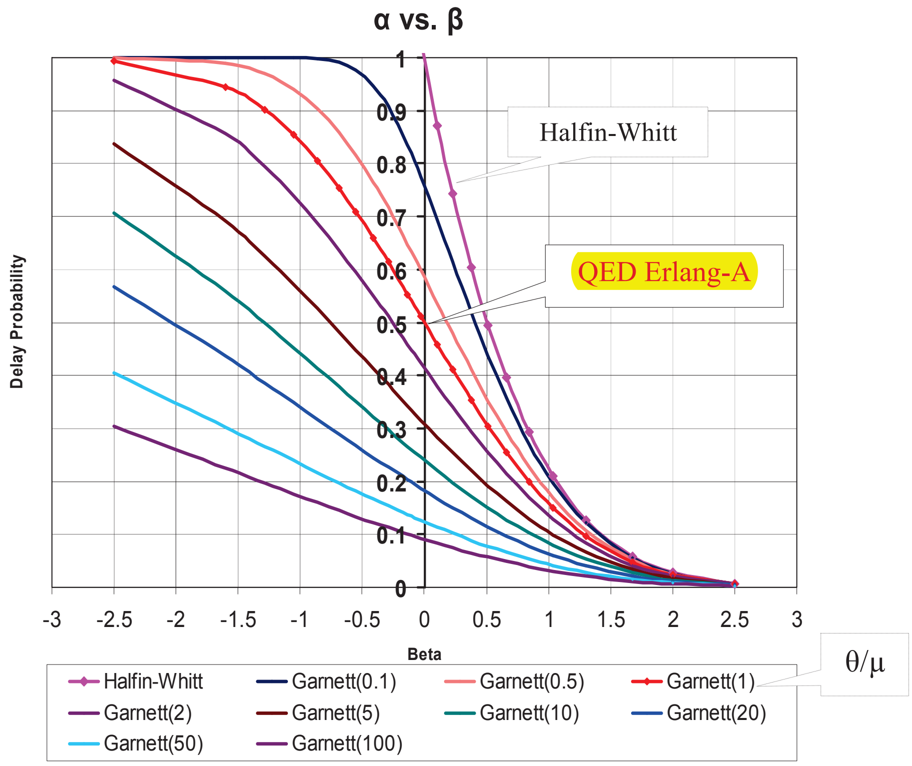
HW/GMR Delay Functions

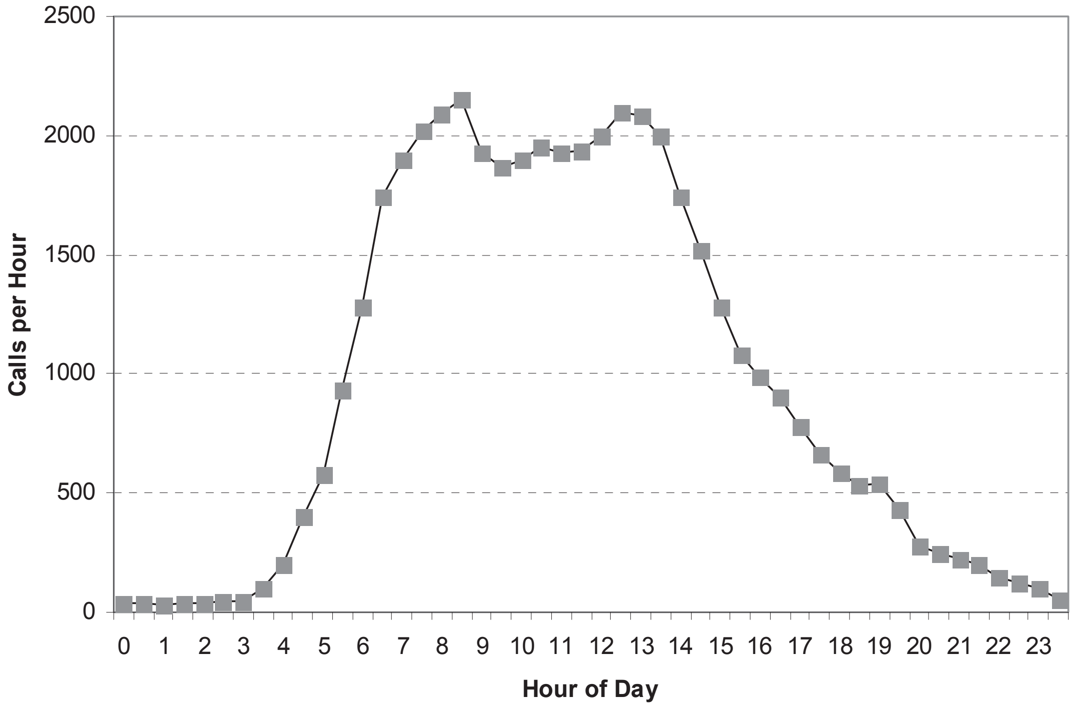


Example: "Real" Call Center

(The "Right Answer" for the "Wrong Reasons")

Time-Varying (two-hump) arrival functions common

(Adapted from Green L., Kolesar P., Soares J. for benchmarking.)



Assume: Service and abandonment times are both

Exponential, with mean 0.1 (6 min.)

Time-Varying Arrivals

Model	$M_t / M / N_t + M$
Parameters	$\lambda(t)$ μ **?** θ

? $N_t = R_t + \beta\sqrt{R_t}$

Time-Varying Arrivals

Model $M_t / M / N_t + \mathrm{M}$

Parameters $\lambda(t)$ μ **?** θ

? $N_t = R_t + \beta\sqrt{R_t}$

Offered Load: $R_t = E\lambda(t-S)\cdot E(S) = E\int_{t-S}^{t}\lambda(u)du$

Average # in $M_t / M / \infty$

Time-Varying Arrivals

Model $M_t / M / N_t + \mathrm{M}$

Parameters $\lambda(t)$ μ **?** θ

? $\mathrm{N_t = R_t + \beta\sqrt{R_t}}$

Offered Load: $R_t = E\lambda(t-S)\cdot E(S) = E\int_{t-S}^{t}\lambda(u)du$

Average # in $M_t / M / \infty$

Gives rise to TIME-STABLE PEFORMANCE

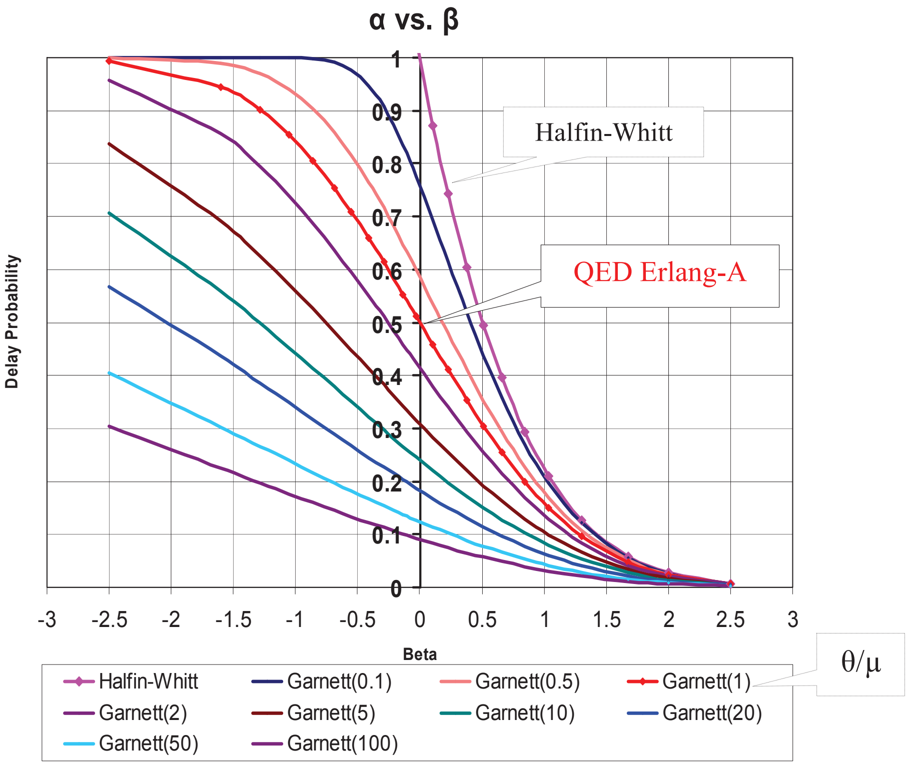
(Why? Think $M_t / M / N_t + \mathrm{M}$ with $\mu = \theta$;

And if $\mu \neq \theta$, or generally:

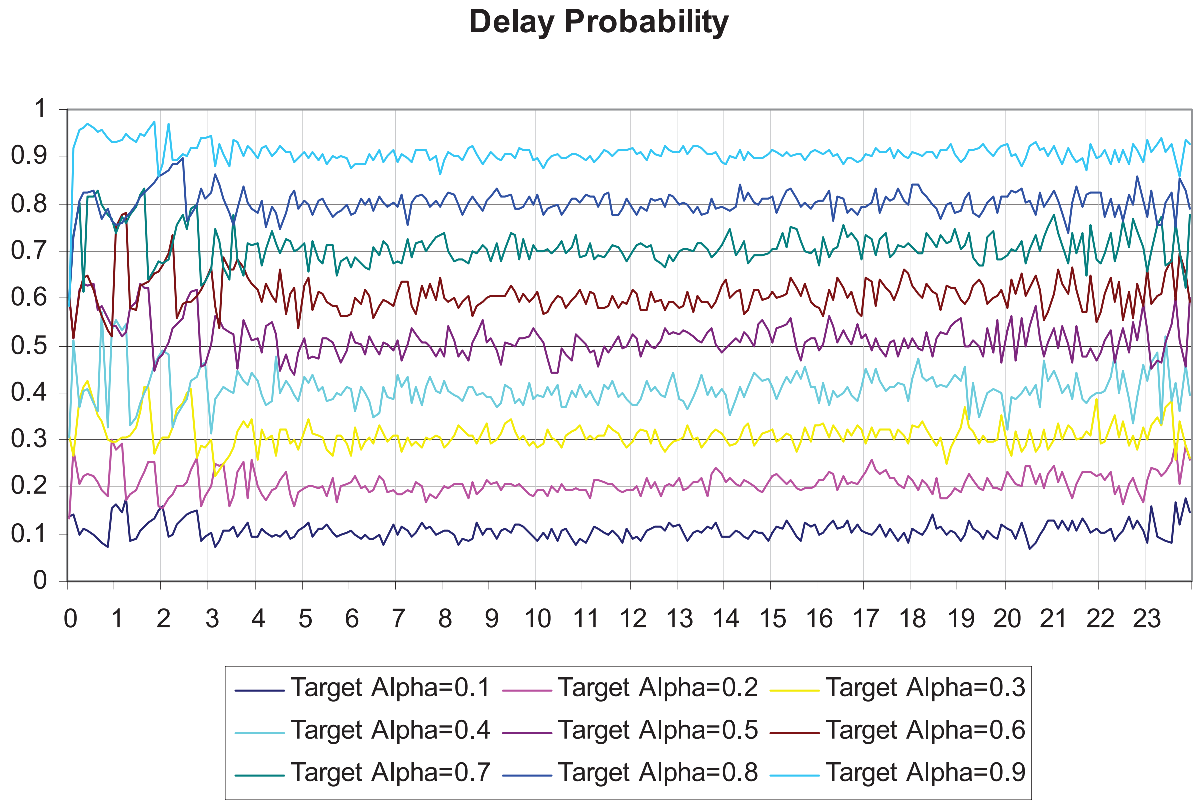
use the Iterative Simulation-Based Staffing Algorithm

in Feldman, M., Massey and Whitt, 2005.)

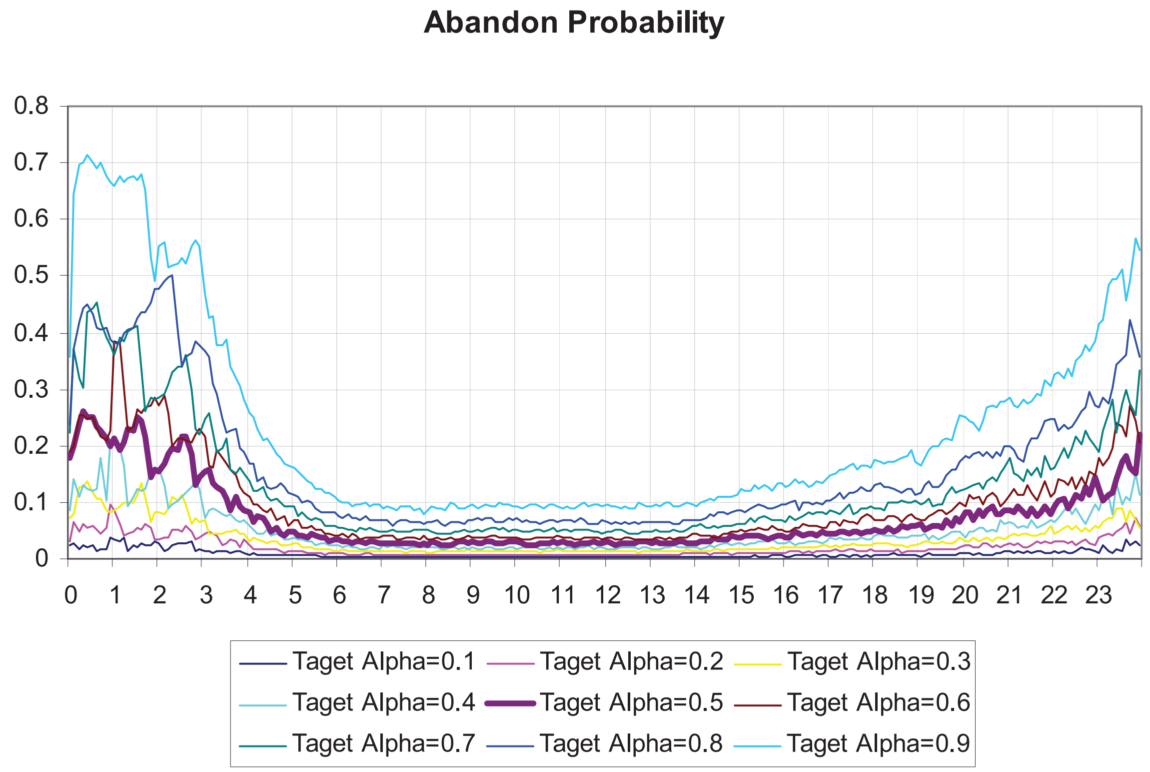
HW/GMR Delay Functions

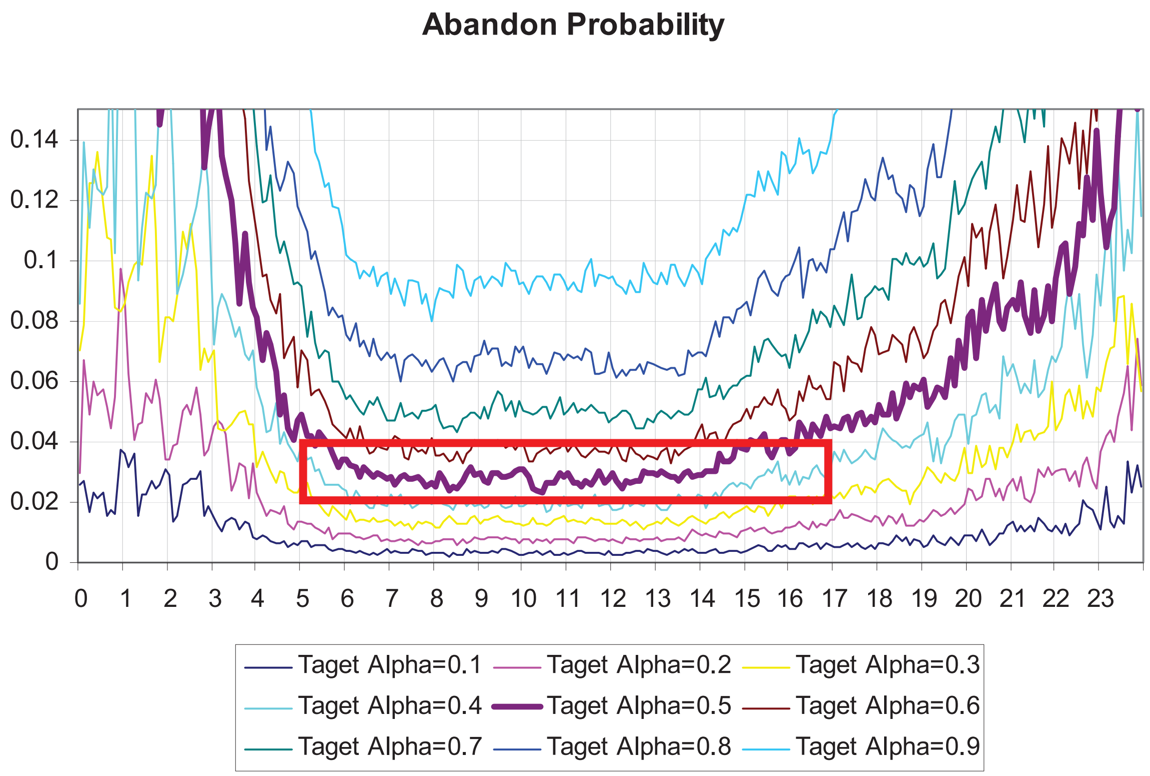


Delay Probability α



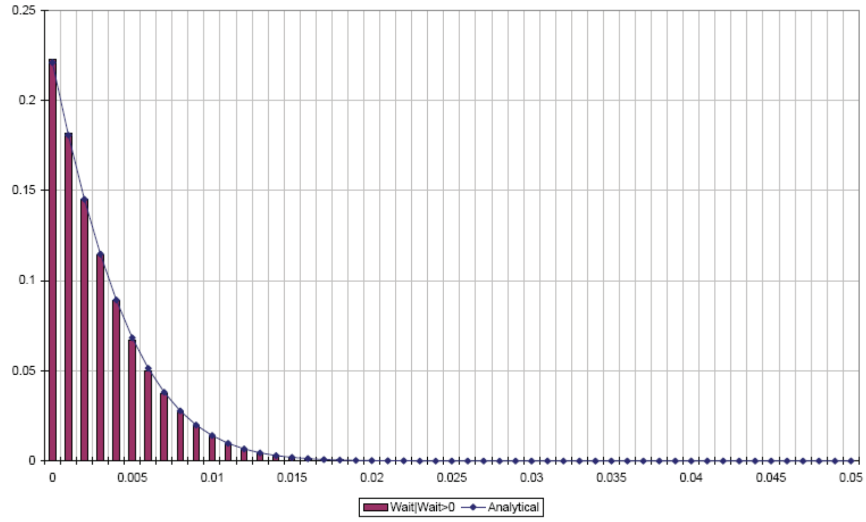
Abandon Probability

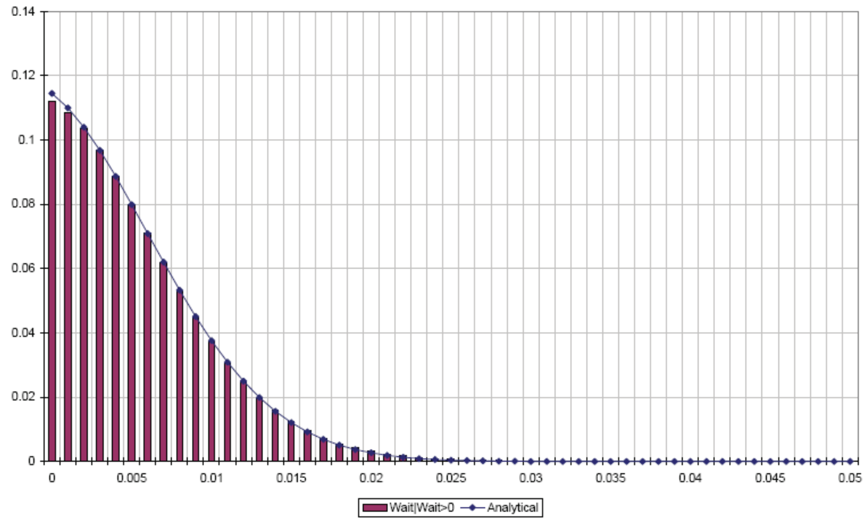


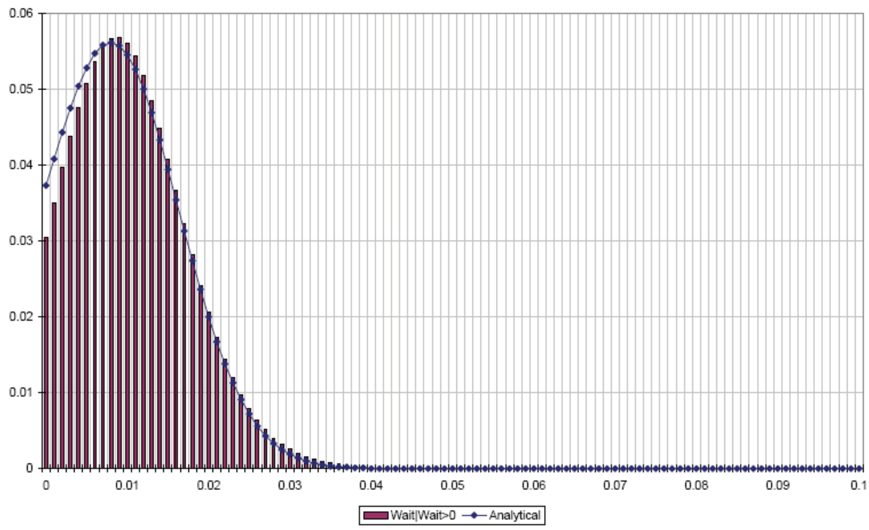


Real Call Center: Empirical waiting time, given positive wait

(1) α=0.1 (*QD*) **(2)** α=0.5 (*QED*) **(3)** α=0.9 (*ED*)







The "Right Answer" (for the "Wrong Reasons")

Prevalent Practice $$N_t = \lceil \lambda(t) \cdot E(S) \rceil$$ (PSA)

"Right Answer" $$N_t \approx R_t + \beta \cdot \sqrt{R_t}$$ (MOL)

$$R_t = E\lambda(t-S) \cdot E(S)$$

The "Right Answer" (for the "Wrong Reasons")

Prevalent Practice $\quad N_t = \lceil \lambda(t) \cdot E(S) \rceil \quad$ (PSA)

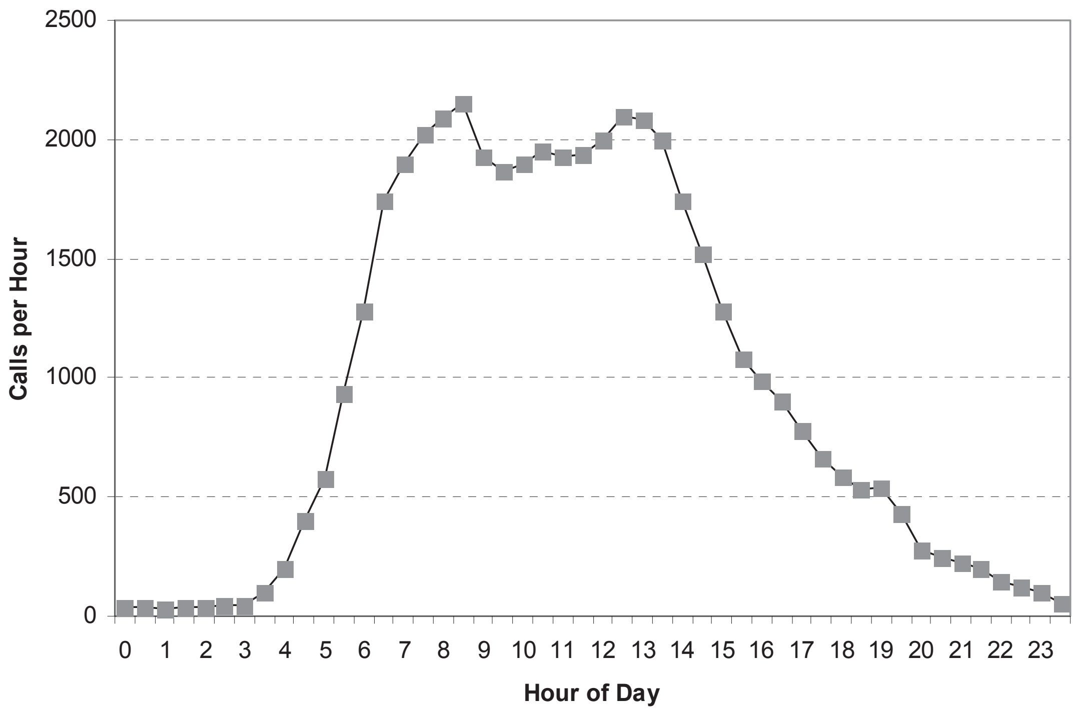
"Right Answer" $\quad N_t \approx R_t + \beta \cdot \sqrt{R_t} \quad$ (MOL)

$$R_t = E\lambda(t - S) \cdot E(S)$$

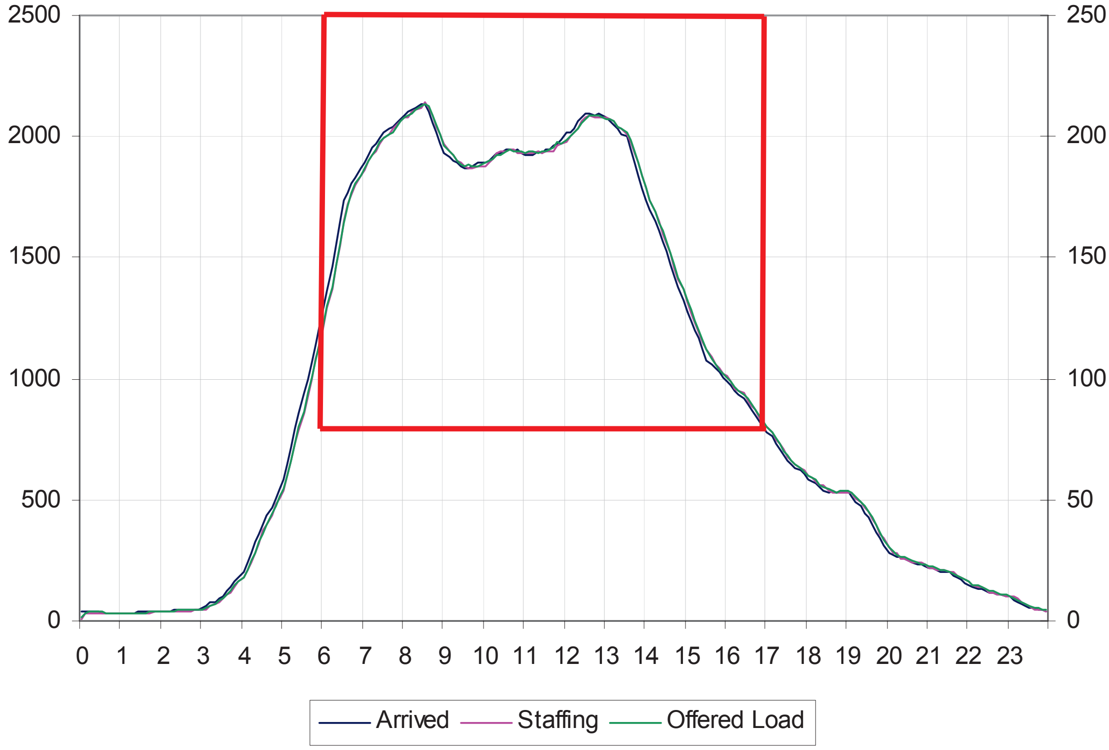
Practice $\approx$ "Right" $\quad \beta \approx 0 \quad$ (QED)

and $\quad \lambda(t) \approx$ stable over service-durations

Practice Improved $\quad N_t = \lceil \lambda[t - E(S)] \cdot E(S) \rceil$



QED Staffing (β=0 iff α=0.5)



The "Right Answer" (for the "Wrong Reasons")

Prevalent Practice $N_t = \lceil \lambda(t) \cdot E(S) \rceil$ (PSA)

"Right Answer" $N_t \approx R_t + \beta \cdot \sqrt{R_t}$ (MOL)

$$R_t = E\lambda(t - S) \cdot E(S)$$

Practice $\approx$ "Right" $\beta \approx 0$ (QED)

and $\lambda(t) \approx$ stable over service-durations

Practice Improved $\boxed{N_t = \lceil \lambda[t - E(S)] \cdot E(S) \rceil}$

When Optimal ? for moderately-patient customers:

1. Satisfization $\Leftrightarrow$ At least 50% to be serve immediately
2. Optimization $\Leftrightarrow$ Customer-Time = 2 x Agent-Salary

Time-Varying Arrivals: $\sqrt{\cdot}$ Safety-Staffing

Model	$M_t / M / N_t + \mathrm{M}$
Parameters	$\lambda(t) \quad \mu \quad \mathbf{?} \quad \theta$

? $\boxed{N_t = R_t + \beta\sqrt{R_t}}$

Time-Varying Arrivals: $\sqrt{\cdot}$ Safety-Staffing

Model $\qquad M_t / M / N_t + \mathrm{M}$

Parameters $\qquad \lambda(t) \quad \mu \quad \mathbf{?} \quad \theta$

? $\qquad \mathrm{N_t} = \mathrm{R_t} + \beta\sqrt{\mathrm{R_t}}$

$\mu = \theta: \quad \mathrm{L_t} \overset{d}{=} \mathrm{Poisson}(\mathrm{R_t}) \overset{d}{\approx} \mathrm{N}(\mathrm{R_t}, \mathrm{R_t})$, since $M_t / M / \infty$

$$R_t = E\lambda(t-S) \cdot E(S) = E \int_{t-S}^{t} \lambda(u)du$$

offered load

Time-Varying Arrivals: $\sqrt{\cdot}$ Safety-Staffing

Model $\quad M_t / M / N_t + \mathrm{M}$

Parameters $\quad \lambda(t) \quad \mu \quad \textbf{?} \quad \theta$

? $\quad N_t = R_t + \beta\sqrt{R_t}$

$\mu = \theta: \quad L_t \overset{d}{=} \text{Poisson}(R_t) \overset{d}{\approx} N(R_t, R_t), \text{ since } M_t / M / \infty$

$$R_t = E\lambda(t - S) \cdot E(S) = E \int_{t-S}^{t} \lambda(u)du$$

offered load

Given $\quad L_t \approx R_t + Z\sqrt{R_t} \quad , \quad Z \overset{d}{=} N(0,1)$

choose $\quad N_t = R_t + \beta\sqrt{R_t}$

$\Rightarrow \quad \alpha = P(W_t > 0) \underset{\text{PASTA}}{\approx} P(L_t \geq N_t) = P(Z \geq \beta) = 1 - \phi(\beta)$

$\Rightarrow \quad \beta = \phi^{-1}(1 - \alpha) \qquad$ time-stable $\quad \alpha \equiv P(W_t > 0) \;\;?$

Time-Varying Arrivals: $\sqrt{\cdot}$ Safety-Staffing

Model $M_t / M / N_t + M$

Parameters $\lambda(t)$ μ **?** θ

? $N_t = R_t + \beta\sqrt{R_t}$

$\mu = \theta$: $L_t \stackrel{d}{=} \text{Poisson}(R_t) \stackrel{d}{\approx} N(R_t, R_t)$, since $M_t / M / \infty$

$$R_t = E\lambda(t - S) \cdot E(S) = E \int_{t-S}^{t} \lambda(u)du$$ offered load

Given $L_t \approx R_t + Z\sqrt{R_t}$, $Z \stackrel{d}{=} N(0,1)$

choose $N_t = R_t + \beta\sqrt{R_t}$

$\Rightarrow$ $\alpha = P(W_t > 0) \underset{\text{PASTA}}{\approx} P(L_t \geq N_t) = P(Z \geq \beta) = 1 - \phi(\beta)$

$\Rightarrow$ $\beta = \phi^{-1}(1 - \alpha)$ time-stable $\alpha \equiv P(W_t > 0)$?

Indeed, but in fact TIME-STABLE PERFORMANCE

Time-Varying Arrivals: $\sqrt{\cdot}$ Safety-Staffing

Model $\quad M_t / M / N_t + M$

Parameters $\quad \lambda(t) \quad \mu \quad ? \quad \theta$

? $\quad N_t = R_t + \beta\sqrt{R_t}$

$$\mu = \theta : \quad L_t \stackrel{d}{=} \text{Poisson}(R_t) \stackrel{d}{\approx} N(R_t, R_t), \ \text{since} \ M_t / M / \infty$$

$$R_t = E\lambda(t-S) \cdot E(S) = E \int_{t-S}^{t} \lambda(u)du \qquad \text{offered load}$$

Given $\quad L_t \approx R_t + Z\sqrt{R_t} \quad , \quad Z \stackrel{d}{=} N(0,1)$

choose $\quad N_t = R_t + \beta\sqrt{R_t}$

$$\Rightarrow \quad \alpha = P(W_t > 0) \underset{\text{PASTA}}{\approx} P(L_t \geq N_t) = P(Z \geq \beta) = 1 - \phi(\beta)$$

$\Rightarrow \quad \beta = \phi^{-1}(1-\alpha) \qquad$ time-stable $\quad \alpha \equiv P(W_t > 0)$?

Indeed, but in fact $\qquad$ TIME-STABLE PERFORMANCE

($\mu \neq \theta$, or generally : Iterative Simulation-Based Algorithm)