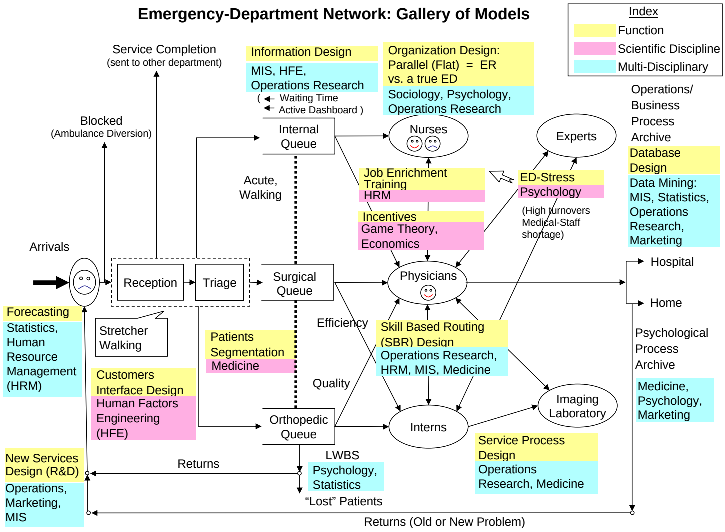
Multi-Trauma

ED-Environment in Israel

Queueing in a “Good" Hospital

Tong-ren Hospital at 6am, Beijing

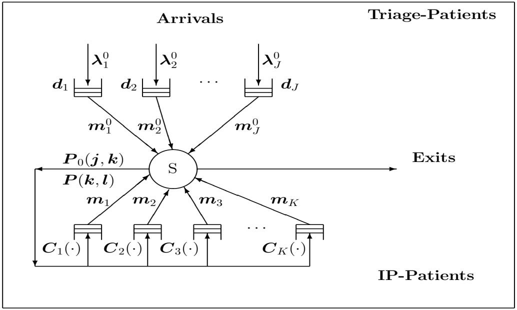
Emergency-Department Network: Gallery of Models



- **Forecasting**, Abandonment = **LWBS**, SBR ≈ **Flow Control**

ED Patient Flow: The Physicians View

with **J. Huang, B. Carmeli**



- **Goal**: Adhere to **Triage-Constraints**, then **release In-Process** Patients
- **Model** = Multi-class Q with Feedback: Min. convex **congestion costs** of IP-Patients, s.t. **deadline constraints** on Triage-Patients.

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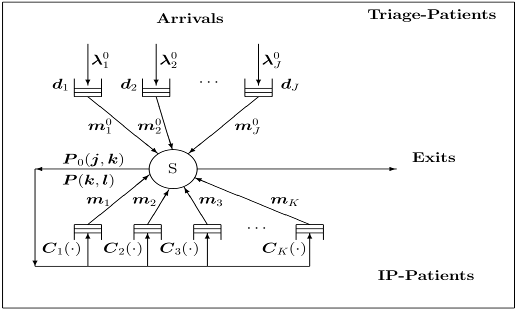
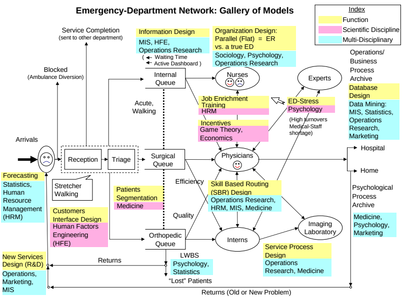


- **Goal**: Adhere to **Triage-Constraints**, then **release In-Process** Patients
- **Model** = Multi-class Q with Feedback: Min. convex **congestion costs** of IP-Patients, s.t. **deadline constraints** on Triage-Patients.
- **Solution**: In **conventional** heavy-traffic, **asymptotic least-cost** s.t. **asymptotic compliance** (as in Plambeck, Harrison, Kumar, who applied admission control):
  - Triage or IP? former, if some deadline is “too" close (least effort)
  - if Triage: Closest deadline (or Van Mieghem’s GLD)
  - if IP: Van Mieghem’s $Gc\mu$, modified for feedback

Emergency-Department Network: Flow Control



- ***Queueing-Science**, w/ **Armony, Marmor, Tseytlin, Yom-Tov**
- ***Fair ED-to-IW Routing** (Patients vs. Staff), w/ **Momcilovic, Tseytlin**
- ***Triage vs. InProcess/Release (Plambeck et al, van Mieghem)** in EDs, w/ **Carmeli, Huang; Shimkin**
- ***Staffing Time-Varying Q's with Re-Entrant Customers (de Vericourt & Jennings)**, w/ **Yom-Tov**
- **The Offered-Load in Fork-Join Nets (Adlakha & Kulkarni)**, w/ **Kaspi, Zaeid**
- **Synchronization Control** of Fork-Join Nets, w/ **Atar, Zviran**

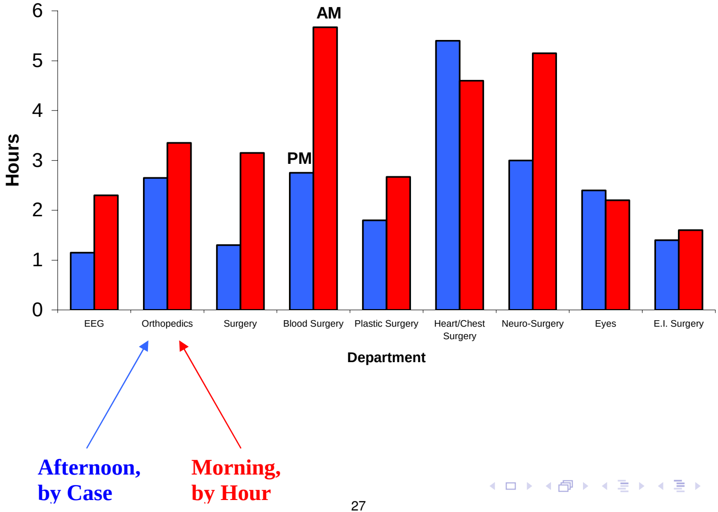
Prerequisite I: Data

Averages Prevalent (and could be useful / interesting).

But I need data at the level of the **Individual Transaction**:
For each service transaction (during a phone-service in a call center, or a patient's visit in a hospital, or browsing in a website, or . . .), its **operational history** = time-stamps of events (events-log files).

Interesting Averages: The Human Factor, or Even “Doctors" Can Manage

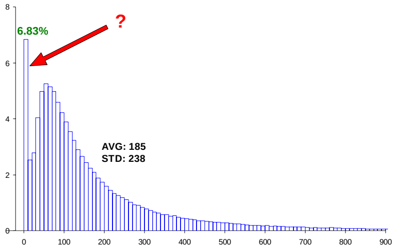
Operations Time - Morning (by Hour) vs. Afternoon (by Case):



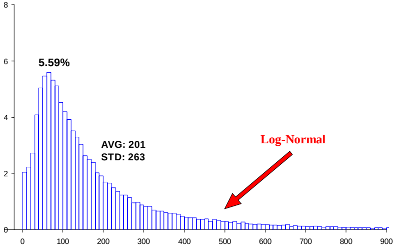
Beyond Averages: The Human Factor

Histogram of Service-Time in an Israeli Call Center, 1999

January-October



November-December



- 6.8% Short-Services:

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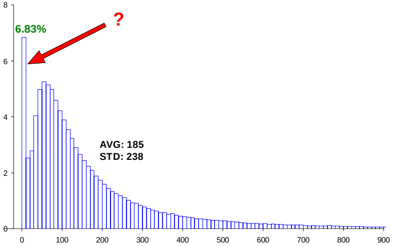

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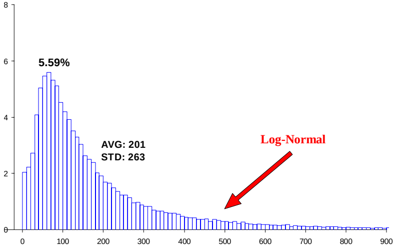
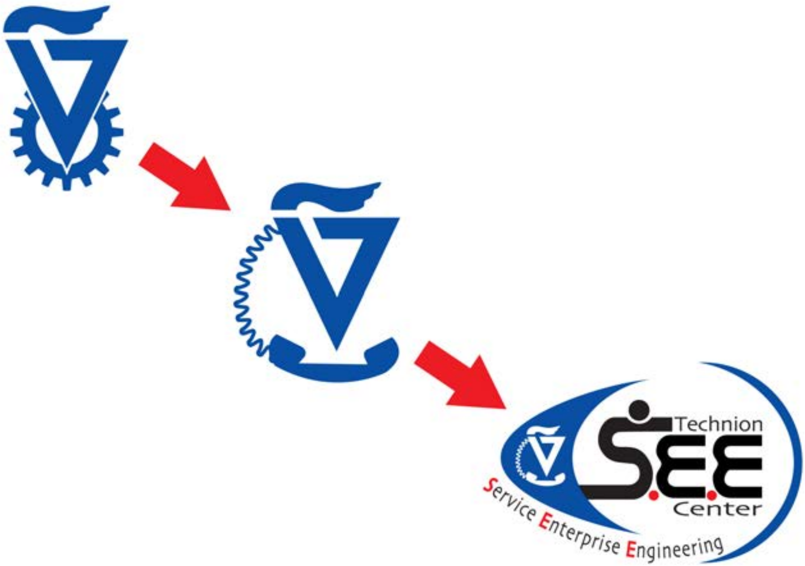


- **6.8% Short-Services:** Agents' "Abandon" (improve bonus, rest), (mis)lead by **incentives**
- **Distributions** must be measured (in **seconds** = **natural scale**)
- **LogNormal** service-durations (???, common, more later)

Pause for a Commercial:

Pause for a Commercial: The Technion SEE Center



Technion SEE = Service Enterprise Engineering

SEELab: Data-repositories for research and teaching

- For example:
  - Bank Anonymous: **1 year, 350K calls by 15 agents** - in 2000.
    Brown, Gans, Sakov, Shen, Zeltyn, Zhao (JASA),
    paved the way to:
  - U.S. Bank: **2.5 years, 220M calls, 40M by 1000 agents**
  - Israeli Cellular: **2.5 years, 110M calls, 25M calls by 750 agents**
  - Israeli Bank: **from January 2010, daily-deposit at a SEESafe**
  - Home (Rambam) Hospital: **4 years, 1000 beds**, ward-level flow
  - 5 EDs: **gathered by the late David Sinreich**, ED arrivals & LOS

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SEEStat: Environment for graphical **EDA** in real-time

- **Universal Design, Internet Access, Real-Time Response.**

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SEEServer: **Free for academic use**

- Register
- Access **U.S. Bank, Bank Anonymous, Home Hospital**

eg. RFID-Based Data: Mass Casualty Event (MCE)

Drill: Chemical MCE, Rambam Hospital, May 2010

Focus on **severely wounded** casualties ($\approx$ 40 in drill)

Note: 20 observers support real-time control (helps validation)

Data Cleaning: MCE with RFID Support

Data-base				Company report		comment
Asset id	order	**Entry** date	**Exit** date	Entry date	Exit date	
4	1	1:14:07 PM		1:14:00 PM		
6	1	12:02:02 PM	12:33:10 PM	12:02:00 PM	12:33:00 PM	
8	1	11:37:15 AM	12:40:17 PM	11:37:00 AM		**exit is missing**
10	1	12:23:32 PM	12:38:23 PM	12:23:00 PM		
12	1	12:12:47 PM	12:35:33 PM		12:35:00 PM	**entry is missing**
15	1	1:07:15 PM		1:07:00 PM		
16	1	11:18:19 AM	11:31:04 AM	11:18:00 AM	11:31:00 AM	
17	1	1:03:31 PM		1:03:00 PM		
18	1	1:07:54 PM		1:07:00 PM		
19	1	12:01:58 PM		12:01:00 PM		
20	1	11:37:21 AM	12:57:02 PM	11:37:00 AM	12:57:00 PM	
21	1	12:01:16 PM	12:37:16 PM	12:01:00 PM		
22	1	12:04:31 PM	12:20:40 PM			first customer is missing
22	2	12:27:37 PM		12:27:00 PM		
25	1	12:27:35 PM	1:07:28 PM	12:27:00 PM	1:07:00 PM	
27	1	12:06:53 PM		12:06:00 PM		
28	1	11:21:34 AM	11:41:06 AM	11:41:00 AM	11:53:00 AM	**exit time instead of entry time**
29	1	12:21:06 PM	12:54:29 PM	12:21:00 PM	12:54:00 PM	
31	1	11:40:54 AM	12:30:16 PM	11:40:00 AM	12:30:00 PM	
31	2	12:37:57 PM	12:54:51 PM	12:37:00 PM	12:54:00 PM	
32	1	11:27:11 AM	12:15:17 PM	11:27:00 AM	12:15:00 PM	
33	1	12:05:50 PM	12:13:12 PM	12:05:00 PM	12:15:00 PM	wrong exit time
35	1	11:31:48 AM	11:40:50 AM	11:31:00 AM	11:40:00 AM	
36	1	12:06:23 PM	12:29:30 PM	12:06:00 PM	12:29:00 PM	
37	1	11:31:50 AM	11:48:18 AM	11:31:00 AM	11:48:00 AM	
37	2	12:59:21 PM		12:59:00 PM		

- Imagine **“Cleaning" 60,000+ customers per day** (call centers) !
- “Psychology" of Data Trust and Transfer (e.g. 2 years till transfer)

Event-Logs in a Call Center (Bank Anonymous)

A Data Sample (Excel worksheet)

vru+line	call_id	customer_id	priority	type	date	vru_entry	vru_exit	vru_time	q_start	q_exit	q_time	outcome	ser_start	ser_exit	ser_time	server
AA0101	44749	27644400	2	PS	990901	11:45:33	11:45:39	6	11:45:39	11:46:58	79	AGENT	11:46:57	11:51:00	243	DORIT
AA0101	44750	12887816	1	PS	990905	14:49:00	14:49:06	6	14:49:06	14:53:00	234	AGENT	14:52:59	14:54:29	90	ROTH
AA0101	44967	58660291	2	PS	990905	14:58:42	14:58:48	6	14:58:48	15:02:31	223	AGENT	15:02:31	15:04:10	99	ROTH
AA0101	44968	0	0	NW	990905	15:10:17	15:10:26	9	15:10:26	15:13:19	173	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44969	63193346	2	PS	990905	15:22:07	15:22:13	6	15:22:13	15:23:21	68	AGENT	15:23:20	15:25:25	125	STEREN
AA0101	44970	0	0	NW	990905	15:31:33	15:31:47	14	00:00:00	00:00:00	0	AGENT	15:31:45	15:34:16	151	STEREN
AA0101	44971	41630443	2	PS	990905	15:37:29	15:37:34	5	15:37:34	15:38:20	46	AGENT	15:38:18	15:40:56	158	TOVA
AA0101	44972	64185333	2	PS	990905	15:44:32	15:44:37	5	15:44:37	15:47:57	200	AGENT	15:47:56	15:49:02	66	TOVA
AA0101	44973	3.06E+08	1	PS	990905	15:53:05	15:53:11	6	15:53:11	15:56:39	208	AGENT	15:56:38	15:56:47	9	MORIAH
AA0101	44974	74780917	2	NE	990905	15:59:34	15:59:40	6	15:59:40	16:02:33	173	AGENT	16:02:33	16:26:04	1411	ELI
AA0101	44975	55920755	2	PS	990905	16:07:46	16:07:51	5	16:07:51	16:08:01	10	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44976	0	0	NW	990905	16:11:38	16:11:48	10	16:11:48	16:11:50	2	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44977	33689787	2	PS	990905	16:14:27	16:14:33	6	16:14:33	16:14:54	21	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44978	23817067	2	PS	990905	16:19:11	16:19:17	6	16:19:17	16:19:39	22	AGENT	16:19:38	16:21:57	139	TOVA
AA0101	44764	0	0	PS	990901	15:03:26	15:03:36	10	00:00:00	00:00:00	0	AGENT	15:03:35	15:06:36	181	ZOHARI
AA0101	44765	25219700	2	PS	990901	15:14:46	15:14:51	5	15:14:51	15:15:10	19	AGENT	15:15:09	15:17:00	111	SHARON
AA0101	44766	0	0	PS	990901	15:25:48	15:26:00	12	00:00:00	00:00:00	0	AGENT	15:25:59	15:28:15	136	ANAT
AA0101	44767	58859752	2	PS	990901	15:34:57	15:35:03	6	15:35:03	15:35:14	11	AGENT	15:35:13	15:35:15	2	MORIAH
AA0101	44768	0	0	PS	990901	15:46:30	15:46:39	9	00:00:00	00:00:00	0	AGENT	15:46:38	15:51:51	313	ANAT
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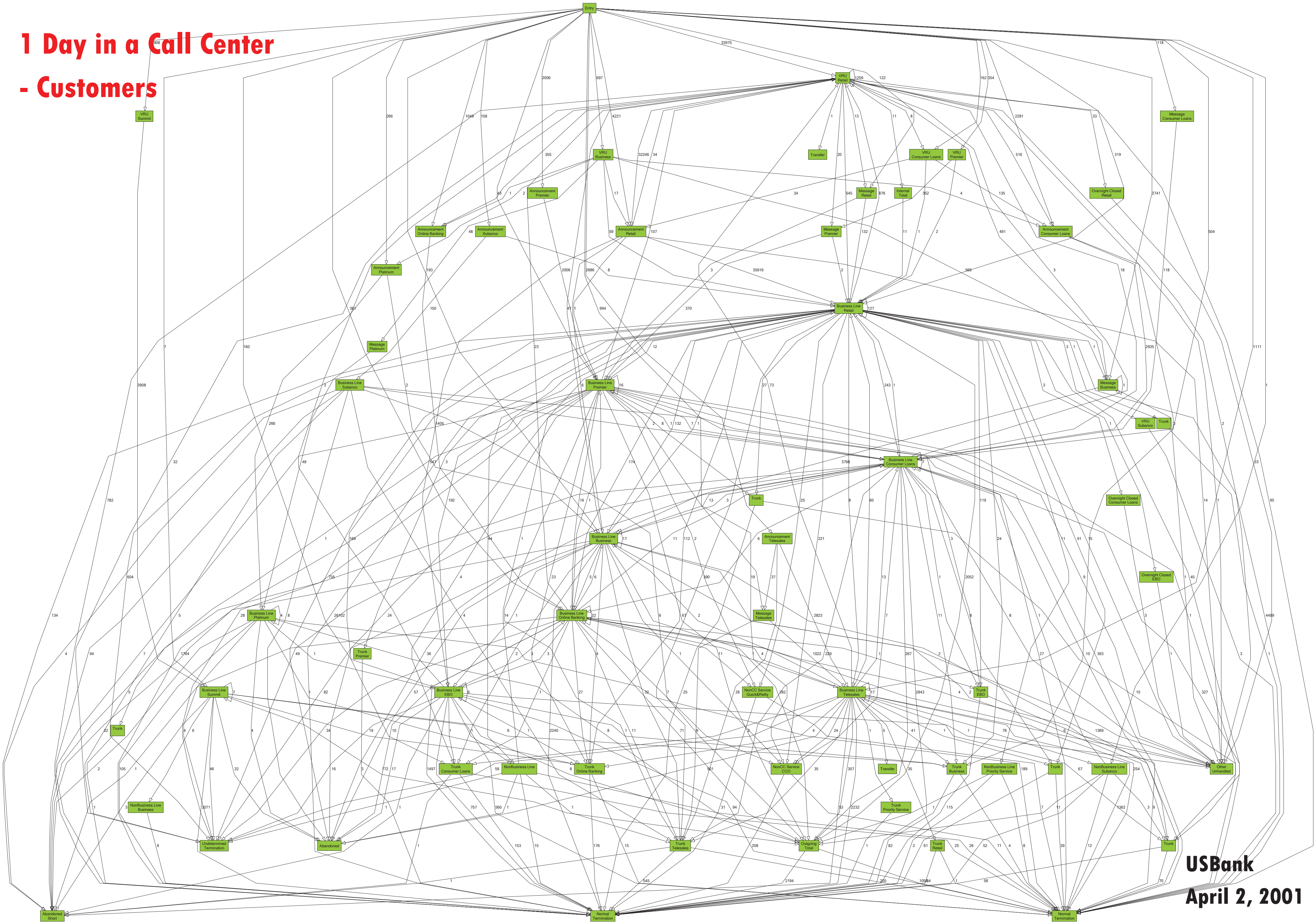
AA0101	44770	0	0	PS	990901	16:14:31	16:14:46	15	00:00:00	00:00:00	0	AGENT	16:14:44	16:16:02	78	BENSION
AA0101	44771	0	0	PS	990901	16:38:59	16:39:12	13	00:00:00	00:00:00	0	AGENT	16:39:11	16:43:35	264	VICKY
AA0101	44772	0	0	PS	990901	16:51:40	16:51:50	10	00:00:00	00:00:00	0	AGENT	16:51:49	16:53:52	123	ANAT
AA0101	44773	0	0	PS	990901	17:02:19	17:02:28	9	00:00:00	00:00:00	0	AGENT	17:02:28	17:07:42	314	VICKY
AA0101	44774	32387482	1	PS	990901	17:18:18	17:18:24	6	17:18:24	17:19:01	37	AGENT	17:19:00	17:19:35	35	VICKY
AA0101	44775	0	0	PS	990901	17:38:53	17:39:05	12	00:00:00	00:00:00	0	AGENT	17:39:04	17:40:43	99	TOVA

- Unsynchronized transition times,

1 Day in a Call Center - Customers

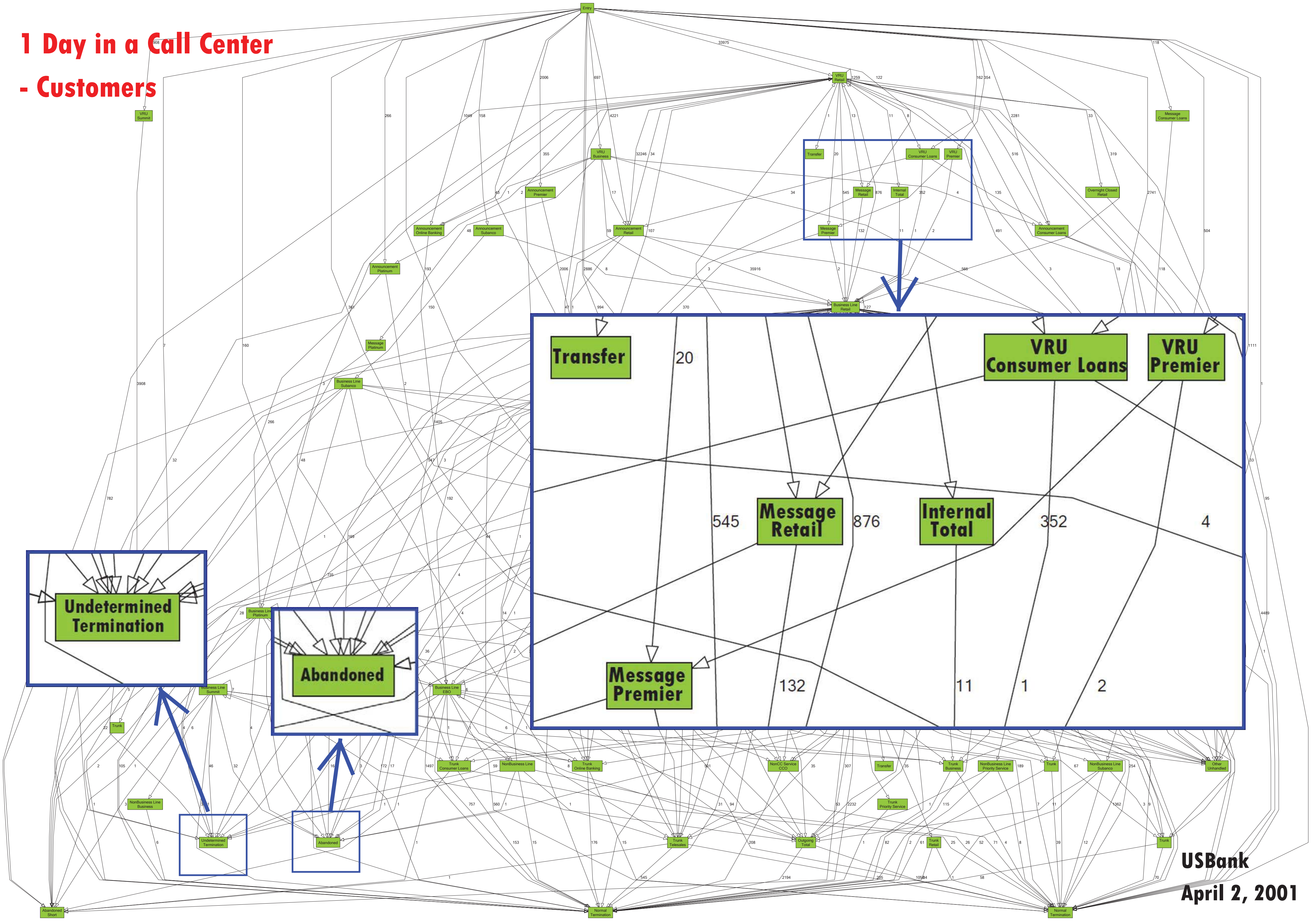


USBank

April 2, 2001

1 Day in a Call Center

- Customers

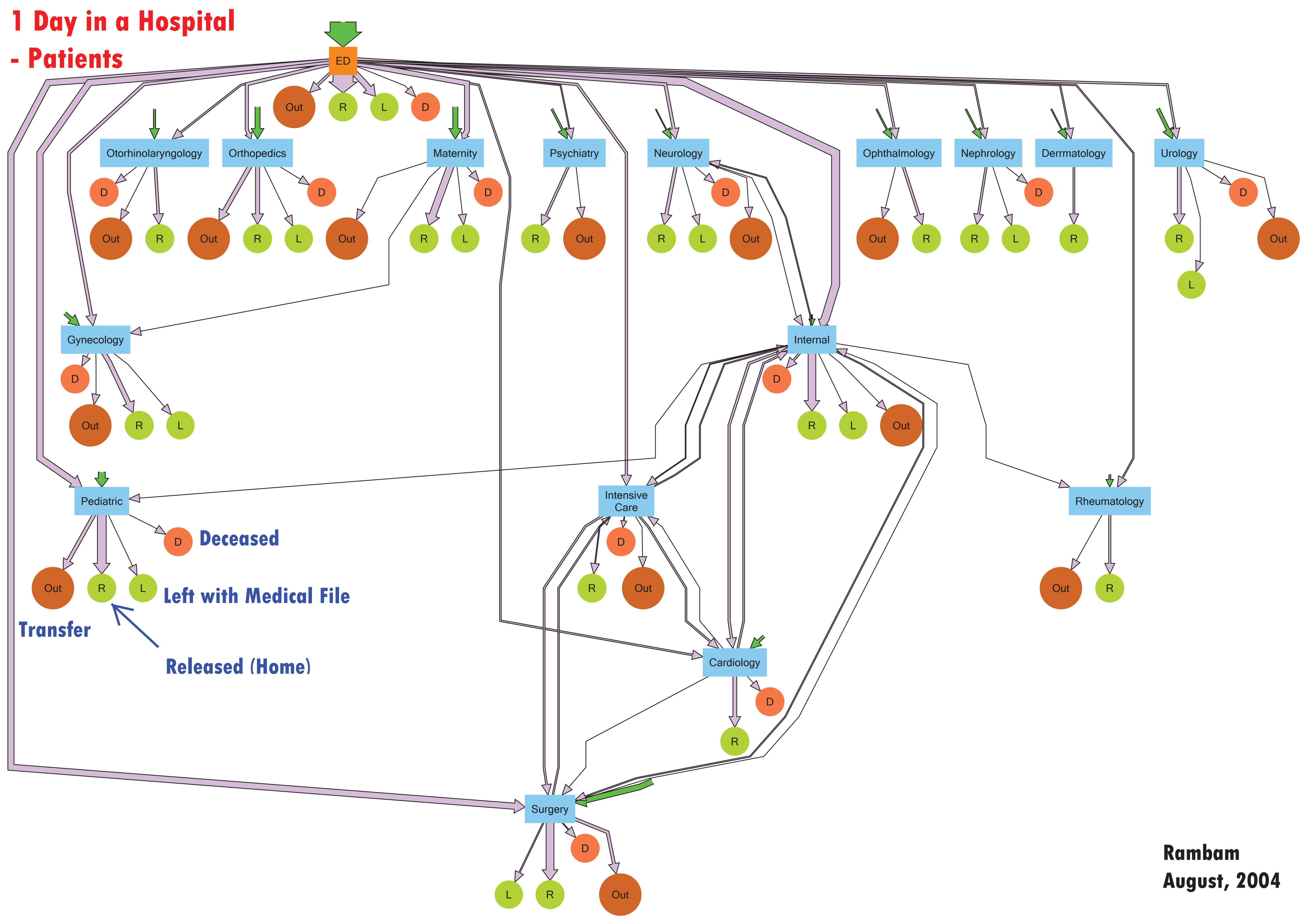


USBank

April 2, 2001

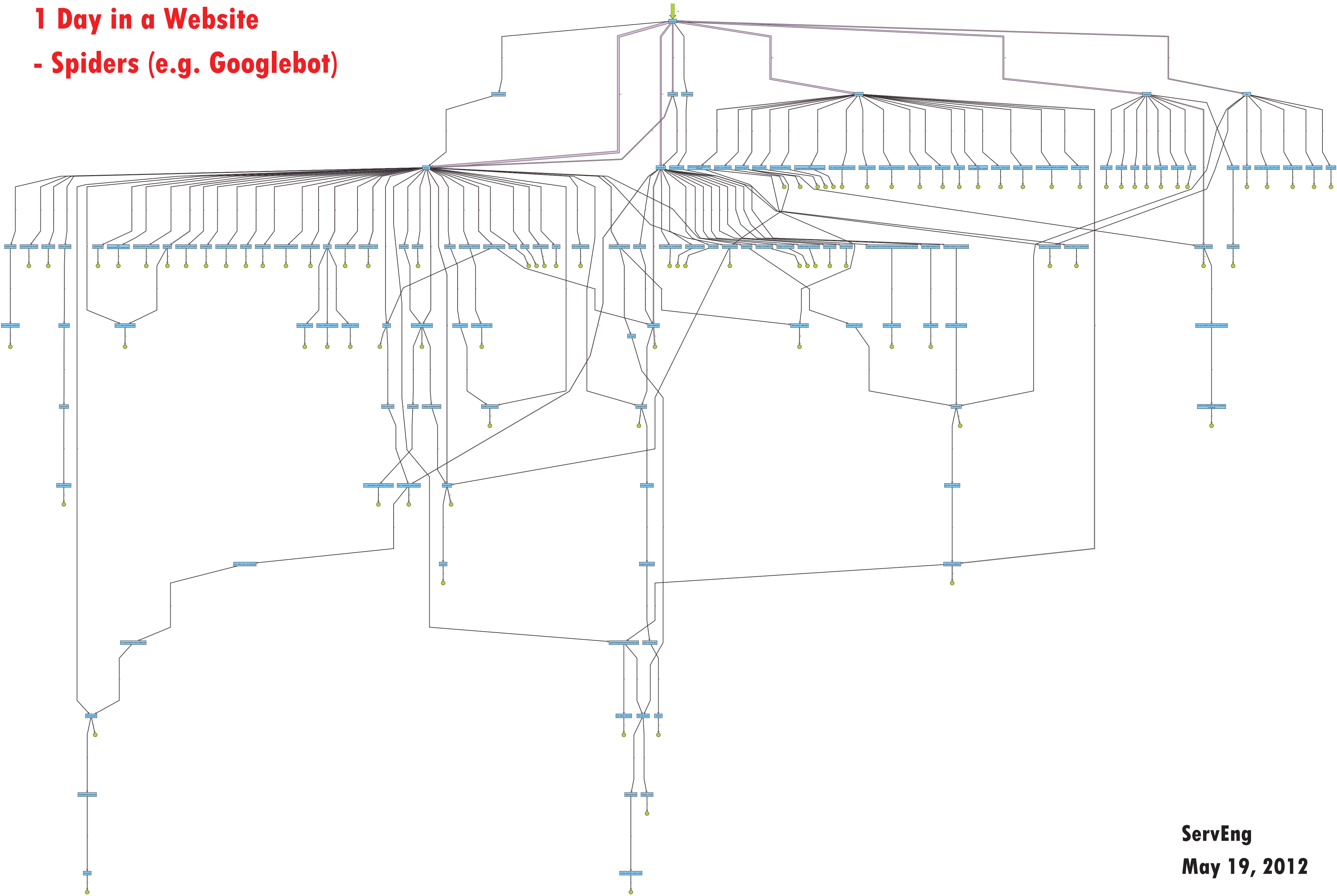

Server Network
- 1 Day
- All Agents
- Activities + Queues
Inbound
Available
Dialer
Idle
Call-Backs
Outgoing
Break
Shifting (209.71)
Sales (329.00)
Virtual Branches (195.46)
Pension (758.00)
General Banking (218.31)
Investments (252.68)
Wrap Up (8.71)
Securities (278.20)
Short Break (12.97)
Paperwork (140.62)
Logged Off
Consulting (562.54)
Sales (177.14)
Wrap Up (7.46)
Paperwork (140.57)
Consulting (208.00)
Medium Break (507.26
Available (7.36)
Managers VIP (227.05)
Paperwork (313.19)
Paperwork (140.97)
Consulting (175.42)
Private Call (27.11)
General Banking (136.89)
Wrap Up (8.11)
Logged Off
Logged In
Idle (185.11)
Virtual Branches (133.67)
Internet Support (114.49)
Managers VIP (149.36)
Message (209.00)
Long Break (1,384.28)
Internet Support (313.66)
General Banking (97.22)
Wrap Up (8.03)
Pension (307.14)
Internet Support (166.50)
Securities (532.00)
Investments (211.41)
ILDUBank
January 24, 2010
from 06:00 to 23:59:59


Server Network
- 1 Day
- All Agents
- Activities + Queues
Sales (329.00)
Virtual Branches (195.46)
Shifting (209.71)
1 General Banking (218)
Pension (758.00)
Inbound
General Banking (218.31)
Investments (252.68)
Securities (278.20)
2 Wrap Up (9)
Available
Consulting (562.54)
Short Break (12.97)
Sales (177.14)
Wrap Up (7.46)
Paperwork (140.57)
3 Paperwork (141)
Logged Off
Consulting (208.00)
Medium Break (507.26
0 Available (7)
Dialer
Managers VIP (227.05)
Paperwork (313.19)
Paperwork (140.97)
Consulting (175.42)
Logged Off
Logged In
Idle (185.11)
Private Call (27.11)
General Banking (136.89)
Wrap Up (8.11)
Virtual Branches (133.67)
Internet Support (114.49)
Managers VIP (149.36)
Message (209.00)
Long Break (1,384.28)
Idle
Call-Backs
Outgoing
Internet Support (313.66)
General Banking (97.22)
Wrap Up (8.03)
Break
Pension (307.14)
Internet Support (166.50)
Securities (532.00)
Investments (211.41)
ILDUBank
January 24, 2010
from 06:00 to 23:59:59


1 Day in a Hospital
- Patients
ED
Out
R
L
D
Otorhinolaryngology
Orthopedics
Maternity
Psychiatry
Neurology
Ophthalmology
Nephrology
Derrmatology
Urology
Gynecology
Internal
Pediatric
Intensive
Care
Rheumatology
Cardiology
Surgery
Deceased
Left with Medical File
Transfer
Released (Home)
Rambam
August, 2004

1 Day in a Website

- Spiders (e.g. Googlebot)



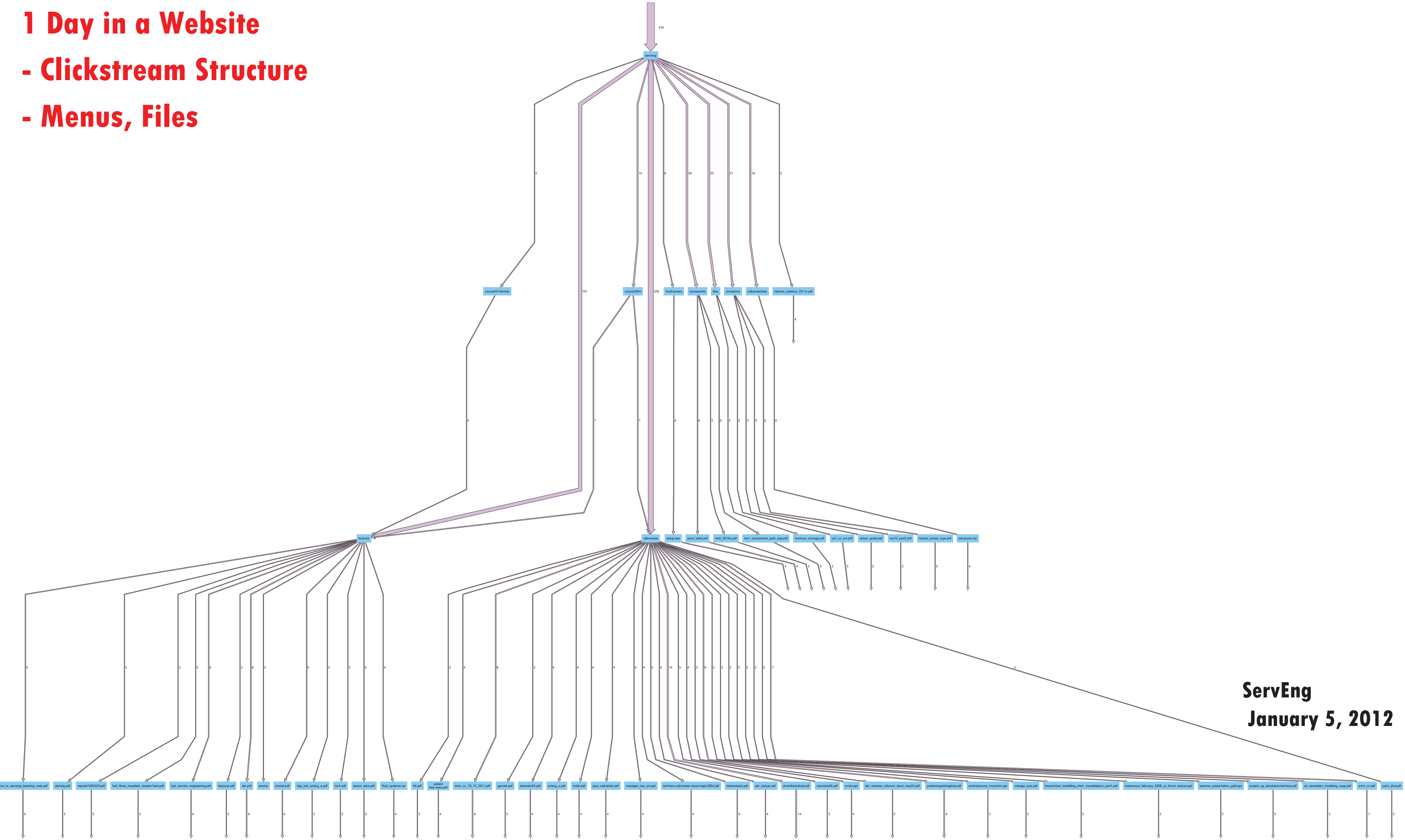
ServEng

May 19, 2012

1 Day in a Website

- Clickstream Structure

- Menus, Files



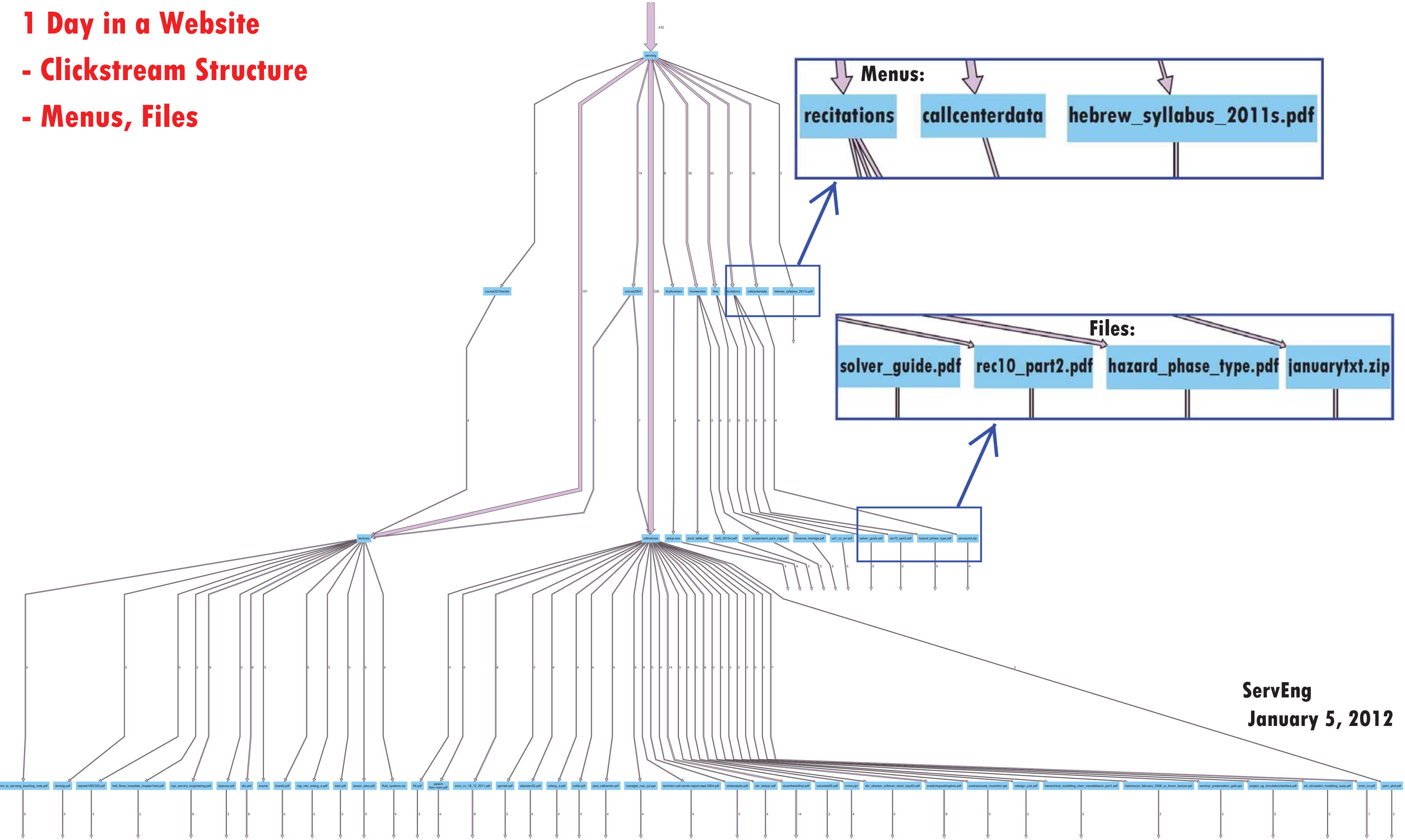
ServEng

January 5, 2012

1 Day in a Website

- Clickstream Structure

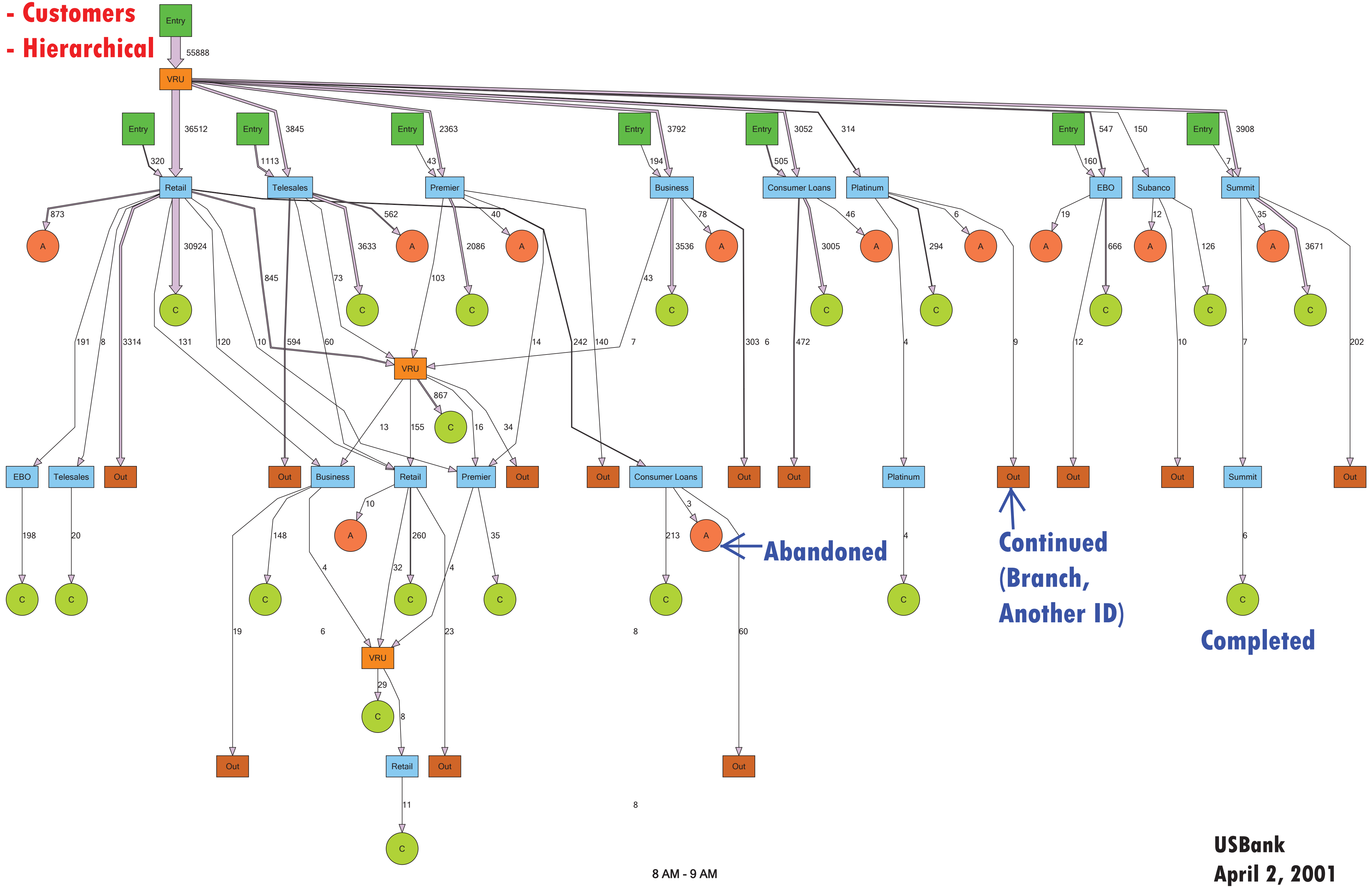
- Menus, Files



ServEng

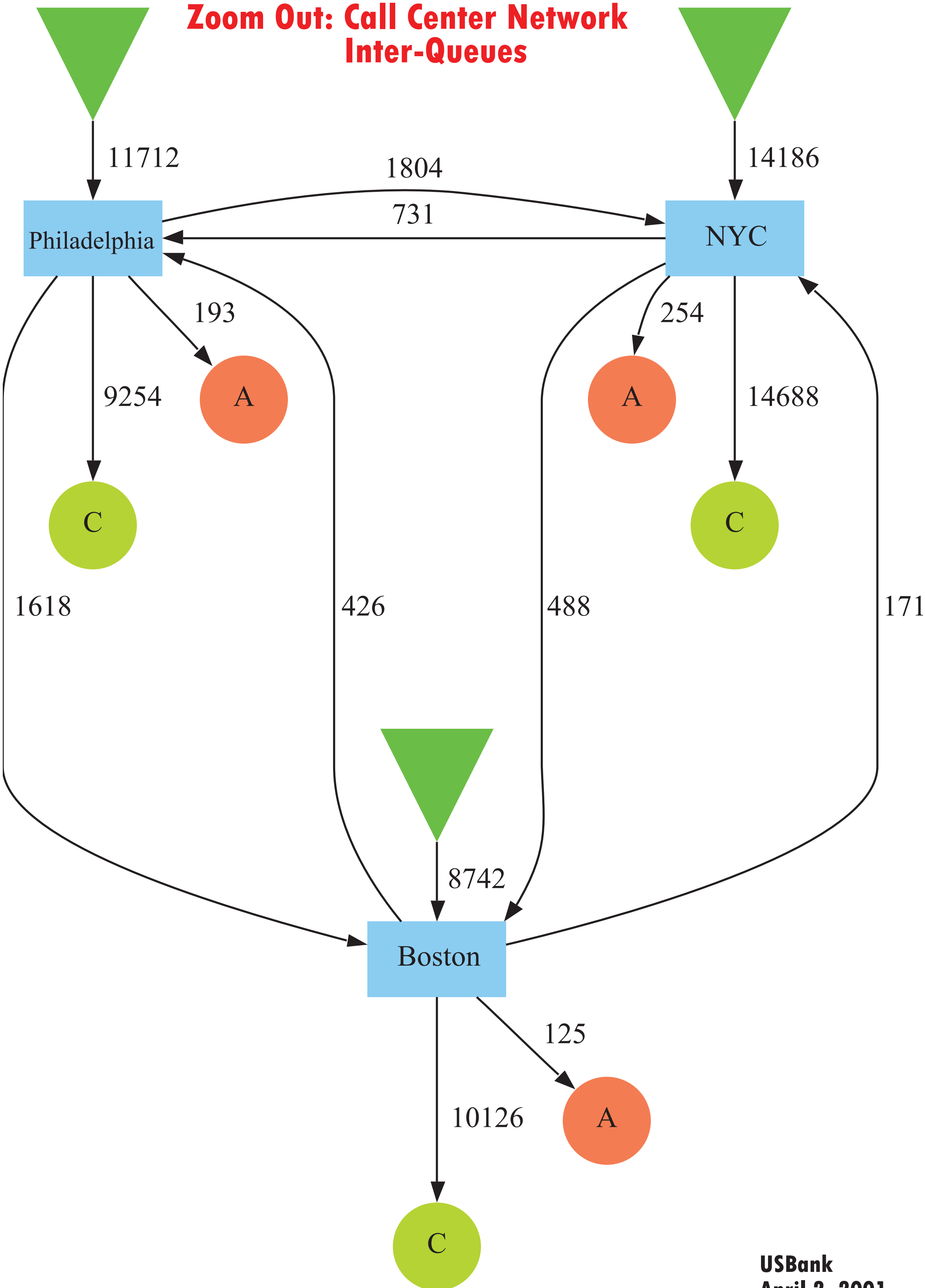
January 5, 2012

1 Hour in a Call Center
- Customers
- Hierarchical



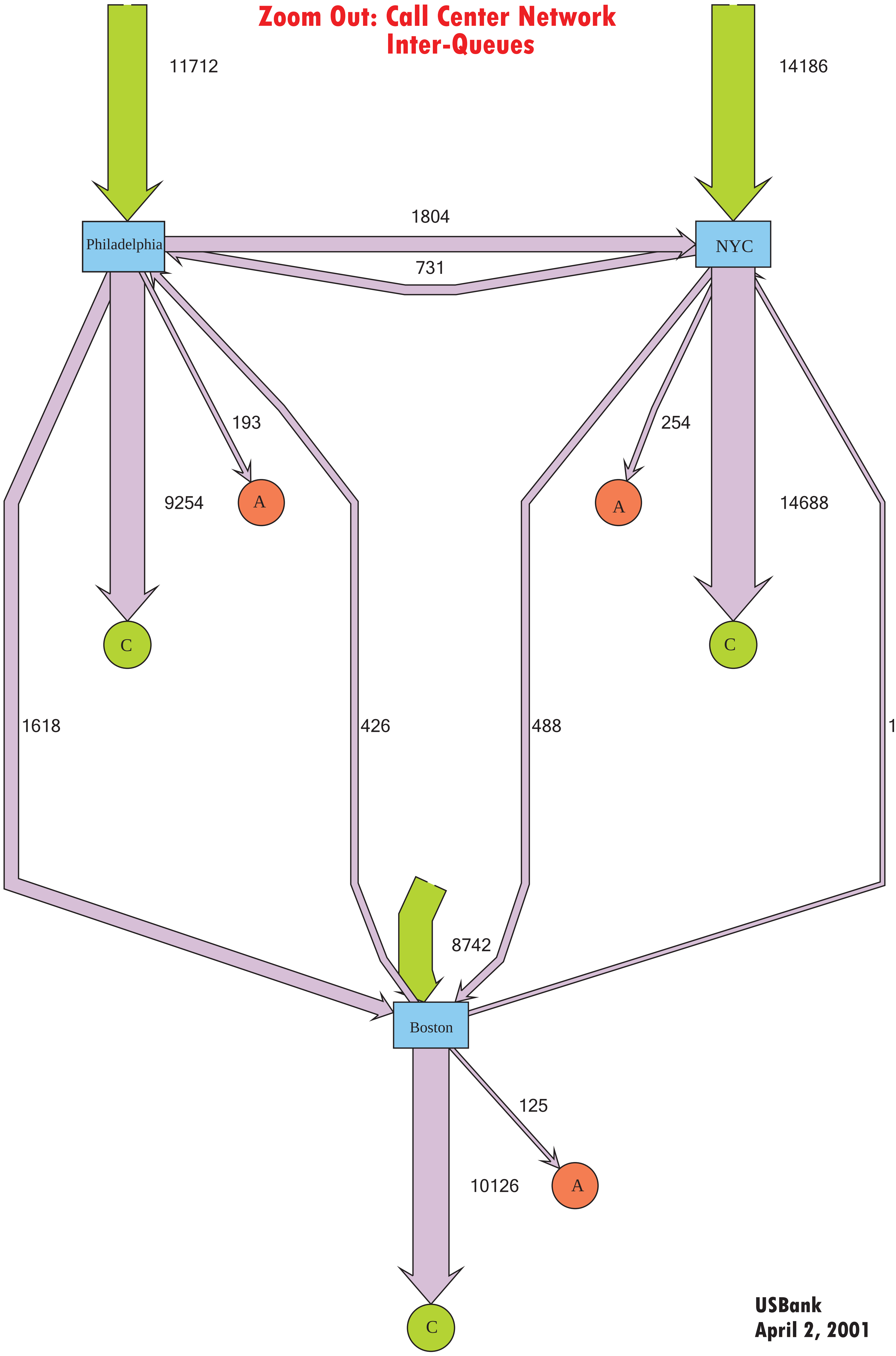
USBank
April 2, 2001

Zoom Out: Call Center Network
Inter-Queues



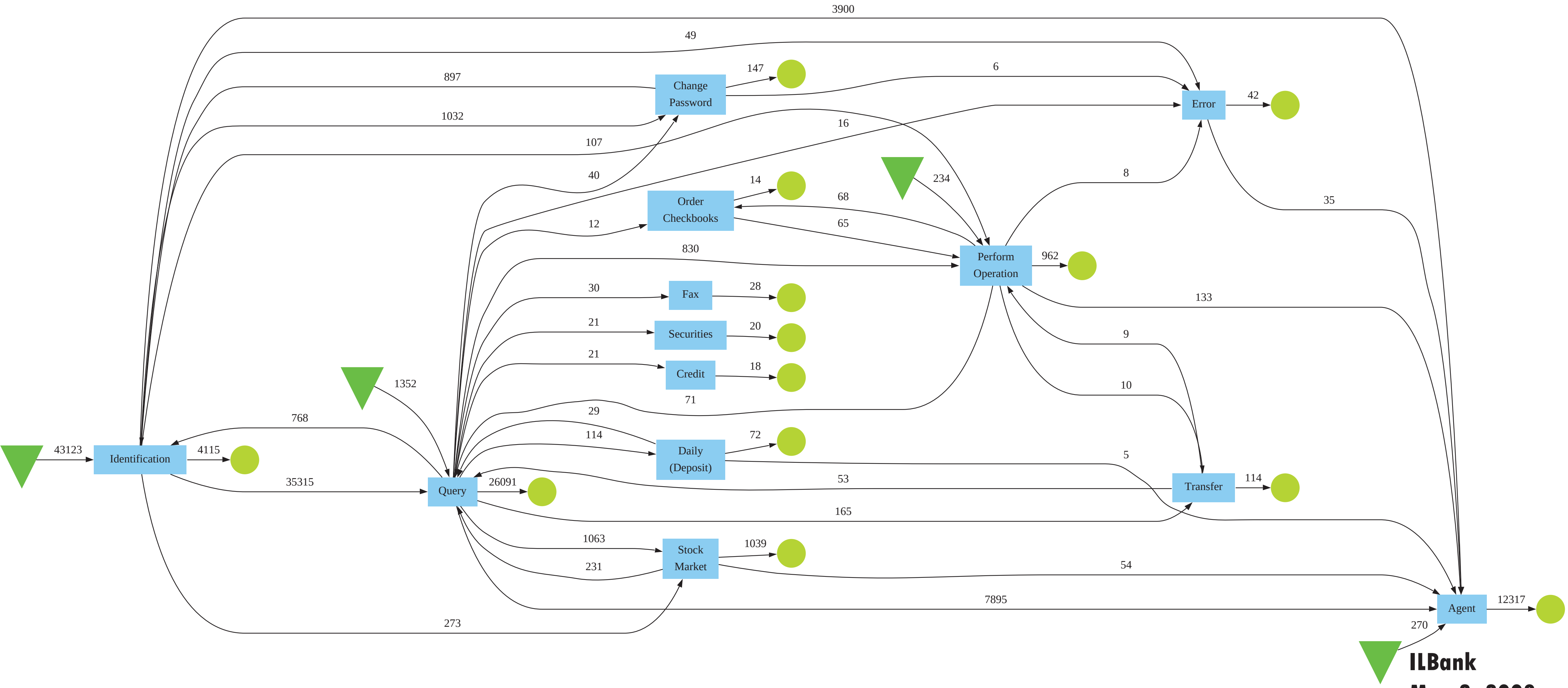
USBank
April 2, 2001

Zoom Out: Call Center Network
Inter-Queues

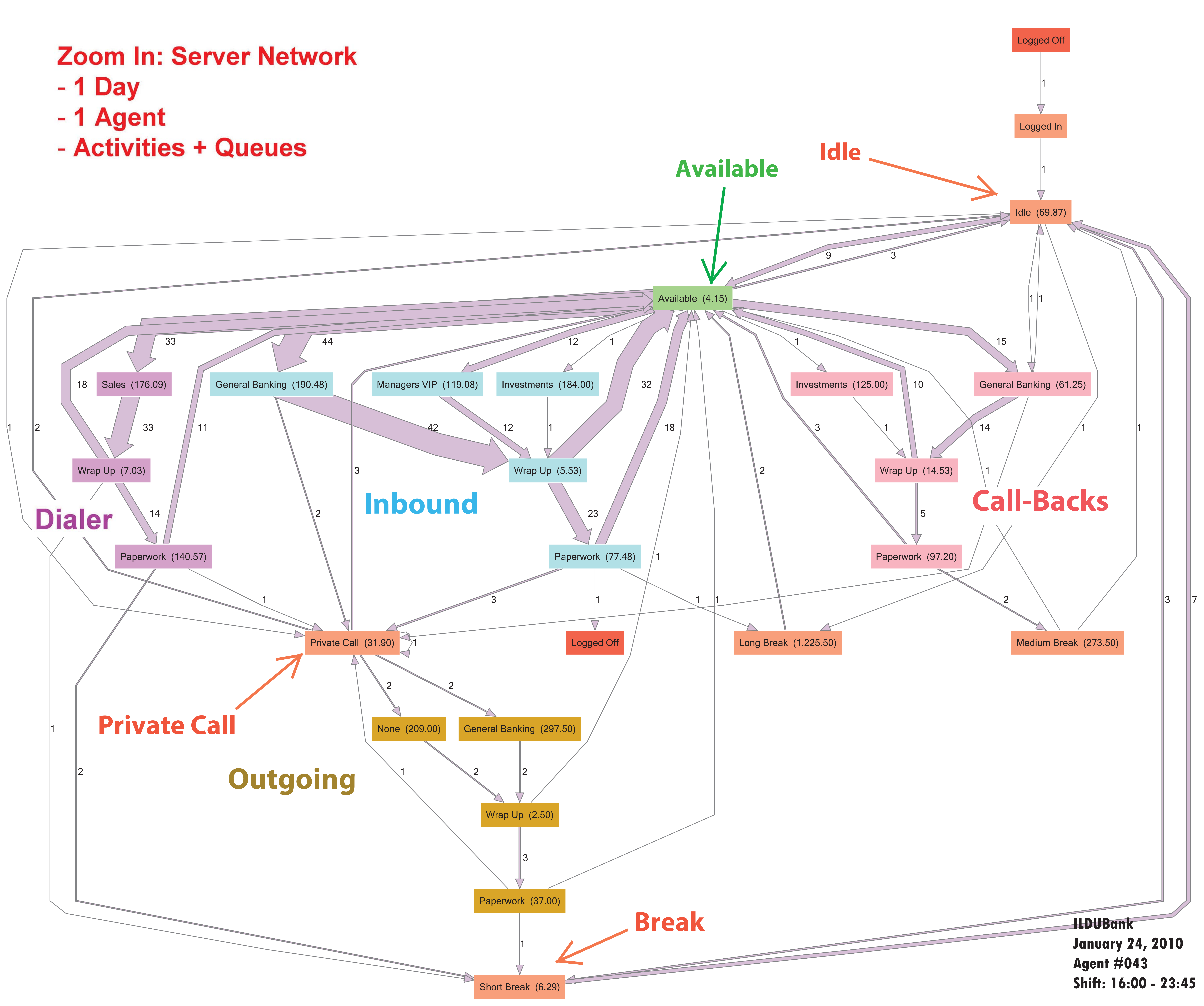


USBank
April 2, 2001

Zoom In: Interactive Voice Response (IVR)

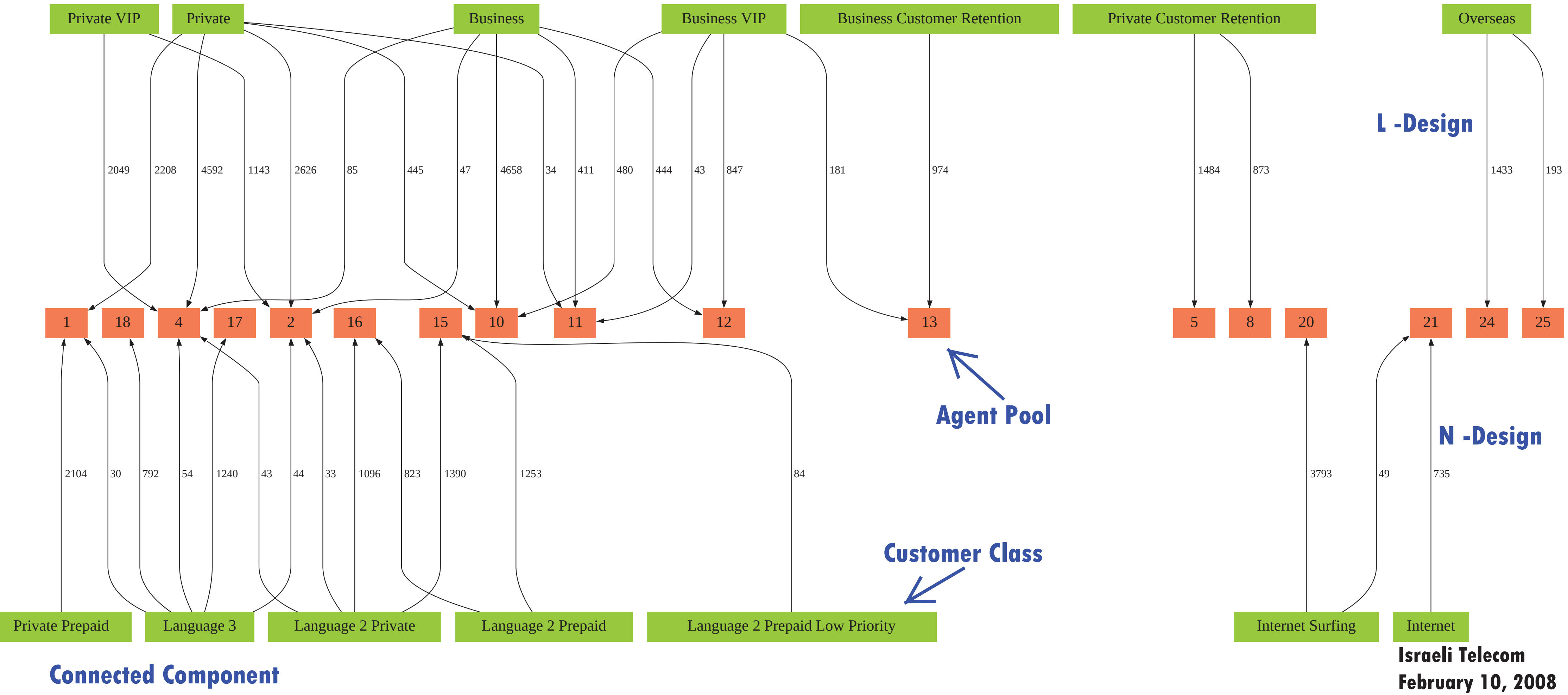


ILBank
May 2, 2008



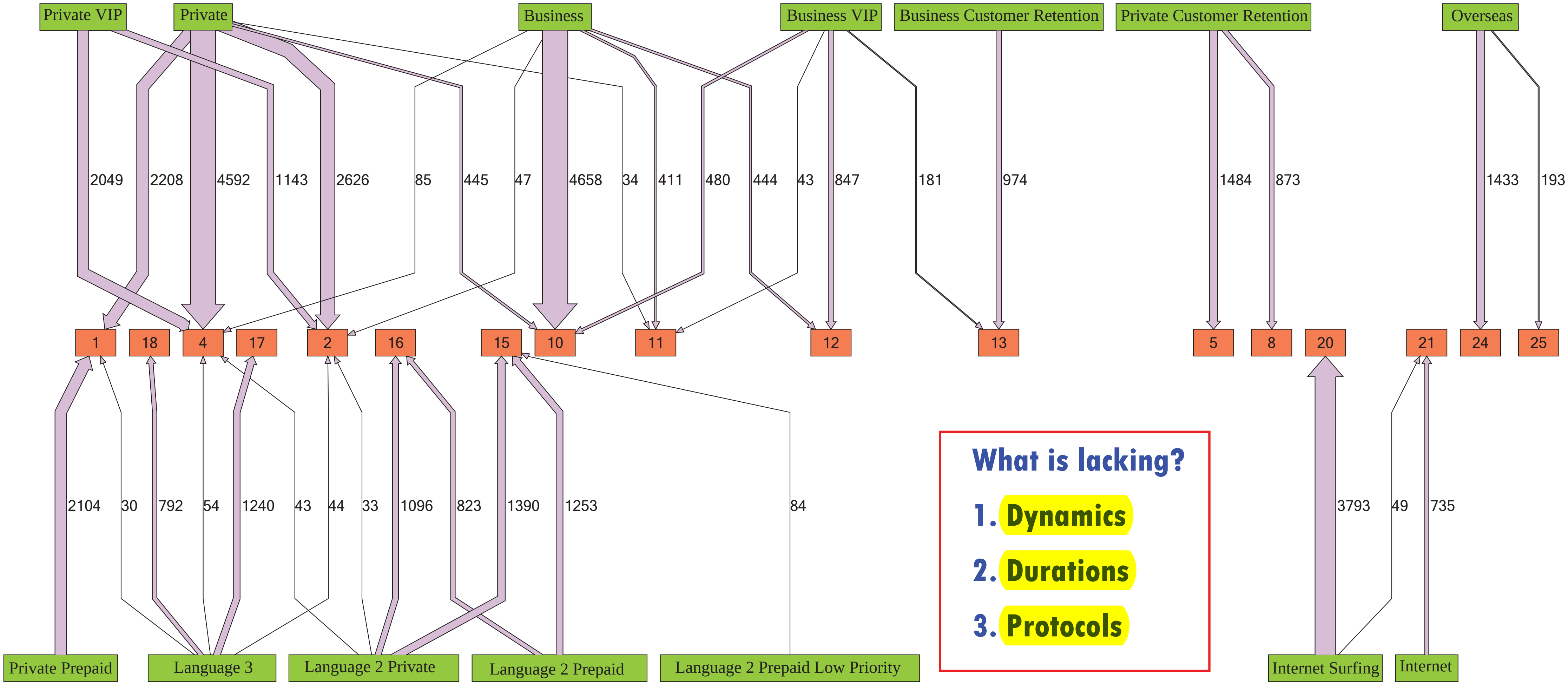
Zoom In: Server Network
- 1 Day
- 1 Agent
- Activities + Queues
Logged Off
Logged In
Idle
Idle (69.87)
Available
Available (4.15)
Sales (176.09)
General Banking (190.48)
Managers VIP (119.08)
Investments (184.00)
Investments (125.00)
General Banking (61.25)
Wrap Up (7.03)
Wrap Up (5.53)
Wrap Up (14.53)
Dialer
Inbound
Call-Backs
Paperwork (140.57)
Paperwork (77.48)
Paperwork (97.20)
Private Call (31.90)
Logged Off
Long Break (1,225.50)
Medium Break (273.50)
None (209.00)
General Banking (297.50)
Private Call
Outgoing
Wrap Up (2.50)
Paperwork (37.00)
Break
Short Break (6.29)
ILDUBank
January 24, 2010
Agent #043
Shift: 16:00 - 23:45

Zoom In: Skills - Based Routing (SBR)
Network Structure (Protocol)



Israeli Telecom
February 10, 2008

Goal: Data-Based Real-Time Simulation (SimNet)



What is lacking?

1. Dynamics
2. Durations
3. Protocols

Israeli Telecom
February 10, 2008

4 Hours in a Call Center - Dynamics

USBank

April 2, 2001

8 AM - 9 AM


Entry
VRU
Entry
Entry
Entry
Entry
Entry
Entry
Entry
Subanco
Premier
Telesales
Business
Retail
Summit
Consumer Loans
EBO
Platinum
A
C
Out
VRU
Premier
Retail
Business
Consumer Loans
EBO
Telesales
Summit
Platinum
VRU
Retail

8 AM - 9 AM


Entry
VRU
Subanco
Premier
Telesales
Business
Retail
Summit
Consumer Loans
EBO
Platinum
A
C
Out

9 AM - 10 AM

Entry
VRU
Entry
Entry
Entry
Entry
Entry
Entry
Entry
Subanco
Premier
Telesales
Business
Retail
Summit
Consumer Loans
EBO
Platinum
A
A
A
A
A
A
A
A
A
C
C
C
C
C
C
C
C
C
VRU
C
Out
Out
Out
Premier
Retail
Out
Out
Business
Consumer Loans
EBO
Telesales
Out
Summit
Out
Out
Out
Out
Platinum
A
A
C
C
C
C
C
C
C
C
VRU
C
Out
Retail
Out
Out
Business
C
C

10 AM - 11 AM

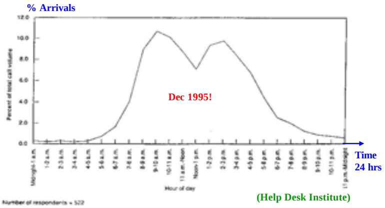

Entry
VRU
Subanco
Premier
Telesales
Business
Retail
Summit
Consumer Loans
EBO
Platinum
A
C
Out

11 AM - 12 PM

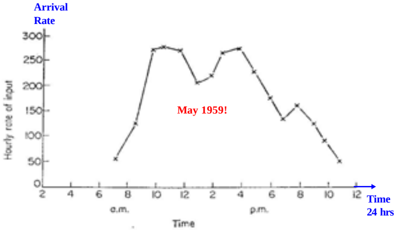
Dynamics: Time-Varying Arrival-Rates

2 Daily Peaks

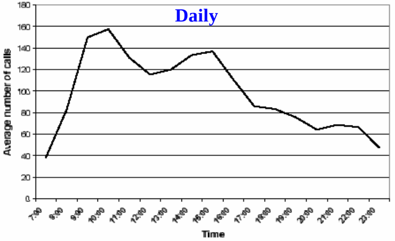
CC: Dec. **1995**, (USA, 700 Helpdesks)



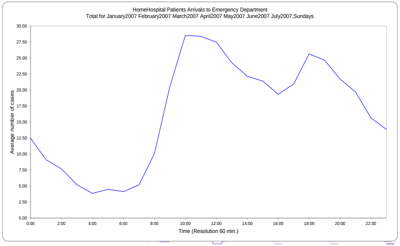
CC: May **1959** (England)



CC: Nov. 1999 (Israel)



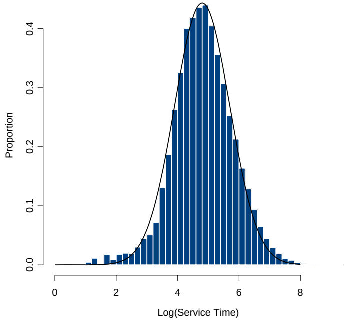
ED: Jan.–July 2007 (Israel)



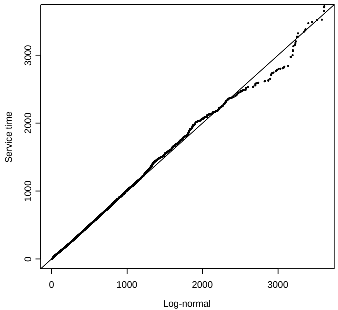
Durations: Phone Calls (2 Surprises)

Israeli Call Center, Nov–Dec, 1999

Log(Service Times)



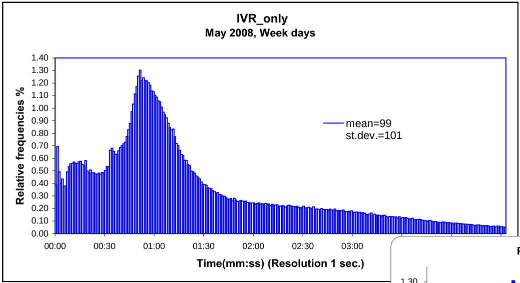
LogNormal QQPlot



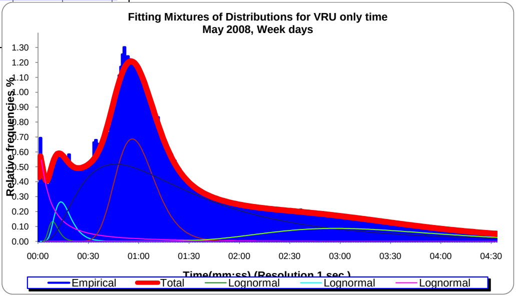
- **Practically Important**: (mean, std)(log) characterization
- **Theoretically Intriguing**: Why LogNormal ? Naturally multiplicative but, in fact, also **Infinitely-Divisible** (Generalized Gamma-Convolutions)

Durations: Answering Machine

Israeli Bank: IVR/VRU Only, May 2008



Mixture: 7 LogNormals



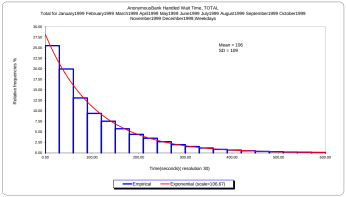
Tree-Network Tomography:
Reconstruct topology
from root and leaves

Durations: Waiting Times in a Call Center

⇒ Protocols

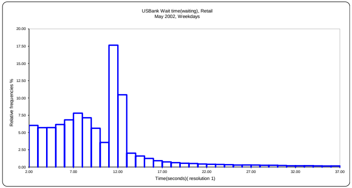
Exponential in Heavy-Traffic (min.)

Small Israeli Bank



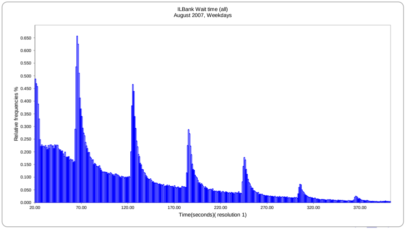
Routing via Thresholds (sec.)

Large U.S. Bank



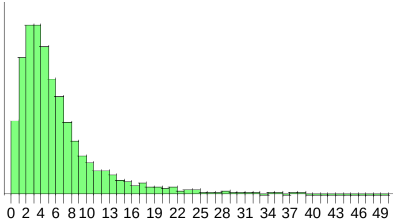
Scheduling Priorities (sec.) [compare Hospital LOS (hours)]

Medium Israeli Bank



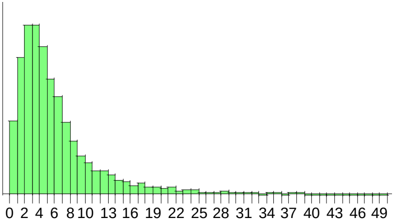
LogNormal & Beyond: Length-of-Stay in a Hospital

Israeli Hospital, in Days: LN

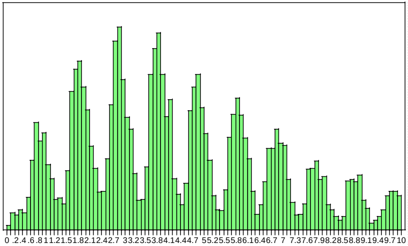


LogNormal & Beyond: Length-of-Stay in a Hospital

Israeli Hospital, in Days: LN

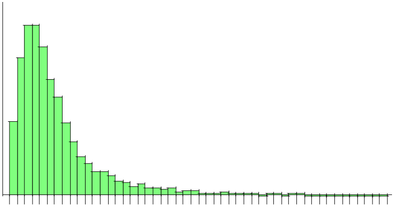


Israeli Hospital, in Hours: Mixture

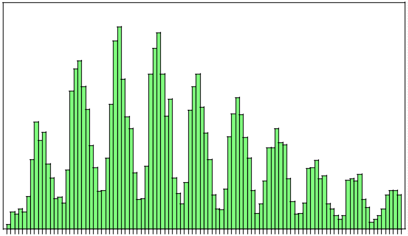


LogNormal & Beyond: Length-of-Stay in a Hospital

Israeli Hospital, in Days: LN



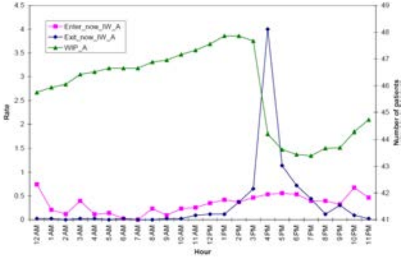
Israeli Hospital, in Hours: Mixture



Explanation: Patients released around **3pm** (1pm in Singapore)

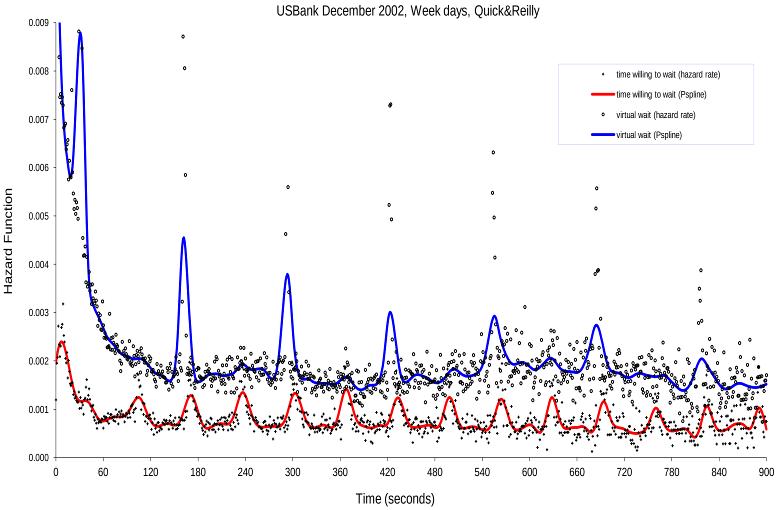
Why Bother ?

- Hourly Scale: Staffing,. . .
- Daily: Flow / Bed Control,. . .



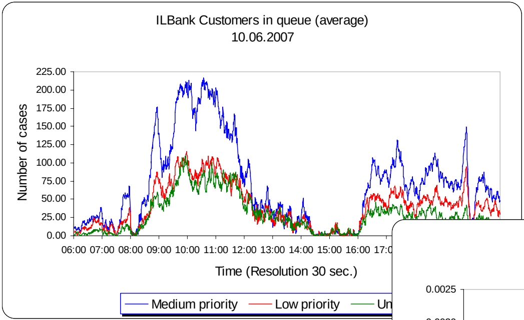
Protocols + Psychology

Patient Customers, Announcements, Priority Upgrades

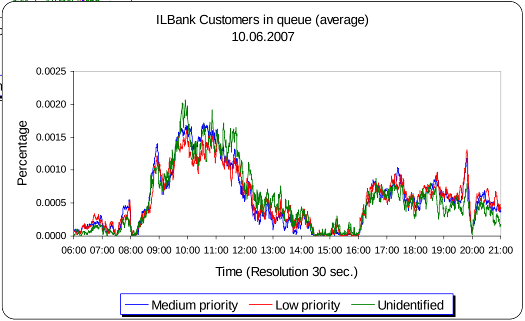


Dynamics: Parsimonious Models (Congestion Laws)

3 Queue-Lengths at 30 sec. resolution (ILBank, 10/6/2007)



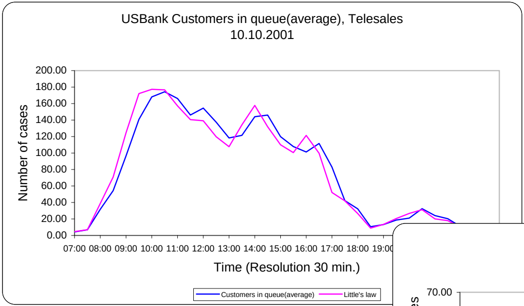
Queue "Shape"

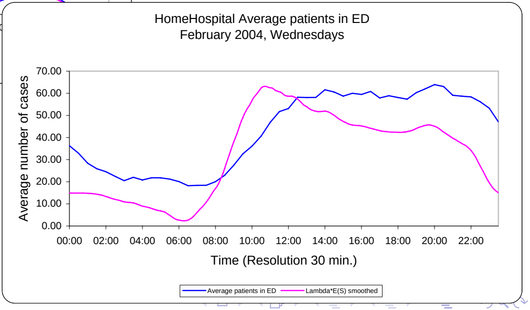


- Area normalized to 100%
- **State-Space Collapse**

Little's Law: Call Center & Emergency Department

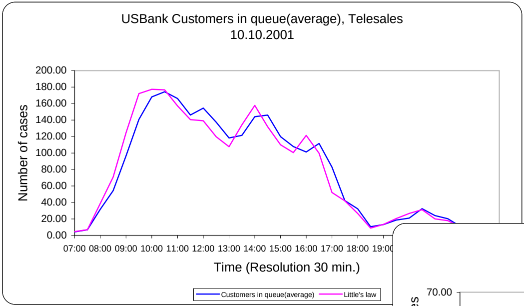
Time-Gap: **# in System** lags behind **Piecewise-Little** ($L = \lambda \times W$)

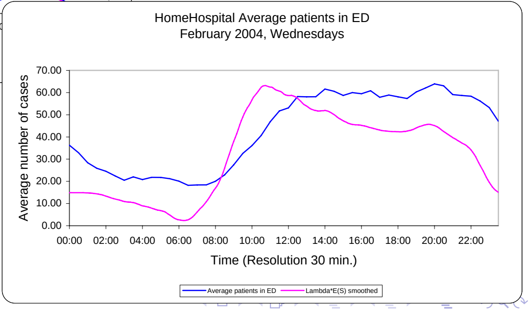




Little's Law: Call Center & Emergency Department

Time-Gap: **# in System** lags behind **Piecewise-Little** ($\boldsymbol{L} = \lambda \times \boldsymbol{W}$)





$\Rightarrow$ **Time-Varying Little's Law**

- **Berstemas & Mourtzinou;**
- **Fralix, Riano, Serfozo;** ...

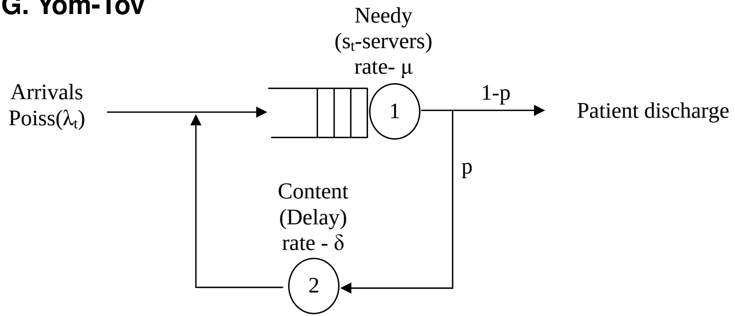
Prerequisite II: Models (FNets)

"Laws of Large Numbers" capture **Predictable** Variability

Deterministic Models: Scale Averages-out **Stochastic Individualism**

The Basic Service-Network Model: Erlang-R

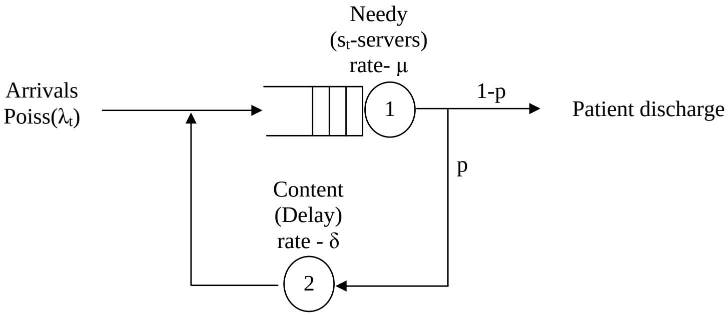
w/ G. Yom-Tov



Erlang-R **(IE: Repairman Problem 50's; CS: Central-Server 60's)** =
2-station "Jackson" Network = (M/M/S, M/M/∞) :

- λ_t – **Time-Varying** **Arrival** rate
- S_t – Number of **Servers** (Nurses / Physicians)
- μ – **Service** rate ($E[\text{Service}] = \frac{1}{\mu}$)
- p – **ReEntrant** (Feedback) fraction
- δ – **Content-to-Needy** rate ($E[\text{Content}] = \frac{1}{\delta}$)

Fluid Model $\leftrightarrow$ (Time-Varying) Erlang-R System



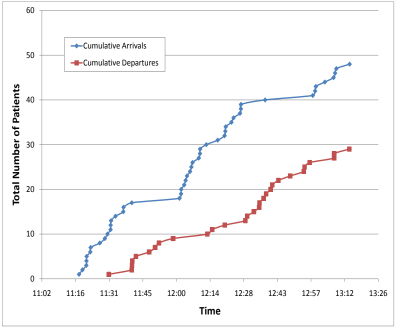
FNet of a 2-station “Jackson" Network:

$$\begin{aligned}
\frac{d}{dt} q_t^1 &= \lambda_t - \mu \cdot \left(q_t^1 \wedge s_t\right) + \delta \cdot q_t^2, \\
\frac{d}{dt} q_t^2 &= p \cdot \mu \cdot \left(q_t^1 \wedge s_t\right) - \delta \cdot q_t^2.
\end{aligned} \tag{1}$$

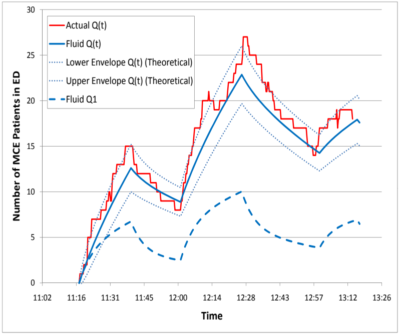
Erlang-R: Fitting a Simple Model to a Complex Reality

Chemical MCE Drill (Israel, May 2010)

Arrivals & Departures (RFID)



Erlang-R (Fluid, Diffusion)

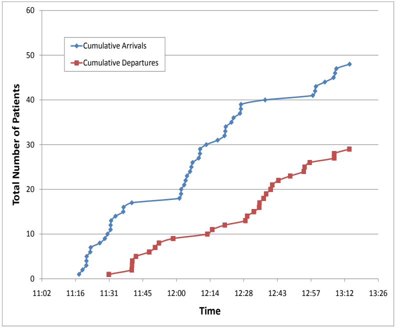


- **Recurrent/Repeated** services in MCE Events: eg. Injection every 15 minutes

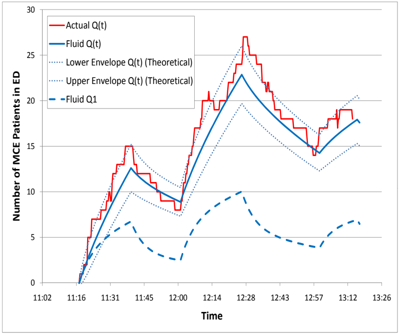
Erlang-R: Fitting a Simple Model to a Complex Reality

Chemical MCE Drill (Israel, May 2010)

Arrivals & Departures (RFID)



Erlang-R (Fluid, Diffusion)



- **Recurrent/Repeated** services in MCE Events: eg. Injection every 15 minutes
- **Fluid (Sample-path)** Modeling, via Functional Strong Laws of Large Numbers
- **Stochastic** Modeling, via Functional Central Limit Theorems
  - ED in **MCE**: Confidence-interval, usefully narrow for **Control**
  - ED in **normal** (**time-varying**) conditions: Personnel **Staffing**

An Asymptotic Framework: Erlang-R in the ED

System = Emergency Department (eg. Rambam Hospital)

- **SimNet** = Customized ED-Simulator (Marmor & Sinreich)
- **QNet** = Erlang-R (time-varying 2-station Jackson; **w/ Yom-Tov**)
- **FNets** = 2-dim dynamical system (Massey & Whitt)
- **DNets** = 2-dim Markovian Service Net (w/ Massey and Reiman)

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Asymptotic Framework

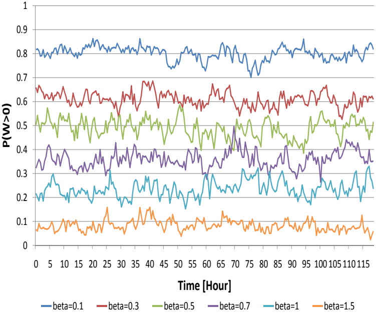
- Data and Measurements
- Fit a simple model (time-varying Erlang-R) to a complex reality (ED Physicians)
- Develop FNets (Offered-Load of Physicians) and (relevant) DNet (if needed)
- Use FNet / DNet for Design ($\sqrt{}$-Staffing), Analysis, ...
- Simulate reality (ED with $\sqrt{}$-staffing of Physicians)
- Validation: stable performance, confidence intervals, ...

Case Study: Emergency Ward Staffing

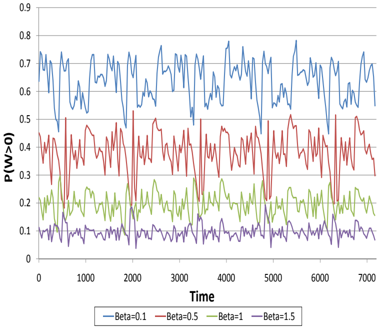
Many-Server ($\uparrow \infty$) Approximations for Small Systems (1-7)

- Staffing resolution: 1 hour
- Lower bound: 1 doctor per type
- Flexible (time-varying square-root) staffing: **Yunan's Lecture**
- Rounding effects $\Rightarrow$ Not all performance levels achievable

90 Servers

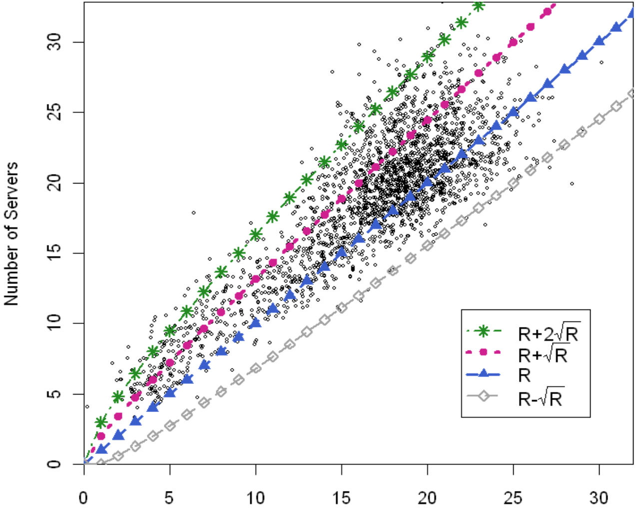


1-7 Doctors



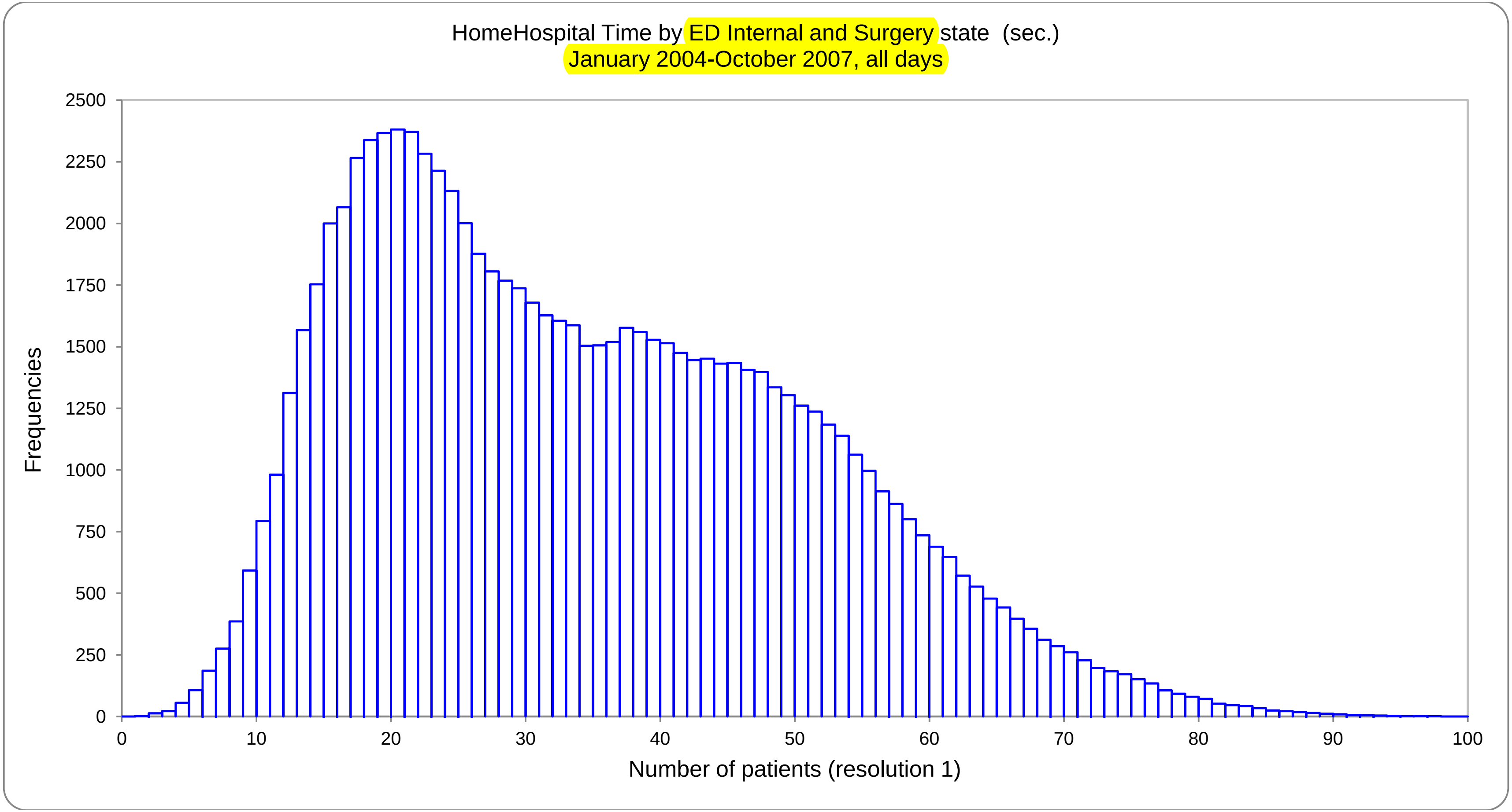
Protocols: Staffing (N) vs. Offered-Load (R $= \lambda \times E(S)$)

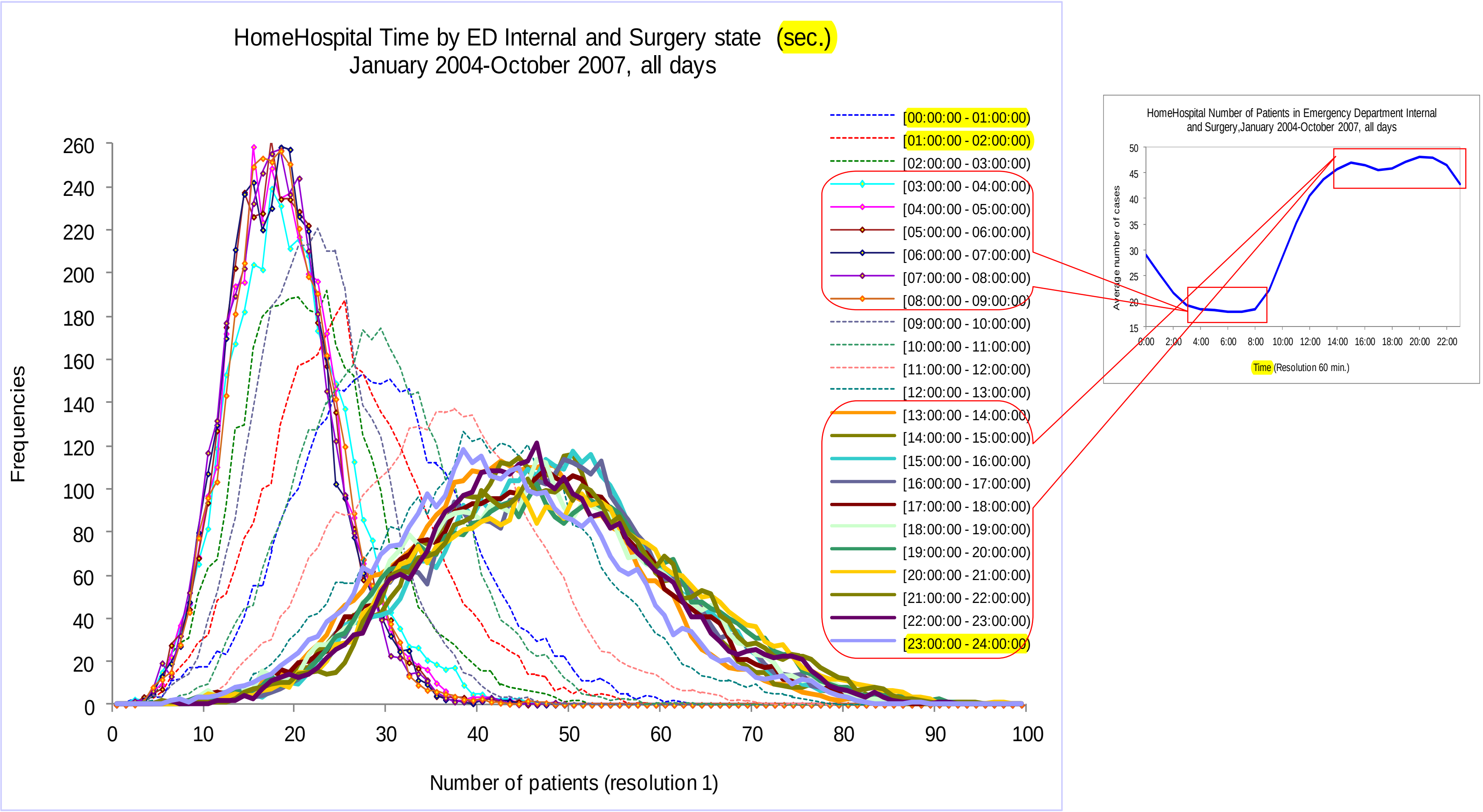
IL Telecom; June-September, 2004; w/ Nardi, Plonski, Zeltyn



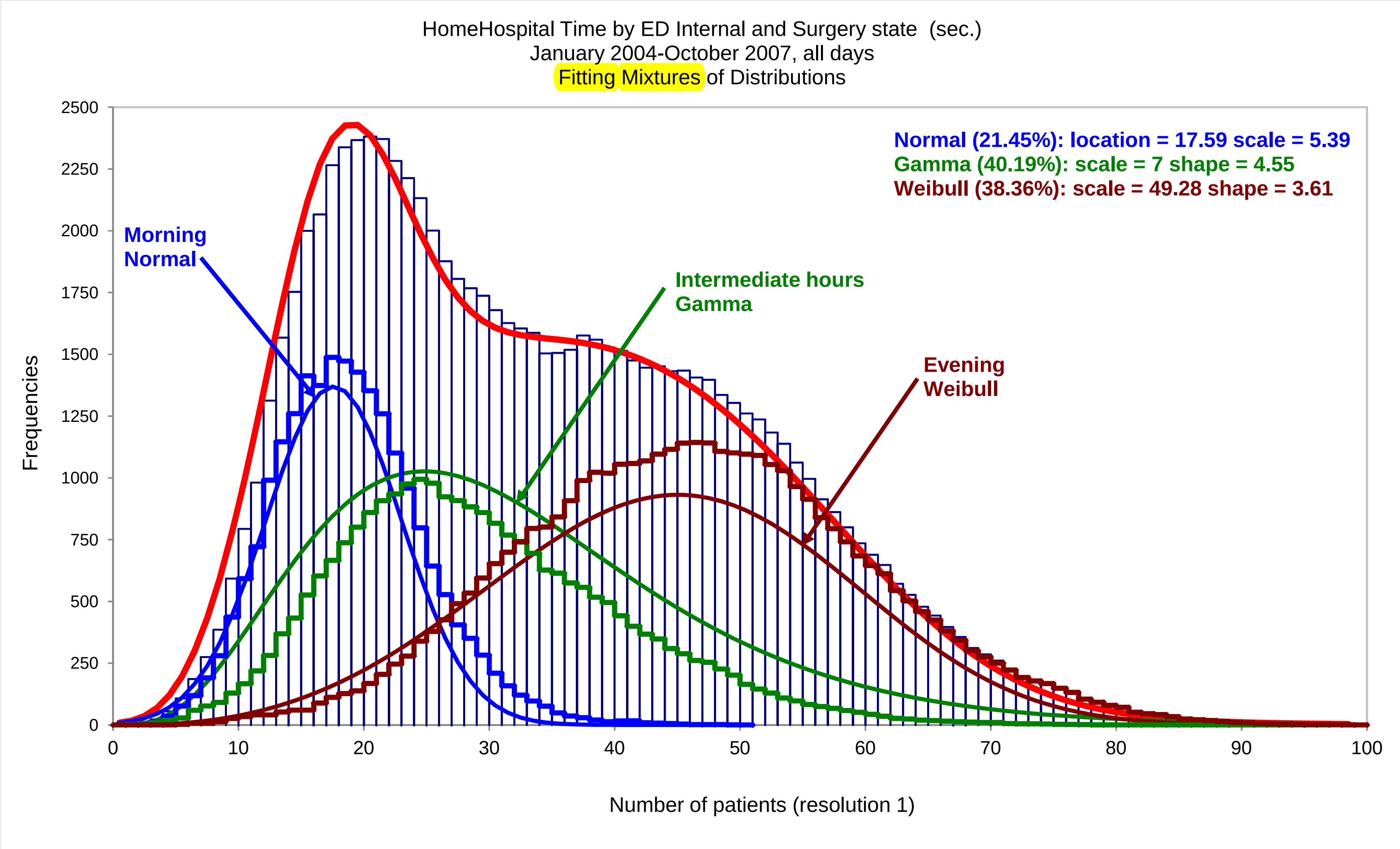
2205 half-hour intervals (13 summer weeks, week-days)

Number of patients in ED





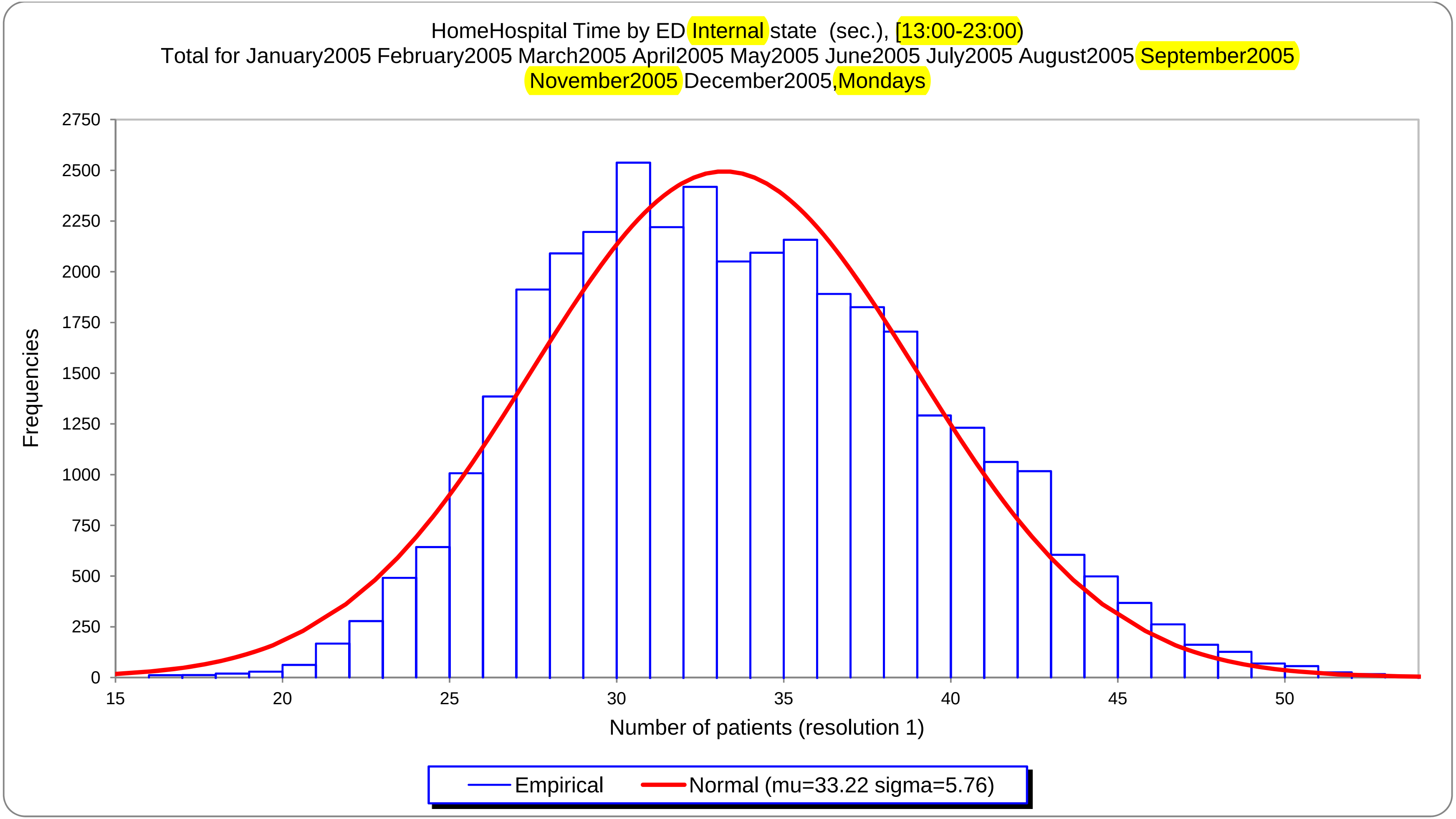
HomeHospital Time by ED Internal and Surgery state (sec.)
January 2004-October 2007, all days
Frequencies
Number of patients (resolution 1)
0
20
40
60
80
100
120
140
160
180
200
220
240
260
10
30
50
70
90
[00:00:00 - 01:00:00)
[01:00:00 - 02:00:00)
[02:00:00 - 03:00:00)
[03:00:00 - 04:00:00)
[04:00:00 - 05:00:00)
[05:00:00 - 06:00:00)
[06:00:00 - 07:00:00)
[07:00:00 - 08:00:00)
[08:00:00 - 09:00:00)
[09:00:00 - 10:00:00)
[10:00:00 - 11:00:00)
[11:00:00 - 12:00:00)
[12:00:00 - 13:00:00)
[13:00:00 - 14:00:00)
[14:00:00 - 15:00:00)
[15:00:00 - 16:00:00)
[16:00:00 - 17:00:00)
[17:00:00 - 18:00:00)
[18:00:00 - 19:00:00)
[19:00:00 - 20:00:00)
[20:00:00 - 21:00:00)
[21:00:00 - 22:00:00)
[22:00:00 - 23:00:00)
[23:00:00 - 24:00:00)
HomeHospital Number of Patients in Emergency Department Internal
and Surgery,January 2004-October 2007, all days
Average number of cases
50
45
40
35
30
25
20
15
0:00
2:00
4:00
6:00
8:00
10:00
12:00
14:00
16:00
18:00
20:00
22:00
Time (Resolution 60 min.)


HomeHospital Time by ED Internal and Surgery state (sec.)
January 2004-October 2007, all days
Fitting Mixtures of Distributions
Normal (21.45%): location = 17.59 scale = 5.39
Gamma (40.19%): scale = 7 shape = 4.55
Weibull (38.36%): scale = 49.28 shape = 3.61
Morning
Normal
Intermediate hours
Gamma
Evening
Weibull
Frequencies
2500
2250
2000
1750
1500
1250
1000
750
500
250
0
0
10
20
30
40
50
60
70
80
90
100
Number of patients (resolution 1)

Time by ED Internal and Surgery state (sec.) **Statistics**	
N	120960000
N(average per day)	86400
Mean	34.1
Standard Deviation	16.34
Variance	266.87
Median	32
Minimum	0
Maximum	99
Skewness	0.524
Kurtosis	-0.44452
Standard Error Mean	0.00149
Interquartile Range	25
Mean Absolute Deviation	13.7
Median Absolute Deviation(MAD)	12
Coefficient of Variation (CV) (%)	47.9
L-moment 2 (half of Gini's Mean Difference)	9.25
L-Skewness	0.121
L-Kurtosis	0.0561
Coefficient of L-variation (L-CV)(%) (Gini's Coefficient)	27.12

Parameter Estimates						
Components	*Mixing Proportions (%)*	*Location*	*Scale*	*Shape*	*Mean*	*Standard Deviation*
1. Normal	21.45	17.59	5.39		17.59	5.39
2. Gamma	40.19		7.00	4.55	31.83	14.99
3. Weibull	38.36		49.28	3.61	44.42	13.679

Goodness-of-Fit Tests			
Tests	*Statistic*	*DF*	*p Value*
Residuals Std	0.011		
Kolmogorov-Smirnov	0.028		<.0001
Cramer-von Mises	14953.04		<.0001
Andersen-Darling	96156.09		<.0001
Chi-Square	460741.3	94	<.0001


HomeHospital Time by ED Internal state (sec.), [13:00-23:00)
Total for January2005 February2005 March2005 April2005 May2005 June2005 July2005 August2005 September2005
November2005 December2005,Mondays
Frequencies
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
15
20
25
30
35
40
45
50
Number of patients (resolution 1)
Empirical
Normal (mu=33.22 sigma=5.76)

Time by ED Internal state (sec.), [13:00-23:00) ***Statistics***	
N	1656000
N(average per day)	36000
Mean	33.22
Standard Deviation	5.756
Variance	33.13
Median	33
Minimum	16
Maximum	54
Skewness	0.282
Kurtosis	-0.31791
Standard Error Mean	0.00447
Interquartile Range	8
Mean Absolute Deviation	4.712
Median Absolute Deviation(MAD)	4
Coefficient of Variation(CV) (%)	17.33
L-moment 2 (half of Gini's Mean Difference)	3.263
L-Skewness	0.0601
L-Kurtosis	0.0924
Coefficient of L-variation(L-CV)(%) (Gini's Coefficient)	9.82

Parameters for Normal Distribution	
Parameter	*Estimate*
mu	33.22
sigma	5.76
mean	33.22
std	5.756

Goodness-of-Fit Tests for Normal Distribution			
Test	*Statistic*	*DF*	*p Value*
Residuals Std	0.025		
Kolmogorov-Smirnov	0.069		<.0001
Cramer-von Mises	1068.72		<.0001
Anderson-Darling	6193.27		<.0001
Chi-Square	>1000	34	<.0001

Internal ED, non-holiday Mondays, 13:00-23:00

Our EDA “implies" (**w/ Armony, Marmor, Tseytlin, Yom-Tov**):

- **Israeli ED census** $\stackrel{d}{=} M/M/\infty$: “Secret" behind $\sqrt{}$-Staffing

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- **U.S. ED census** $\stackrel{d}{=} M/M/B/B$, which can be (has been) used to analyze Ambulance Diversion (ED Blocking)

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- **U.S. ED census** $\stackrel{d}{=}$ $M/M/B/B$, which can be (has been) used to analyze Ambulance Diversion (ED Blocking)
- Puzzle (getting ahead): Is the ED an Erlang-A system with $\mu = \theta$? then the "effective number of servers" can be, perhaps, deduced via those LWBS = Left Without Being Seen (or LAMA)

Simple models at the service of complex realities

Prerequisite II: Models (DNets, QED Q's)

Traditional Queueing Theory predicts that **Service-Quality** and **Servers' Efficiency** **must** be traded off against each other.

For example, **M/M/1** (single-server queue): **91%** server's utilization goes with

$$\textbf{Congestion Index} = \frac{E[Wait]}{E[Service]} = \mathbf{10},$$

and only 9% of the customers are served immediately upon arrival.

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Yet, **heavily-loaded** queueing systems with **Congestion Index = 0.1** (Waiting one order of magnitude less than Service) are prevalent:

- **Call Centers**: Wait **"seconds"** for **minutes** service;
- **Transportation**: Search **"minutes"** for **hours** parking;
- **Hospitals**: Wait **"hours"** in ED for **days** hospitalization in IW's.

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- EDs + IWs: **?**

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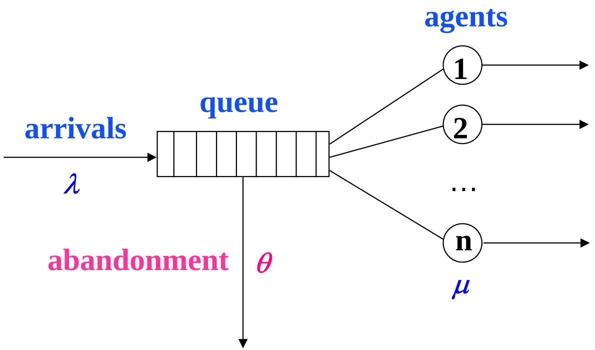
- CCs: **50%** served "immediately" & **90%** utilization ⇒ **QED**
- EDs + IWs: **?** Multiple scales! IW-"Beds" (10's) are QED while IW-Doctors (1's) are in conventional heavy-traffic (hours wait for minutes service), hence the bottlenecks

The Basic Staffing Model: Erlang-A (M/M/N + M)



Erlang-A **(Palm 1940's)** = **Birth & Death Q**, with parameters:

- λ – **Arrival** rate (Poisson)
- μ – **Service** rate (Exponential; $E[S] = \frac{1}{\mu}$)
- θ – **Patience** rate (Exponential, $E[\text{Patience}] = \frac{1}{\theta}$)
- N – Number of **Servers** (Agents).

Erlang-A: Practical Relevance?

Experience:

- Arrival process **not pure Poisson** (time-varying, σ^2 too large)
- Service times **not Exponential** (typically close to LogNormal)
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- Customers and Servers **not homogeneous** (classes, skills)
- Customers return for service (after busy, abandonment; dependently; **P. Khudiakov, M. Gorfine, P. Feigin**)
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Question: **Is Erlang-A Relevant?**

YES ! **Fitting a Simple Model to a Complex Reality**, both **Theoretically** and **Practically**

Asymptotic Landscape: 9 Operational Regimes, and then some

Erlang-A, w/ I. Gurvich & J. Huang

Erlang-A μ & θ fixed	Conventional scaling			Many-Server scaling			NDS scaling		
	Sub	Critical	Over	QD	QED	ED	Sub	Critical	Over
Offered load per server	$\frac{1}{1+\delta}$	$1-\frac{\beta}{\sqrt{n}}$	$\frac{1}{1-\gamma}$	$\frac{1}{1+\delta}$	$1-\frac{\beta}{\sqrt{n}}$	$\frac{1}{1-\gamma}$	$\frac{1}{1+\delta}$	$1-\frac{\beta}{n}$	$\frac{1}{1-\gamma}$
Arrival rate λ	$\frac{\mu}{1+\delta}$	$\mu-\frac{\beta}{\sqrt{n}}\mu$	$\frac{\mu}{1-\gamma}$	$\frac{n\mu}{1+\delta}$	$n\mu-\beta\mu\sqrt{n}$	$\frac{n\mu}{1-\gamma}$	$\frac{n\mu}{1+\delta}$	$n\mu-\beta\mu$	$\frac{n\mu}{1-\gamma}$
# servers	1			n			n		
Time-scale	n			1			n		
Impatience rate	θ/n			θ			θ/n		
Staffing level	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu}(1+\frac{\beta}{\sqrt{n}})$	$\frac{\lambda}{\mu}(1-\gamma)$	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu}+\beta\sqrt{\frac{\lambda}{\mu}}$	$\frac{\lambda}{\mu}(1-\gamma)$	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu}+\beta$	$\frac{\lambda}{\mu}(1-\gamma)$
Utilization	$\frac{1}{1+\delta}$	$1-\sqrt{\frac{\theta}{\mu}}\frac{h(\hat{\beta})}{\sqrt{n}}$	1	$\frac{1}{1+\delta}$	$1-\sqrt{\frac{\theta}{\mu}}\frac{\hat{h}(\hat{\beta})}{\sqrt{n}}$	1	$\frac{1}{1+\delta}$	$1-\sqrt{\frac{\theta}{\mu}}\frac{h(\hat{\beta})}{n}$	1
$\mathbb{E}(Q)$	$\frac{1}{\delta(1+\delta)}$	$\sqrt{n}g(\hat{\beta})$	$\frac{n\mu\gamma}{\theta(1-\gamma)}$	$\frac{1}{\delta}\varrho_n$	$\sqrt{n}g(\hat{\beta})\alpha$	$\frac{n\mu\gamma}{\theta(1-\gamma)}$	$o(1)$	$ng(\hat{\beta})$	$\frac{n^2\mu\gamma}{\theta(1-\gamma)}$
$\mathbb{P}(Ab)$	$\frac{1}{n}\frac{1}{\delta}\frac{\theta}{\mu}$	$\frac{\theta}{\sqrt{n}\mu}g(\hat{\beta})$	γ	$\frac{1}{n}\frac{(1+\delta)}{\delta}\frac{\theta}{\mu}\varrho_n$	$\frac{\theta}{\sqrt{n}\mu}g(\hat{\beta})\alpha$	γ	$o(\frac{1}{n^2})$	$\frac{\theta}{n\mu}g(\hat{\beta})$	γ
$\mathbb{P}(W_q>0)$	$\frac{1}{1+\delta}$	≈ 1		ϱ_n	$\alpha\in(0,1)$	≈ 1	≈ 0	≈ 1	
$\mathbb{P}(W_q>T)$	$\frac{1}{1+\delta}e^{-\frac{\delta}{1+\delta}\mu T}$	$1+O(\frac{1}{\sqrt{n}})$	$1+O(\frac{1}{n})$	≈ 0		$f(T)$	≈ 0	$\frac{\bar{\Phi}(\hat{\beta}+\sqrt{\theta\mu}T)}{\bar{\Phi}(\hat{\beta})}$	$1+O(\frac{1}{n})$
Congestion $\frac{\mathbb{E}W_q}{\mathbb{E}S}$	$\frac{1}{\delta}$	$\sqrt{n}g(\hat{\beta})$	$n\mu\gamma/\theta$	$\frac{1}{n}\frac{(1+\delta)}{\delta}\varrho_n$	$\frac{\alpha}{\sqrt{n}}g(\hat{\beta})$	$\frac{\mu\gamma}{\theta}$	$o(\frac{1}{n})$	$g(\hat{\beta})$	$n\mu\gamma/\theta$

- **Conventional**: Ward & Glynn (03, $G/G/1+G$)
- **Many-Server**:
  - QED: Halfin-Whitt (81), Garnett-M-Reiman (02)
  - ED: Whitt (04)
  - NDS: Atar (12)

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Arrival rate λ	$\frac{\mu}{1+\delta}$	$\mu-\frac{\beta}{\sqrt{n}}\mu$	$\frac{\mu}{1-\gamma}$	$\frac{n\mu}{1+\delta}$	$n\mu-\beta\mu\sqrt{n}$	$\frac{n\mu}{1-\gamma}$	$\frac{n\mu}{1+\delta}$	$n\mu-\beta\mu$	$\frac{n\mu}{1-\gamma}$
# servers	1			n			n		
Time-scale	n			1			n		
Impatience rate	θ/n			θ			θ/n		
Staffing level	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu}(1+\frac{\beta}{\sqrt{n}})$	$\frac{\lambda}{\mu}(1-\gamma)$	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu}+\beta\sqrt{\frac{\lambda}{\mu}}$	$\frac{\lambda}{\mu}(1-\gamma)$	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu}+\beta$	$\frac{\lambda}{\mu}(1-\gamma)$
Utilization	$\frac{1}{1+\delta}$	$1-\sqrt{\frac{\theta}{\mu}}\frac{h(\hat{\beta})}{\sqrt{n}}$	1	$\frac{1}{1+\delta}$	$1-\sqrt{\frac{\theta}{\mu}}\frac{\hat{h}(\hat{\beta})}{\sqrt{n}}$	1	$\frac{1}{1+\delta}$	$1-\sqrt{\frac{\theta}{\mu}}\frac{h(\hat{\beta})}{n}$	1
$\mathbb{E}(Q)$	$\frac{1}{\delta(1+\delta)}$	$\sqrt{n}g(\hat{\beta})$	$\frac{n\mu\gamma}{\theta(1-\gamma)}$	$\frac{1}{\delta}\varrho_n$	$\sqrt{n}g(\hat{\beta})\alpha$	$\frac{n\mu\gamma}{\theta(1-\gamma)}$	$o(1)$	$ng(\hat{\beta})$	$\frac{n^2\mu\gamma}{\theta(1-\gamma)}$
$\mathbb{P}(Ab)$	$\frac{1}{n}\frac{1}{\delta}\frac{\theta}{\mu}$	$\frac{\theta}{\sqrt{n}\mu}g(\hat{\beta})$	γ	$\frac{1}{n}\frac{(1+\delta)}{\delta}\frac{\theta}{\mu}\varrho_n$	$\frac{\theta}{\sqrt{n}\mu}g(\hat{\beta})\alpha$	γ	$o(\frac{1}{n^2})$	$\frac{\theta}{n\mu}g(\hat{\beta})$	γ
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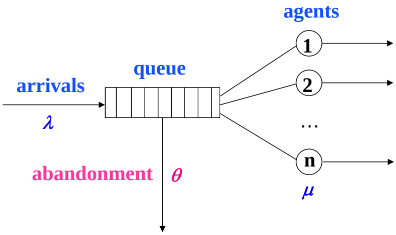
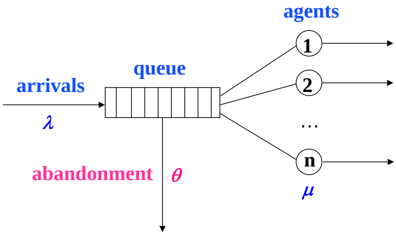
- Conventional: Ward & Glynn (03, $G/G/1+G$)
- Many-Server:
  - QED: Halfin-Whitt (81), Garnett-M-Reiman (02)
  - ED: Whitt (04)
  - NDS: Atar (12)
- **"Missing"**: ED+QED; Hazard-rate scaling (M/M/N+G); Time-Varying, Non-Parametric; Moderate- and Large-Deviation; Networks; Control

Universal Approximations: Erlang-A (M/M/N + M)



w/ **I. Gurvich & J. Huang**

Universal Approximations: Erlang-A (M/M/N + M)



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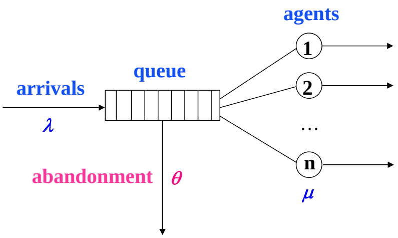

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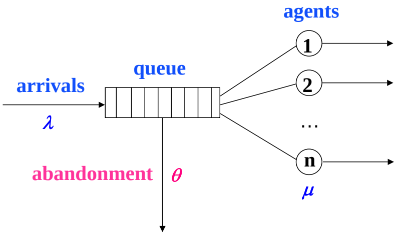

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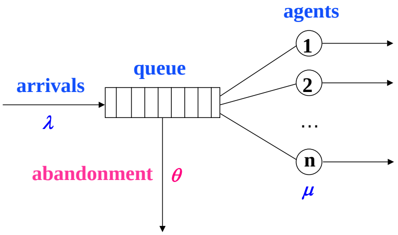

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eg. $\mu = \theta$: $\dot{x} = \lambda - \mu \cdot x$, $Y =$ OU process

Accuracy increases as $\lambda \uparrow \infty$ (no additional assumptions)

Value of Universal Approximation

- Tractable - closed-form stable expressions
- Accurate - more than heavy traffic limits
- Robust - all many-server regimes, and beyond, with hardly any assumptions
- Value
  - Performance Analysis
  - Optimization (Staffing)
  - Inference (**w/ G. Pang**)
  - Simulation (**w/ J. Blanchet**)
- Limitation: Steady-State (but working on it)

Why does it work so well?

Coupling **“Busy" + “Idle" Excursions** of B&D and the corresponding Diffusion (durations order $\frac{1}{\sqrt{\lambda}}$)

Universal Diffusion: Tractability

- Density function of $Y(\infty) - n$:

$$\pi(x) = \begin{cases} \frac{\sqrt{\mu}}{\sqrt{\lambda}} \frac{\phi(\sqrt{\mu}(x/\sqrt{\lambda}+\beta/\mu))}{\Phi(\beta/\sqrt{\mu})} p(\beta, \mu, \theta), & \text{if } x \leq 0, \\ \frac{\sqrt{\theta}}{\sqrt{\lambda}} \frac{\phi(\sqrt{\theta}(x/\sqrt{\lambda}+\beta/\theta))}{1-\Phi(\beta/\sqrt{\theta})} (1 - p(\beta, \mu, \theta)), & \text{if } x > 0, \end{cases}$$

Here $\beta := (n\mu - \lambda)/\sqrt{\lambda}$
and

$$p(\beta, \mu, \theta) = \left[1 + \sqrt{\frac{\mu}{\theta}} \frac{\phi(\beta/\sqrt{\mu})}{\Phi(\beta/\sqrt{\mu})} \frac{1 - \Phi(\beta/\sqrt{\theta})}{\phi(\beta/\sqrt{\theta})} \right]^{-1}.$$

Universal Approximation: Accuracy

- Δ^λ is the "balancing" state, obtained by solving

$$\lambda = \mu(n \wedge \Delta^\lambda) + \theta(\Delta^\lambda - n)^+.$$

Solution: $\Delta^\lambda = \frac{\lambda}{\mu} - \left(\frac{\lambda}{\mu} - n\right)^+ \left(1 - \frac{\mu}{\theta}\right).$

Specifically: QD = $\frac{\lambda}{\mu}$; ED = $n + \frac{1}{\theta}(\lambda - n\mu)$; QED = $n + \mathcal{O}(\sqrt{\lambda})$)

- Centered processes (excursions):

$$\tilde{Q}^\lambda(\cdot) = Q(\cdot) - \Delta^\lambda, \quad \tilde{Y}^\lambda(\cdot) = Y(\cdot) - \Delta^\lambda.$$

Theorem
For f bounded by an m-degree polynomial ($m \geq 0$):

$$\mathbb{E}f(\tilde{Q}^\lambda(\infty)) - \mathbb{E}f(\tilde{Y}^\lambda(\infty)) = \mathcal{O}(\sqrt{\lambda}^{m-1}).$$

- **Accuracy**: higher than heavy-traffic *limits*

Universal Approximation: Why 2λ?

- Semi-martingale representation of the B&D process: Fluid + Martingale
- Predictable quadratic variation:

$$\int_0^t [\lambda + \mu(Q_s \wedge n) + \theta(Q_s - n)^+] ds$$

- In steady-state, arrival rate $\equiv$ departure rate:

$$\lambda = \mathbb{E}[\mu(Q_s \wedge n) + \theta(Q_s - n)^+]$$

- Expectation of the predictable quadratic variation:

$$\mathbb{E} \int_0^t [\lambda + \mu(Q_s \wedge n) + \theta(Q_s - n)^+] ds = 2\lambda t$$

- dMartingale$_t \approx \sqrt{2\lambda} \cdot$ dBrownian$_t$

Reconciling Steady-State and Time-Varying Models

- **Challenge**: Accommodate time-varying demand (routine)
- **Prerequisite**: **Flexible Capacity**
  - As in Call Centers and to a degree in Healthcare,
  - In contrast to **rigid** (fixed) staffing level during a shift: doomed to alternate between overloading and underloading

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- **History**:
  - **Jennings, M., Reiman, Whitt** (1996): Emergence of the phenomenon, via infinite-server heuristics
  - **Feldman, M., Massey, Whitt** (2008): Stabilize delay probability via QED staffing (justified theoretically only for Erlang-A with $\mu = \theta$)
  - **Liu and Whitt** (ongoing): Stabilize abandonment probability by ED staffing, via a corresponding network, theoretically and empirically
  - **Huang, Gurvich, M.** (ongoing): QED theory

The Offered-Load $R(t), t \geq 0 \quad (R(t) \leftrightarrow R)$

Empirically (in SEEStat):

- Process: $\boldsymbol{L(\cdot)}$ = **Least** number of **servers** that guarantees **no delay**.
- **Offered-Load** Function $\boldsymbol{R(\cdot)} = E[L(\cdot)]$

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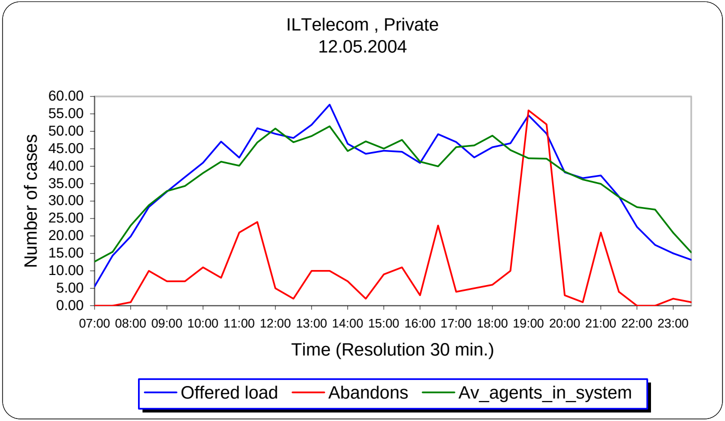


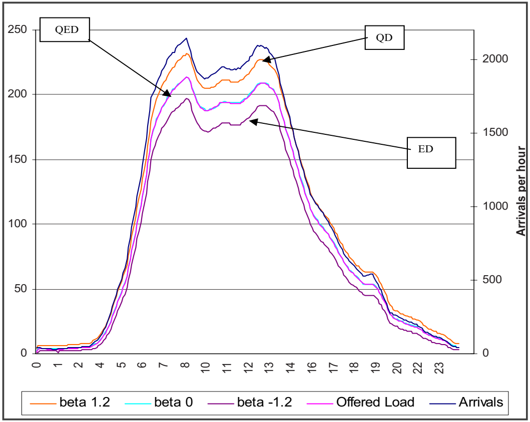
Time-Varying Arrival Rates

Square-Root Staffing:

$$N(t) \;=\; R(t) \;+\; \beta\sqrt{R(t)}\,, \quad -\infty < \beta < \infty.$$

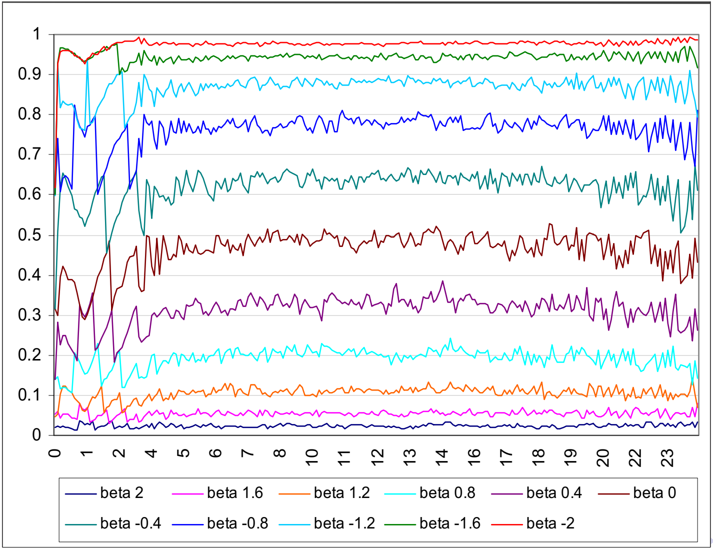
$R(t)$ is the **Offered-Load** at time t $\quad$ ($R(t) \neq \lambda(t) \times$ E[S])

Arrivals, Offered-Load and Staffing



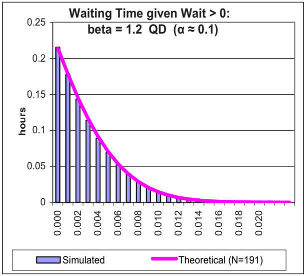
Time-Stable Performance of Time-Varying Systems

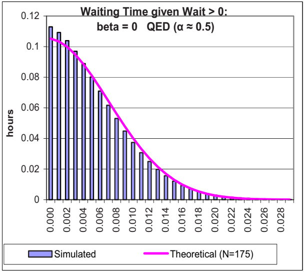
Delay Probability = as in the **Stationary Erlang-A / R**

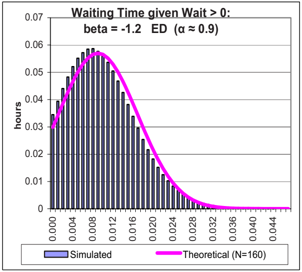


Time-Stable Performance of Time-Varying Systems

Waiting Time, Given Waiting: Empirical vs. Theoretical Distribution







- **Empirical**: Simulate **time-varying** $M_t/M/N_t + M$ $(\lambda(t), N(t) = R(t) + \beta\sqrt{R(t)})$
- **Theoretical**: Naturally-corresponding **stationary** Erlang-A, with QED β-staffing (some **Averaging** Principle?)
- **Generalizes** up to a single-station within a complex network (eg. Doctors in an Emergency Department, modeled as Erlang-R).

Calculating the Offered-Load $R(t)$, Theoretically

- Offered-Load Process: $\boldsymbol{L(\cdot)}$ = **Least** number of **servers** that guarantees **no delay**.
- **Offered-Load** Function $\boldsymbol{R(t)} = E[L(t)],\ t \geq 0$.

  Think $M_t/G/N_t^? + G$ vs. $\boldsymbol{M_t/G/\infty}$: **Ample-Servers**.

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Four (all useful) representations, capturing **"workload before t"**:

$$R(t) = E[L(t)] = \int_{-\infty}^{t} \lambda(u) \cdot P(S > t - u)du = E\Big[A(t) - A(t - S)\Big] =$$

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QED-c: $\boldsymbol{N_t = R_t + \beta R_t^c}$, $1/2 \leq c < 1$; ($c = 1$ separate analysis).

Extending the Notion of the “Offered-Load"

- **Business** (Banking Call-Center): Offered **Revenues**
- **Healthcare** (Maternity Wards): Fetus in stress
  - 2 patients (Mother + Child) = high **operational** and **cognitive** load
  - Fetus dies $\Rightarrow$ **emotional** load dominates
- $\Rightarrow$
  - Offered **Operational** Load
  - Offered **Cognitive** Load
  - Offered **Emotional** Load
  - $\Rightarrow$ **Fair** Division of Load (Routing) between 2 Maternity Wards: One attending to complications <u>before</u> birth, the other to complications <u>after</u> birth, and both share normal birth

ServNets: Data-Based Online Automatic Creation

w/ V. Trofimov, E. Nadjharov, I. Gavako = Technion SEELab

- **ServNets = QNets, SimNets, FNets, DNets**
- **SimNets** of Service Systems = **Virtual Realities**
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- **Why not in Service Science / Engineering / Management ?**

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- **Moreover, address the Reproducibility and Proprietary Crisis in Scientific Research**

Data-Based Creation ServNets: some Technicalities

- ServNets = **QNets, SimNets, FNets, DNets**
- **Graph Layout**: Adapted from but significantly extends Graphviz (AT&T, 90's); eg. *edge-width*, which must be restricted to *poly-lines*, since there are “no parallel Bezier (Cubic) curves $(B_n(p) = E_p F[B(n,p)], 0 \leq p \leq 1)$
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- Draws data directly from **SEELab** data-bases:
  - Relational DBs (Large! eg. USBank Full Binary = 37GB, Summary Tables = 7GB)
  - Structure: Sequence of events/states, which (due to size) partitioned (yet integrated) into days (eg. call centers) or months (eg. hospitals)
  - Differs from industry DBs (in call centers, hospitals, websites)