# Uncertainty in the Demand for Service: The Case of Call Centers and Emergency Departments

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March 16, 2009

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# Outline

# Motivation

**Standard assumption in service system modeling:** arrival process is Poisson with known parameters.

**Example of call centers:** known arrival rates for each basic interval (say, half-hour).

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- Derive Poisson parameters from historical data.
- Plug parameters into a queueing model ($M|M|n$, $M|M|n+M$, Skills-Based Routing models, ...).
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**Is standard Poisson assumption valid? As a rule it is not**, one observes larger variability of the arrival process than the one expected from the Poisson hypothesis.

# Research Outline

- Design model for overdispersed arrival rate.
- Plug arrival model into $M|M|n+G$ queueing model.
- Derive asymptotic results relevant for real-life staffing problems.
- Validate our approach via analysis of real data.

# Related Work


Henderson S.
*Input model uncertainty: Why do we care and what should we do about it?*. 2003.

Steckley S., Henderson S. and Mehrotra V.
*Forecast errors in service systems*. 2007.

Koole G. and Jongbloed G.
*Managing uncertainty in call centers using poisson mixtures*. 2001.

Halfin S. and Whitt W.
*Heavy-traffic limits for queues with many exponential servers*. 1981.

Zeltyn S. and Mandelbaum A.
*Call centers with impatient customers: Exact analysis and many-server asymptotics of the $M|M|n+G$ queue*. 2005.

Feldman Z., Mandelbaum A., Massey W. A. and Whitt W.
*Staffing of time-varying queues to achieve time-stable performance*. 2007.

# Model Definition

**The $M^?|M|n+G$ Queue:**

- $\lambda$ - **Expected** arrival rate of a Poisson arrival process.
- $\mu$ - Exponential service rate.
- $n$ service agents.
- $G$ - Patience distribution. Assume that the patience density exists at the origin and its value $g_0$ is strictly positive.

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**Random Arrival Rate:** Let $X$ be a random variable with cdf $F$, $E[X]=0$, and finite $\sigma(X)$. Assume that the arrival rate varies from interval to interval in an i.i.d. fashion:

$$\Lambda \;=\; \lambda \;+\; \lambda^c X, \quad c \leq 1,$$

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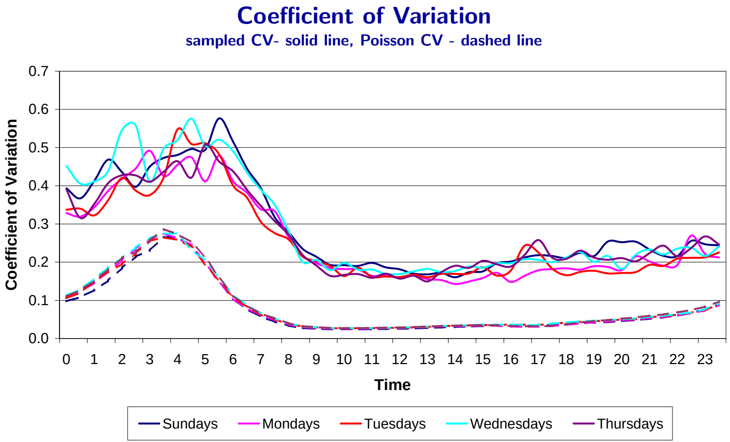
$$\Lambda = \lambda + \lambda^c X, \quad c \le 1,$$

- $c \le 1/2$: Conventional variability $\sim$ QED staffing regime.
- $1/2 < c < 1$: Moderate variability $\sim$ QED-c regime (**new**).
- $c = 1$: Extreme variability $\sim$ ED regime.

# Financial Call Center: Data Description

- Israeli Bank.
- Arrival counts to the Retail queue are studied.
- 263 regular weekdays ranging between April 2007 and April 2008.
- Holidays with different daily patterns are excluded.
- Each day is divided into 48 half-hour intervals.

# Financial Call Center: Over-Dispersion Phenomenon



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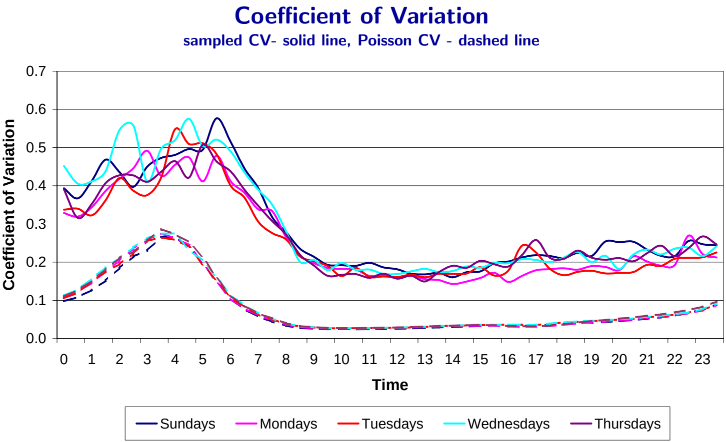


Poisson CV $= 1/\sqrt{\text{mean arrival rate}}$.

Sampled CV's $\gg$ Poisson CV's

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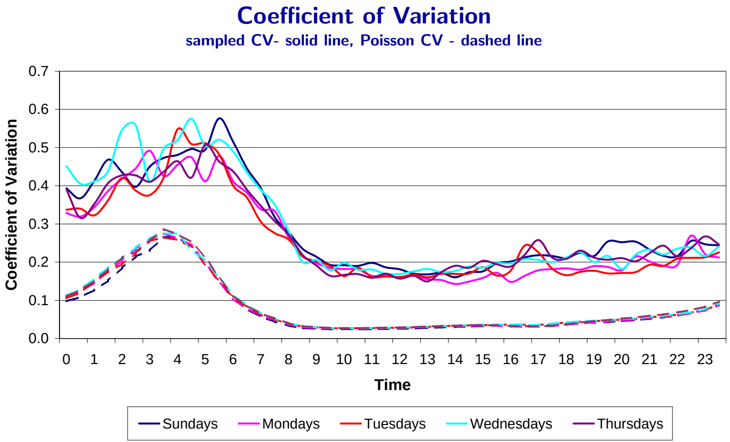


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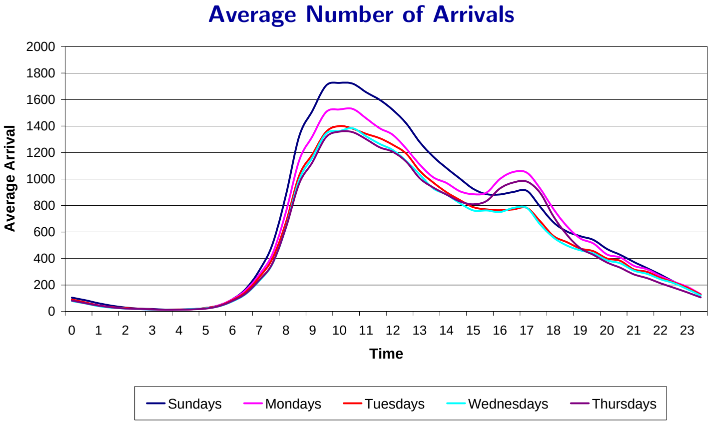
Sampled CV's $\gg$ Poisson CV's $\Rightarrow$ **Over-Dispersion**

# Financial Call Center: Arrival Rates



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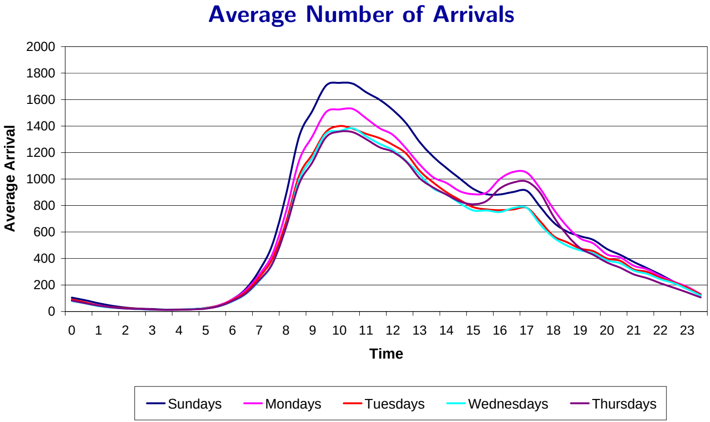


**(1)** Sundays;

**(2)** Mondays;

**(3)** Tuesdays and Wednesdays;

**(4)** Thursdays;

# Financial Call Center: Relation between Mean and Standard Deviation

Consider a Poisson mixture variable $Y$ with random rate $\Lambda = \lambda + \lambda^c \cdot X$, where $E[X] = 0$, finite $\sigma(X) > 0$ and $1/2 < c \leq 1$. Then,

$$Var(Y) = \lambda^{2c} \cdot Var(X) + \lambda + \lambda^c \cdot E(X)$$

and

$$\lim_{\lambda \to \infty} (\ln(\sigma(Y)) - c \ln(\lambda)) = \ln(\sigma(X)).$$

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and

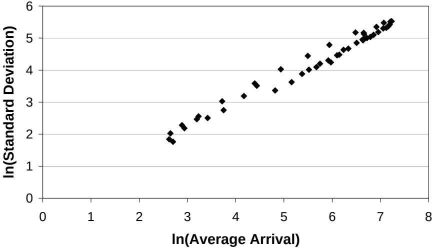
$$\lim_{\lambda \to \infty} (\ln(\sigma(Y)) - c \ln(\lambda)) \; = \; \ln(\sigma(X)).$$

Therefore, for large $\lambda$,

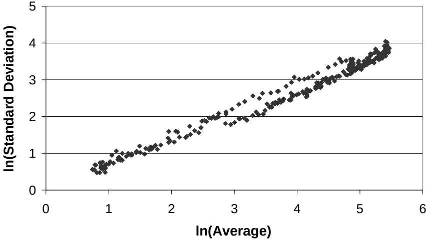
$$\ln(\sigma(Y)) \; \approx \; c \cdot \ln(\lambda) + \ln(\sigma(X)).$$

# Financial Call Center: Fitting Regression Model

Tue-Wed, 30 min resolution




Tue-Wed, 5 min resolution



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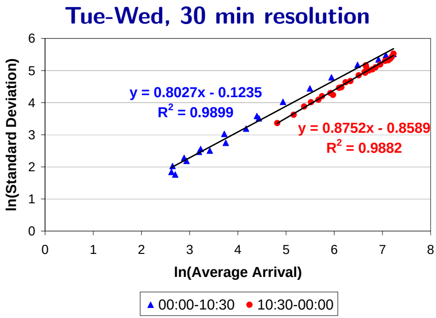
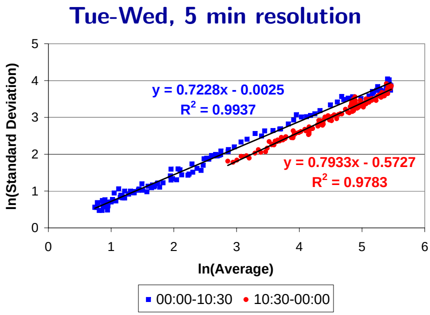



**Results:**

- Two clusters exist: midnight-10:30am and 10:30am-midnight.
- Very good fit ($R^2 > 0.97$).
- Significant linear relations for different weekdays and time-resolution (5-30 min):

$$\ln(\sigma(Y)) = c \cdot \ln(\lambda) + \ln(\sigma(X)).$$

# Financial Call Center: Gamma Poisson Mixture Model

(Jongbloed and Koole ['01])

Assume a prior Gamma distribution for the arrival rate

$$\Lambda \stackrel{d}{=} Gamma(a, b),$$

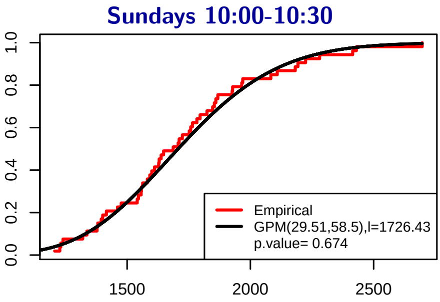
and denote $E[\Lambda] \stackrel{\triangle}{=} \lambda$.

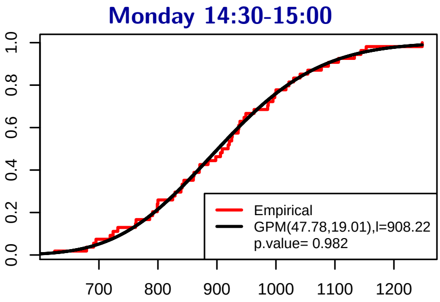
Then, the distribution of $Y \stackrel{d}{=} Poisson(\Lambda)$ is Negative Binomial.

1. Maximum likelihood estimators of $a$ and $b$.
2. Goodness of fit test including FDR control method to correct the multiple comparisons.

# Financial Call Center: Gamma Poisson Mixture Model

$$\mathrm{H}_0 : \; \Lambda_\lambda \stackrel{d}{=} Gamma(a_\lambda, b_\lambda)$$



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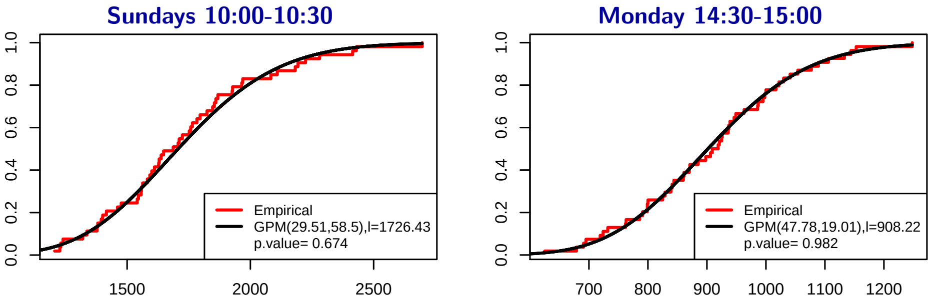



**Results:**

- Very good fit.
- Only 13 hypotheses are rejected (out of 192).

# Financial Call Center: Outline of Additional Results

$$\lambda + \lambda^c \cdot X_\lambda \stackrel{d}{=} Gamma(a_\lambda, b_\lambda)$$

Under Gamma assumption and convergence of $\sigma(X_\lambda)$ to a constant $\sigma(X)$:

- Relation between our main model and Gamma Poisson mixture model is established.
  Significant linear relations:

$$\ln(b_\lambda) \;=\; (2c-1)\cdot \ln(\lambda) + \ln(\sigma^2(X)).$$

- Distribution of $X$ is derived. It is **asymptotically normal** given $\lambda \to \infty$:

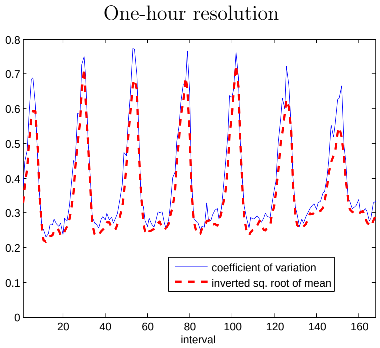
$$X_\lambda \xrightarrow{\mathcal{D}} Norm(0, \sigma^2(X))$$

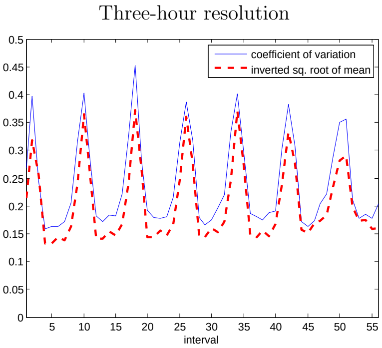
# Emergency Department: Data Description

- Israeli Emergency Department.
- 194 weeks between from January 2004 till October 2007 (five war weeks are excluded from data).
- The analysis is performed using two resolutions: hourly arrival rates (168 intervals in a week) and three-hour arrival rates (56 intervals in a week).
- Holidays are not excluded (results with excluded holidays are similar).

# Emergency Department: Over-Dispersion Phenomenon

## Coefficient of Variation



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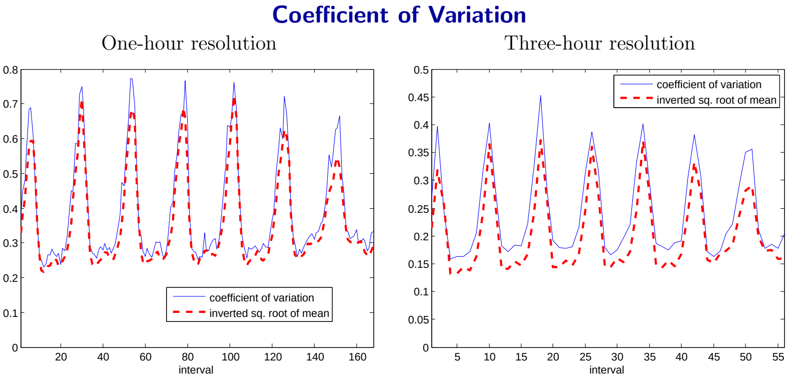


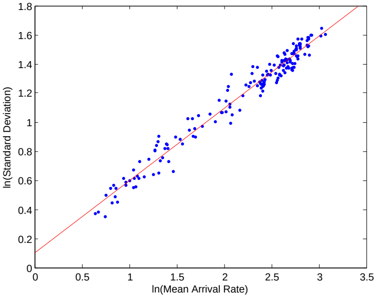

- Moderate over-dispersion.
- $c = 1/2$ seems to be reasonable assumption for hourly resolution.

# Emergency Department: Fitting Regression Model

## ln($\sigma$) versus ln($\lambda$) plots

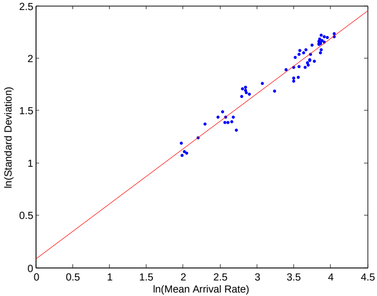
One-hour resolution

$y = 0.497x + 0.102,\ R^2 = 0.968$



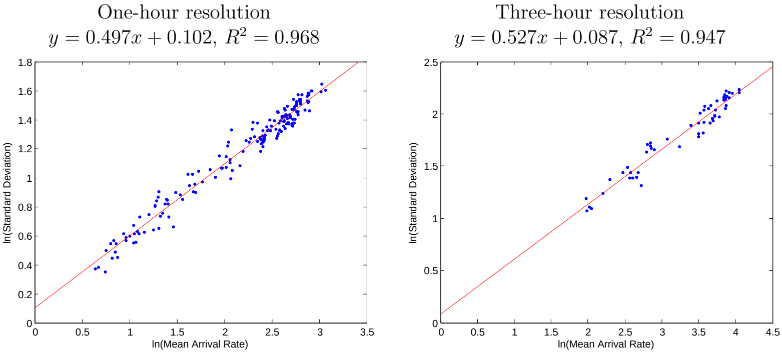

Three-hour resolution

$y = 0.527x + 0.087,\ R^2 = 0.947$



# Emergency Department: Fitting Regression Model

## $\ln(\sigma)$ versus $\ln(\lambda)$ plots




- A linear pattern with the slope that is very close to 0.5 (derivation of asymptotic relation is based on $c > 1/2$).
- Overdispersion is observed at daily level $\Rightarrow$ scaling problem should be studied. (Dependence of $c$ on the interval length.)

# QED-$c$ Regime: Fixed Arrival Rate

## QED-$c$ staffing rule:

$$n \;=\; \frac{\lambda}{\mu} + \beta\left(\frac{\lambda}{\mu}\right)^{c} + o(\sqrt{\lambda}), \quad \beta \in \mathbb{R}, \;\; c \in (1/2, 1).$$

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Assume an $M|M|n+G$ queue with **fixed arrival rate** $\lambda$.
Take $\lambda$ to $\infty$:

- $\beta > 0$: Over-staffing.
- $\beta < 0$: Under-staffing.

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For both cases we provide asymptotically equivalent expressions (or bounds) for $P\{W_q > 0\}$, $P\{Ab\}$ and $E[V]$, where
$W_q$ - waiting time, $V$ - offered wait (wait given infinite patience).

**Proofs:** based on $M|M|n+G$ building blocks from Zeltyn and Mandelbaum['05], carried out via the Laplace Method for asymptotic calculation of integrals.

# QED-$c$ Regime: Fixed Arrival Rate

## Square-Root Staffing versus QED-$c$ staffing

| $R = \frac{\lambda}{\mu}$ | $\beta$ | SRS[1] ($c = 1/2$) | QED-c Staffing $c = 0.6$ | | $c = 0.75$ | | $c = 0.9$ | |
|---|---|---|---|---|---|---|---|---|
| 100 | 0.5 | 105 | 108 | (+3%) | 116 | (+10%) | 132 | (+25%) |
| | 1 | 110 | 116 | (+5%) | 132 | (+20%) | 163 | (+48%) |
| | 1.5 | 115 | 124 | (+8%) | 147 | (+28%) | 195 | (+69%) |
| 500 | 0.5 | 511 | 521 | (+2%) | 553 | (+8%) | 634 | (+24%) |
| | 1 | 522 | 542 | (+4%) | 606 | (+16%) | 769 | (+47%) |
| | 1.5 | 534 | 562 | (+5%) | 659 | (+23%) | 903 | (+69%) |
| 1000 | 0.5 | 1016 | 1032 | (+2%) | 1089 | (+7%) | 1251 | (+23%) |
| | 1 | 1032 | 1063 | (+3%) | 1178 | (+14%) | 1501 | (+46%) |
| | 1.5 | 1047 | 1095 | (+5%) | 1267 | (+21%) | 1752 | (+67%) |

[1] Square-Root-Staffing: $n = R + \beta\sqrt{R}$

# QED-$c$ Regime: Random Arrival Rate

## Theorem

Assume random arrival rate $\Lambda = \lambda + \lambda^c \mu^{1-c} X, \quad c \in (1/2, 1)$, $E[X] = 0$, finite $\sigma(X) > 0$, and staffing according to the QED-$c$ staffing rule with the corresponding $c$. Then, as $\lambda \to \infty$,

**a.** Delay probability: $P_{\Lambda,n}\{W_q > 0\} \sim 1 - F(\beta)$.[a]

**b.** Abandonment probability: $$P_{\Lambda,n}\{Ab\} \sim \frac{E[X-\beta]_+}{n^{1-c}}.$$

**c.** Average offered waiting time: $$E_{\Lambda,n}[V] \sim \frac{E[X-\beta]_+}{n^{1-c} \cdot g_0}.$$

[a] $f(\lambda) \sim g(\lambda)$ denotes that $\lim_{\lambda\to\infty} f(\lambda)/g(\lambda) = 1$.

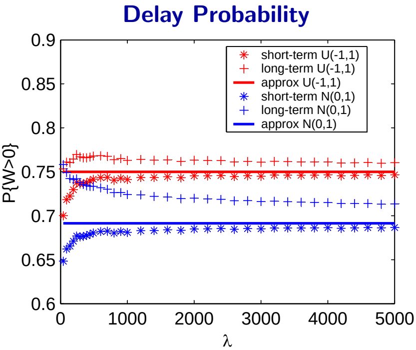
**Proofs:** based on conditioning on values of $X$ and results for QED-$c$ staffing rule.

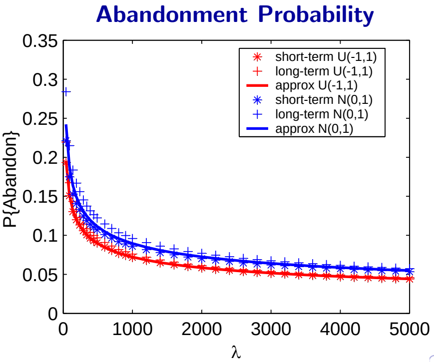
# QED-$c$ Regime: Numerical Experiments

**Examples:** Consider two distributions of $X$

- Uniform distribution on [-1,1],
- Standard Normal distribution.

$$\beta = -0.5, \;\; c = 0.7$$



# QED-$c$ Regime: Practical Guidelines

- Determine "uncertainty coefficient $c$" via regression analysis.
- Check if Gamma Poisson mixture model is reasonable.
- Assume that $X$ is asymptotically normal, calculate standard deviation from regression model.
- Apply our QED-$c$ asymptotic results in order to determine appropriate staffing.

# Constraint Satisfaction Problem

## Formulation of the Problem:

Define the **optimal staffing level** by

$$n^*_\lambda = \underset{n}{\operatorname{argmin}} \left\{ P_{\Lambda,n}\{W_q > 0\} \leq \alpha \right\}.$$

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In addition, $n_\lambda$ is **asymptotically optimal** if

$$|n^*_\lambda - n_\lambda| = o(f(\lambda)),$$

$f(\lambda)$ is defined separately for every special case..

# ED Regime: $c = 1$, Discrete Random Arrival Rate

Assume $\Lambda = \lambda X$, where $X$ is a discrete random variable which takes values $x_1 > x_2 > \ldots > x_I > 0$, with probabilities $p_1, p_2, \ldots, p_I$, respectively. In addition, let $E[X] = 1$, $\sigma(X) < \infty$ and $\lambda \to \infty$.

## Theorem

**a.** The optimal staffing level satisfies

$$n^* = \frac{\lambda x_k}{\mu} + \beta^* \sqrt{\frac{\lambda x_k}{\mu}} + o(\sqrt{\lambda});$$

where $k = \operatorname*{argmin}_s \left\{ \sum_{i=1}^{s} x_i p_i \geq \alpha \right\}$; $\alpha^* \triangleq \frac{\alpha - \sum_{i=1}^{k-1} x_i p_i}{x_k p_k}$;

and $\beta^*$ is the unique solution of the equation

$$\alpha^* = \left[ 1 + \sqrt{\frac{g_0}{\mu}} \cdot \frac{h(\beta\sqrt{\mu/g_0})}{h(-\beta)} \right]^{-1}.$$

# ED Regime: Staffing

## Theorem

**b.** Introduce the staffing level

$$n_\beta^* = \left\lceil \frac{\lambda x_k}{\mu} + \beta^* \sqrt{\frac{\lambda x_k}{\mu}} \right\rceil .$$

Then the staffing level $n_\beta^*$ is both *asymptotically feasible*, as well as *asymptotically optimal* with $f(\lambda) = \sqrt{\lambda}$.

# ED Regime: Performance Measures

**Theorem**

Under the staffing level $n^*_\beta$, as $\lambda \to \infty$,

**a.** The abandonment probability:

$$P_{\Lambda,n^*_\beta}\{Ab\} \;\sim\; \sum_{i=1}^{k-1} p_i\,(x_i - x_k).$$

**b.** Mean server's utilization:

$$E_{\Lambda,n^*_\beta}[U] \;\sim\; \sum_{i=1}^{k} p_i + \sum_{i=k+1}^{I} p_i \cdot \frac{x_i}{x_k}.$$

**c.** Assume that for each $i < k$ the equation $G(y) = 1 - \frac{x_k}{x_i}$ has a unique solution $y_i^*$, and $g(y_i^*) > 0$. Then, the average waiting time

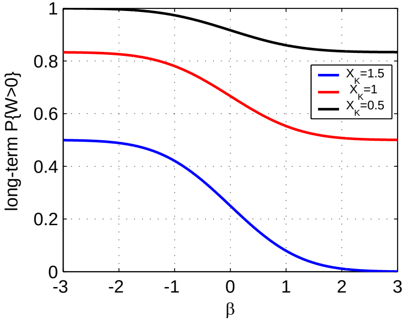
$$E_{\Lambda,n^*_\beta}[W_q] \;\sim\; \sum_{i=1}^{k-1} \left[ x_i p_i \cdot \int_0^{y_i^*} \bar{G}(u)du \right].$$

# ED Regime: Numerical Experiment

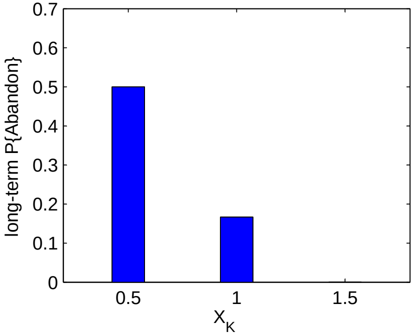
**Example:** $X$ takes the values 1.5, 1 and 0.5, with probability $1/3$ for each.

$$n = \left\lceil \frac{\lambda x_k}{\mu} + \beta\sqrt{\frac{\lambda x_k}{\mu}} \right\rceil$$

**Delay Probability**

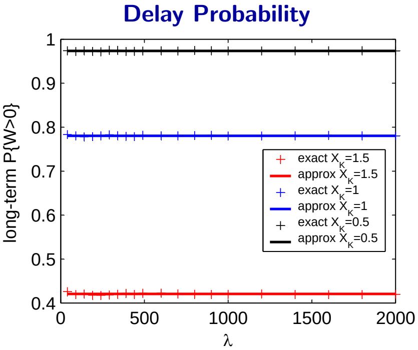


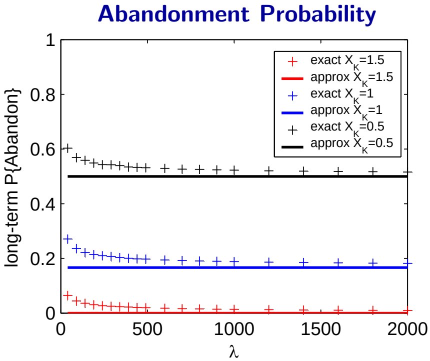

**Abandonment Probability**



# ED Regime: Numerical Experiment

$$\beta = -1$$



# Outline of Additional Results

- **Queueing Theory.** Asymptotic performance measures derived and constraint satisfaction problems solved for:
  - QED regime ($c = 1/2$).
  - ED regime ($c = 1$), continuous distribution of $X$.
- **Numerical Experiments.** Very good fit between asymptotic results and the exact ones.

# Time - Varying Queues

- **Iterative Staffing Algorithm (ISA)**, a simulation code developed by Feldman et al. ['07] with the features of random arrival rate in the time-varying $M|M|n + G$ queue.
- **Goal:** determine time-dependent staffing levels aiming to achieve a time-stable delay probability.

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- **Goal:** determine time-dependent staffing levels aiming to achieve a time-stable delay probability.
- **Example:** $c=1$, Discrete Random Arrival Rate
  1. $\Lambda(t) = \lambda(t) \cdot X$, where

     $$\lambda(t) = 100 - 20 \cdot \cos(t), \quad \text{and} \quad X = \begin{cases} 1.5 & w.p.\ 1/3 \\ 1 & w.p.\ 1/3 \\ 0.5 & w.p.\ 1/3 \end{cases}$$

  2. Service time and patience are distributed exponentially with mean 1 ($\mu = \theta = 1$).

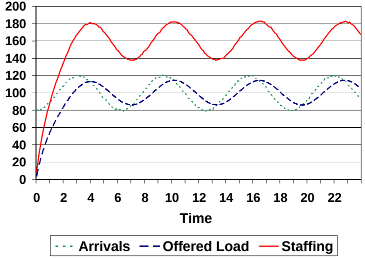
# Time - Varying Queues

## Arrivals, Offered Load and Staffing Level

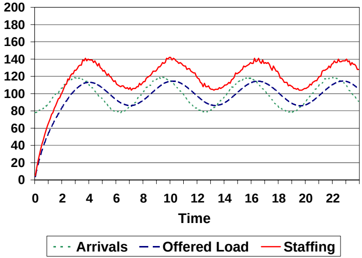
$$\mathbf{R_t} = E\left[\int_{t-S}^{t} \lambda(u)du\right]$$

$$\mathbf{R_t^X} = E\left[E\left[\int_{t-S}^{t} (\lambda(u)\cdot X)du\right]\right] = \mathbf{R_t}.$$

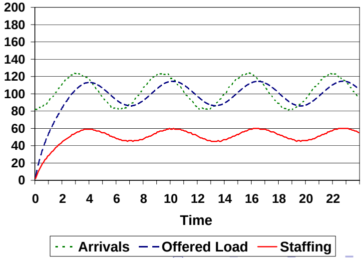
### Target $\alpha$=0.1




### Target $\alpha$=0.5

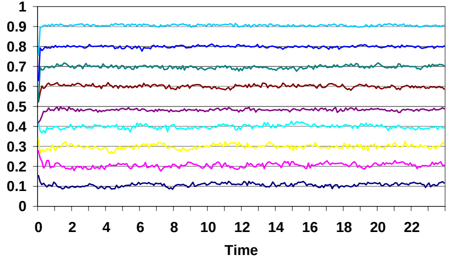



### Target $\alpha$=0.9

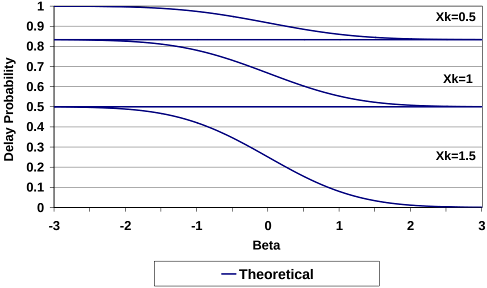


# Time - Varying Queues

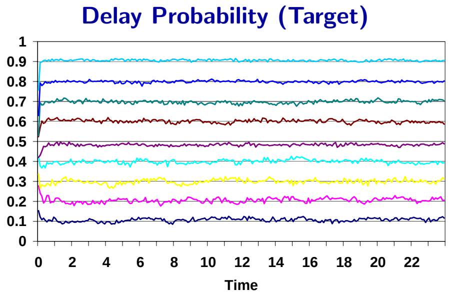
## Delay Probability (Target)



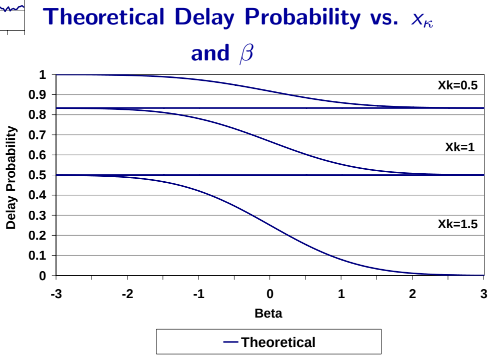

## Theoretical Delay Probability vs. $x_\kappa$ and $\beta$



# Time - Varying Queues

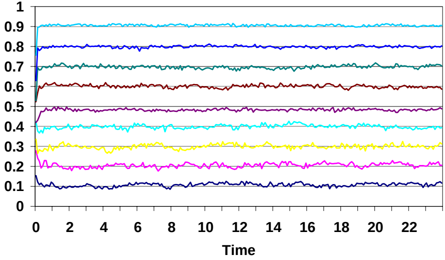



Stable delay probability



# Time - Varying Queues

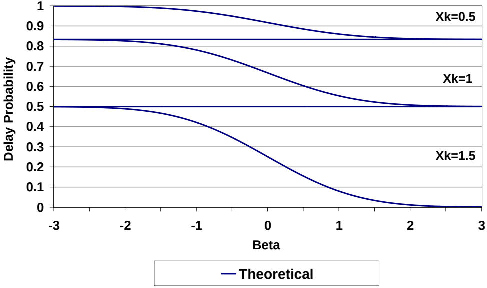
## Delay Probability (Target)




$$n = R \cdot x_k + \beta\sqrt{R \cdot x_k}, \quad R = \lambda/\mu$$

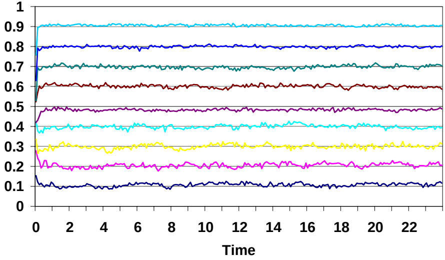
Stable delay probability

## Theoretical Delay Probability vs. $x_\kappa$ and $\beta$



# Time - Varying Queues

## Delay Probability (Target)

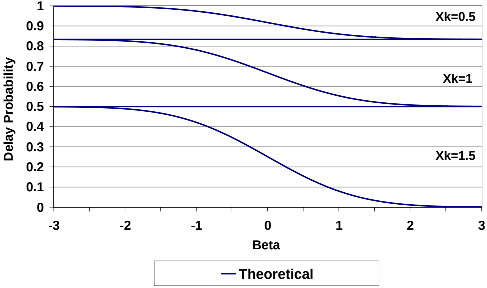



Stable delay probability

$$n = R \cdot x_k + \beta\sqrt{R \cdot x_k}, \quad R = \lambda/\mu$$

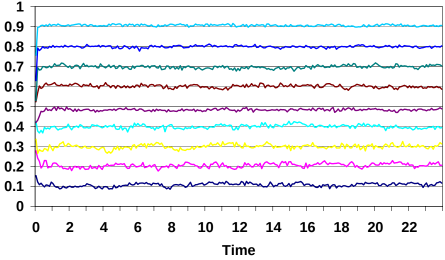
$$n_t \;=\; R_t \cdot x_k + \beta_t\sqrt{R_t \cdot x_k} \quad ?$$

## Theoretical Delay Probability vs. $x_\kappa$ and $\beta$



# Time - Varying Queues

### Delay Probability (Target)



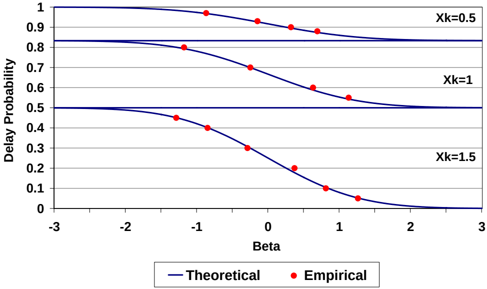

Stable delay probability

$$n = R \cdot x_k + \beta\sqrt{R \cdot x_k}, \quad R = \lambda/\mu$$

$$n_t \;=\; R_t \cdot x_k + \beta_t\sqrt{R_t \cdot x_k} \quad ?$$

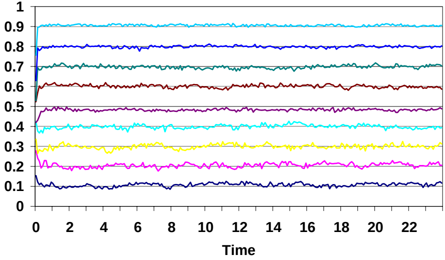
**Stable** $\beta_t = \dfrac{n_t - R_t \cdot x_k}{\sqrt{R_t \cdot x_k}} = \beta$

### Theoretical and Empirical Delay Probability vs. $x_\kappa$ and $\beta$



# Time - Varying Queues

**Delay Probability (Target)**



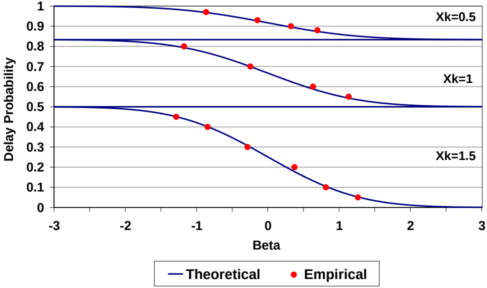

Stable delay probability

$$n = R \cdot x_k + \beta\sqrt{R \cdot x_k}, \quad R = \lambda/\mu$$

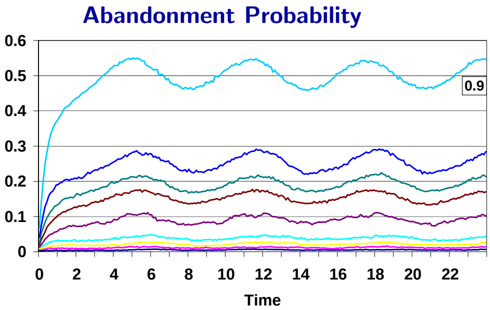
$$n_t = R_t \cdot x_k + \beta\sqrt{R_t \cdot x_k} \quad \checkmark$$

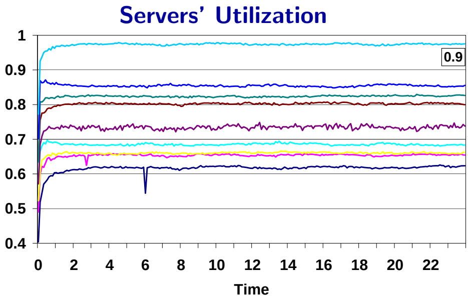
**Stable** $\beta_t = \dfrac{n_t - R_t \cdot x_k}{\sqrt{R_t \cdot x_k}} = \beta$

**Theoretical and Empirical Delay Probability vs. $x_\kappa$ and $\beta$**

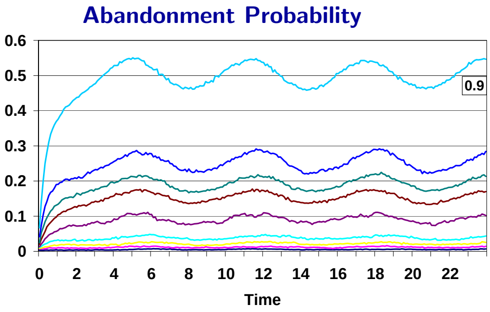


# Time - Varying Queues

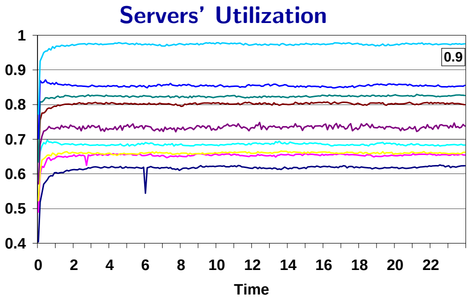


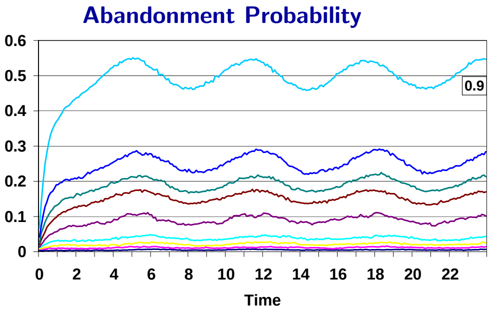
# Time - Varying Queues




For large values of the target delay probability the abandon probability become less time-stable.

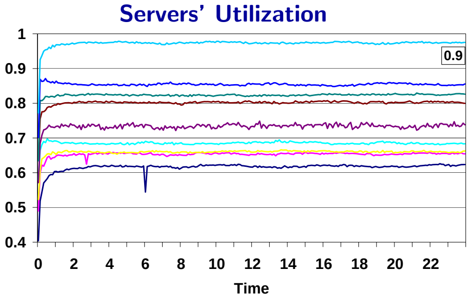


# Time - Varying Queues




For large values of the target delay probability the abandon probability become less time-stable.

Servers utilizations are found to be quite stable.



# Future Research Challenges

- Incorporating **forecasting errors** into our model (in the spirit of Steckley et al., 2007).
- **Scaling problem:** dependence of $c$ on the basic interval duration.
- **ISA:** achieving time-stable performance measures (probability to abandon, average wait).
- Validation of $M^?|M|n+M$ (or $M^?|M|n+G$) model in call center environment (and probably other service systems).

# Thank You