# QED Q's:

# Quality- and Efficiency-Driven Queues

## with a focus on Call/Contact Centers

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Based on joint work with Students, Colleagues, . . .

**Technion SEE Lab: P. Feigin, S. Zeltyn, V. Trofimov**, RA's, . . .

# Contents

# Main Messages

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**5. Service Science / Management / Engineering** are emerging **Academic Disciplines**. For example, universities and

USA NSF (SEE), IBM (SSME), Germany IAO (ServEng), ...

# Background Material (Downloadable)

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- Trofimov, Feigin, M., Ishay, Nadjharov: "DataMOCCA: Models for Call/Contact Center Analysis." Technion Report, 2004-2006.

- M. "Call Centers: Research Bibliography with Abstracts." Version 7, December 2006.

# Queueing Science: Data-Based QED's Q's

**Traditional Queueing Theory** predicts that **Service-Quality** and **Servers' Efficiency** **must** be traded off against each other.

For example, **M/M/1 in heavy-traffic**: **91%** server's utilization goes with

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**Yet**, heavily-loaded queueing systems with **Congestion Index = 0.1** (Waiting one order of magnitude less than Service) are prevalent:

- **Call Centers**: Wait **"seconds"** for **minutes** service;
- **Transportation**: Search **"minutes"** for **hours** parking;
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and, moreover, a significant fraction are not delayed in queue. (For example, in well-run call-centers, **50%** served "immediately", along with over **90%** agents' utilization, is not uncommon ) **?**

# Prerequisite: Data

**Averages Prevalent.**

But I need data at the level of the **Individual Transaction**: For each service transaction (during a phone-service in a call center, or a patient's stay in a hospital), its **operational history** = time-stamps of events.

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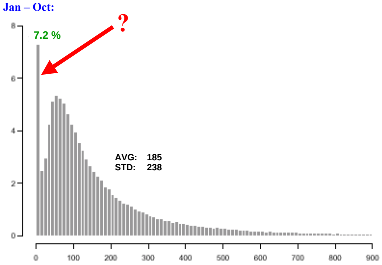
Sources: **"Service-floor"** (vs. Industry-level, Surveys, . . .)

- Administrative (Court, via "paper analysis")
- Face-to-Face (Bank, via bar-code readers)
- Telephone (Call Centers, via ACD / CTI)
- Expanding:
  - Hospitals (via RFID)
  - IVR (VRU), internet, chat (multi-media)
  - Operational + Financial + Marketing / Clinical history

# Beyond Averages (+ The Human Factor)

## Histogram of Service Times in an Israeli Call Center

January-October

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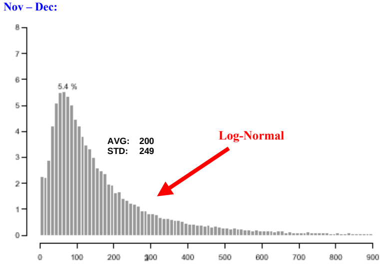


- **7.2% Short-Services:**

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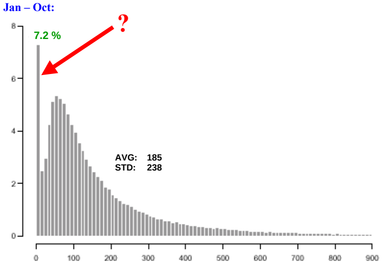


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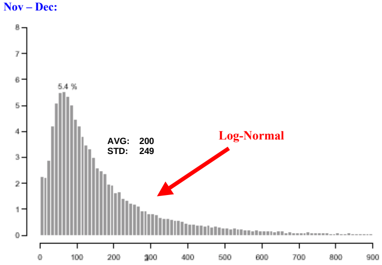



- **7.2% Short-Services:** Agents' “Abandon" (improve bonus, rest)
- **Distributions**, not only Averages, must be measured.
- **Lognormal** service times prevalent in call centers (Why?)

# Present Focus: Call Centers

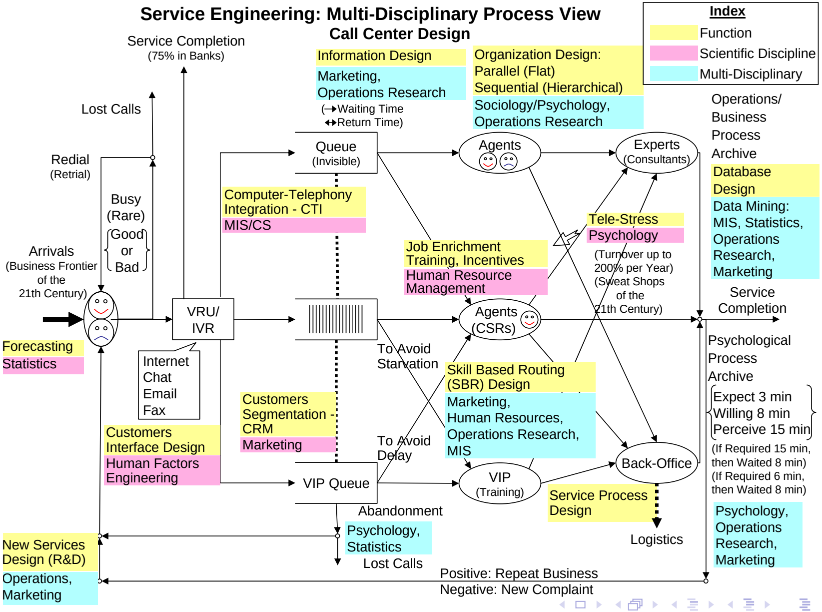
## U.S. Statistics (Relevant Elsewhere)

- Over 60% of annual business volume via the telephone
- 100,000 – 200,000 call centers
- 3 – 6 million employees (**2% – 4% workforce**)
- 1000’s agents in a “single" call center = 70 % costs.
- 20% annual growth rate
- \$200 – \$300 billion annual expenditures

# Call-Center Environment: Service Network

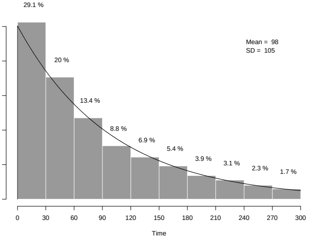
# Call-Centers: “Sweat-Shops of the 21st Century"

# Call-Center Network: Gallery of Models

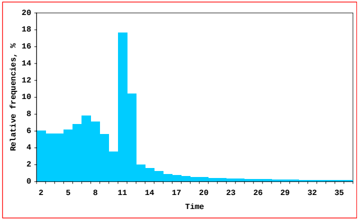


# Beyond Averages: Waiting Times in a Call Center

## Small Israeli Bank




## Large U.S. Bank

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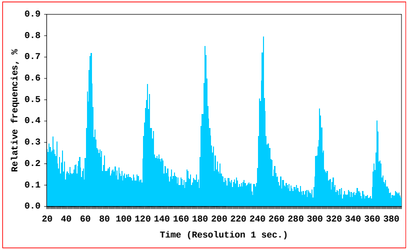

# The “Anatomy of Waiting" for Service

Common Experience:

- Expected to wait 5 minutes, Required to 10,
- Felt like 20, Actually waited 10,
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An attempt at “Modeling the Experience":

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| **1.** | Time that a customer | **expects** to wait | |
| **2.** | | **willing** to wait | ((Im)Patience: $\tau$) |
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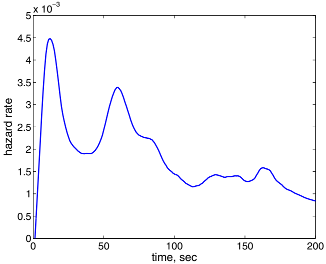
**Experienced** customers $\Rightarrow$ Expected = Required
**"Rational"** customers $\Rightarrow$ Perceived = Actual.

Then left with $\boldsymbol{(\tau, V)}$.

# Call Center Data: Hazard Rates (Un-Censored)

## (Im)Patience Time $\tau$

**Israel**

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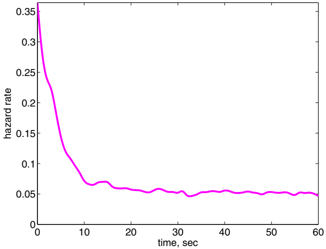

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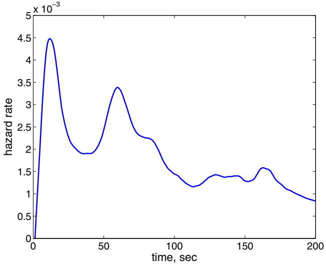


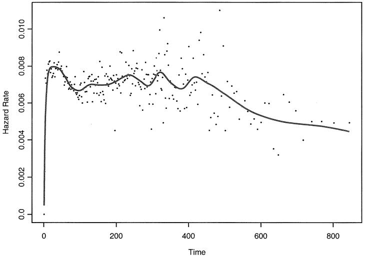


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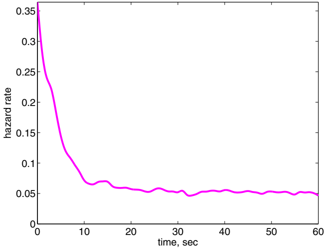


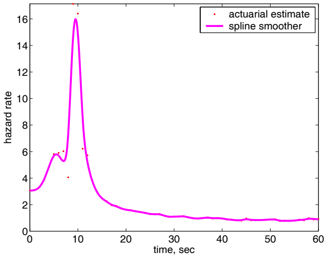

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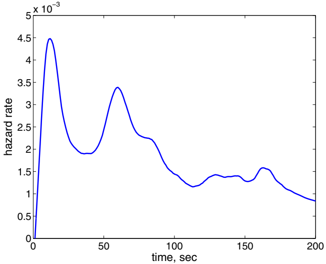


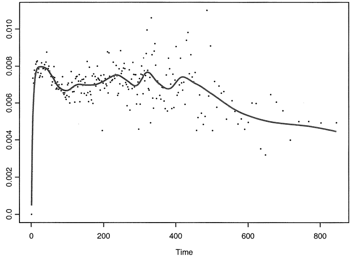


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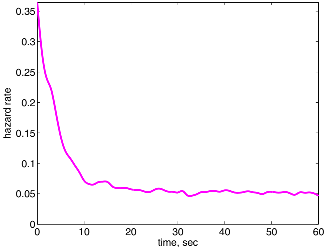
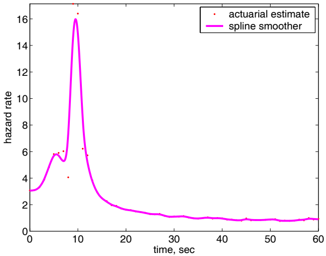
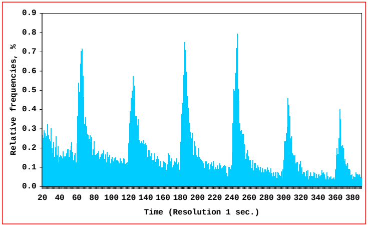



**Note**: 5% abandoning $\Rightarrow$ 95% (im)patience-observations **censored** !

# A “Waiting-Times" Puzzle at a Large Israeli Bank




**Peaks Every 60 Seconds. Why?**

- Human: **Voice-announcement** every 60 seconds.

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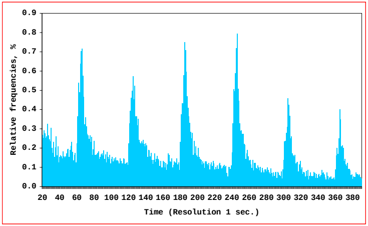
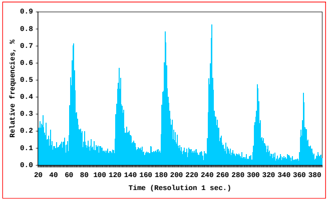


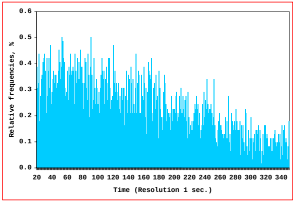

**Peaks Every 60 Seconds. Why?**

- Human: **Voice-announcement** every 60 seconds.
- System: **Priority-upgrade** (unrevealed) every 60 sec's (Theory?)

**Served Customers**




**Abandoning Customers**



# Models for Performance Analysis

- **(Im)Patience**: r.v. $\tau$ = Time a customer is **willing to wait**
- **Offered-Wait**: r.v. $V$ = Time a customer is **required to wait**
  (= Waiting time of a customer with infinite patience).
- **Abandonment** $= \{\tau \leq V\}$
- **Service** $= \{\tau > V\}$
- **Actual Wait** $W_q = \min\{\tau, V\}$.

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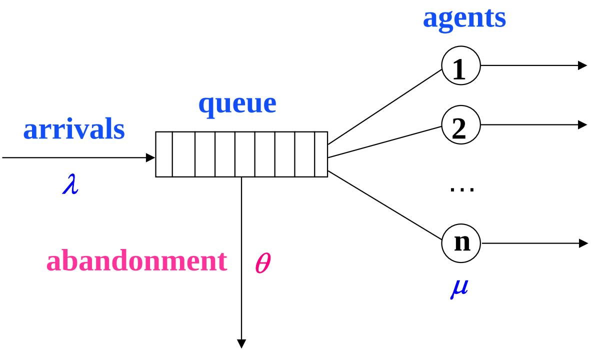
Modeling: $\tau$ = **input** to the model, $V$ = **output**.

Operational Performance-Measure calculable in terms of $(\tau, V)$:

- eg. **Avg. Wait** = $E[\min\{\tau, V\}]$ ( $E[W_q|\text{Served}] = E[V|\tau > V]$ )
- eg. **% Abandon** = $P\{\tau \leq V\}$ ( $P\{5 \text{ sec} < \tau \leq V\}$ )

Application: **Staffing – How Many Agents?** (then: When? Who?)

# The Basic Staffing Model: Erlang-A (M/M/N + M)




**Erlang-A** **(Palm 1940's)** = **Birth & Death Q, with parameters:**

- $\lambda$ – **Arrival** rate (Poisson)
- $\mu$ – **Service** rate (Exponential)
- $\theta$ – **Impatience** rate (Exponential)
- $n$ – Number of **Service-Agents**.

# Testing the Erlang-A Primitives

- **Arrivals**: Poisson?
- **Service-durations**: Exponential?
- **(Im)Patience**: Exponential?

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- **Arrivals**: Poisson?
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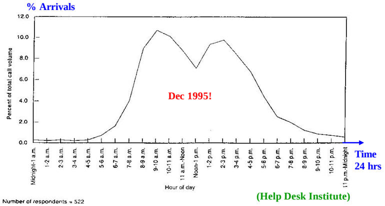
- Primitives independent?
- Customers / Servers Heterogeneous?
- Service discipline FCFS?
- . . . ?

**Validation**: Support? Refute?

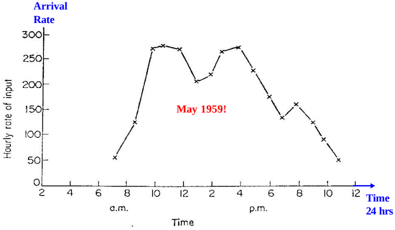
# Arrivals to Service: only Poisson-Relatives

## Arrival Rate to Three Call Centers

Dec. **1995** (U.S. 700 Helpdesks)




May **1959** (England)

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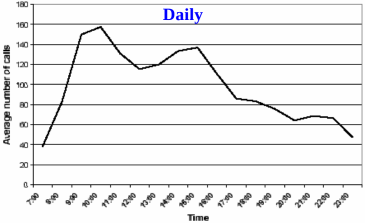

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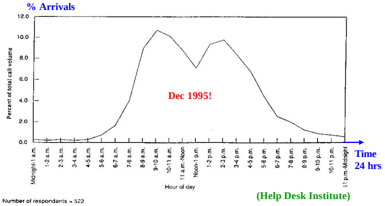


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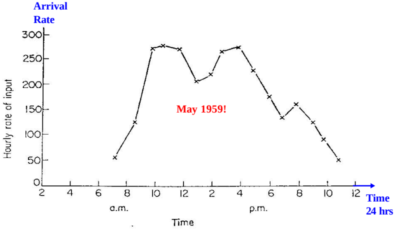


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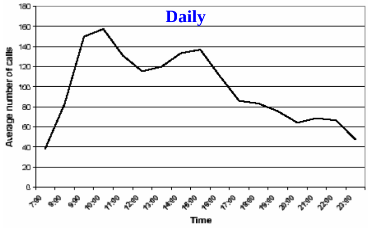
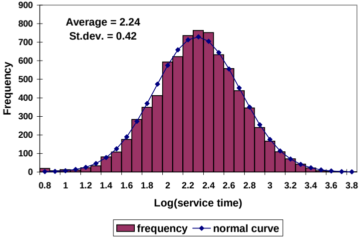



Observation:

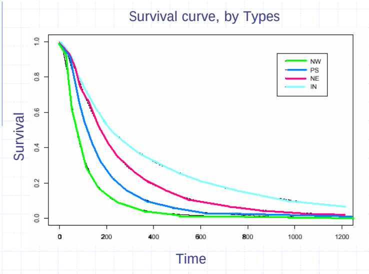
**Peak Loads** at **10:00 & 15:00**

# Service Durations: LogNormal Prevalent

## Israeli Bank Log-Histogram




## Survival-Functions by Service-Class




- **New** Customers: 2 min (NW);
- **Regulars**: 3 min (PS);
- **Stock**: 4.5 min (NE);
- Tech-Support: 6.5 min (IN).

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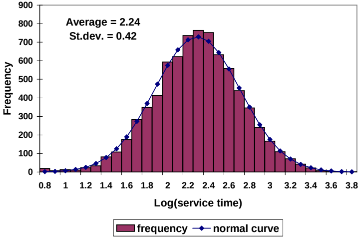


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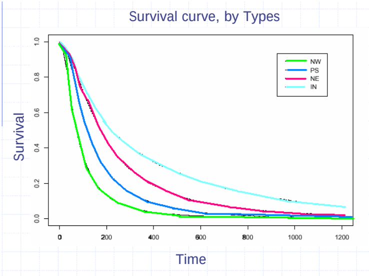


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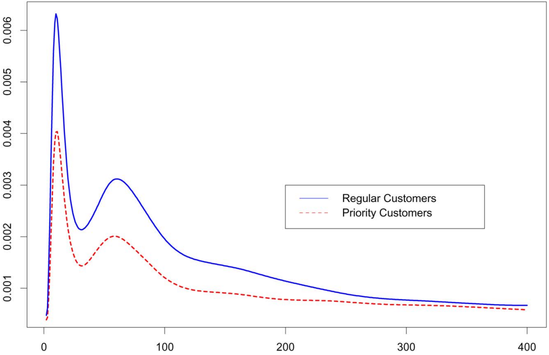
Observation: **VIP** require **longer service** times.

# (Im)Patience while Waiting (Palm 1943-53)

**Irritation $\propto$ Hazard Rate of (Im)Patience Distribution**
**Regular over VIP Customers – Israeli Bank**



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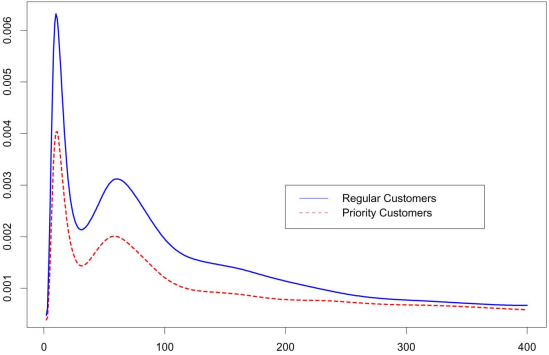


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- **Call-by-Call Data (DataMOCCA)** required (& Un-Censoring).

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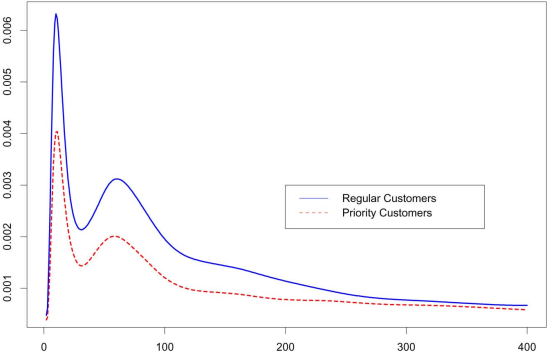


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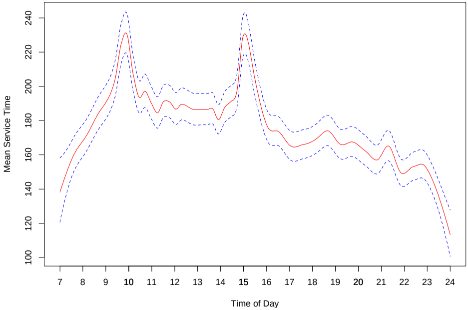
Observation: **VIP** are **more patient** (Needy)

# A “Service-Time" Puzzle at an Israeli Bank

# Inter-related Primitives

## Average Service Time over the Day – Israeli Bank

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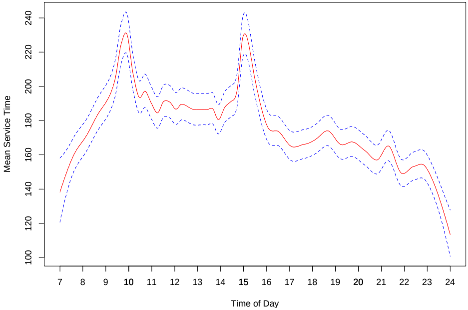


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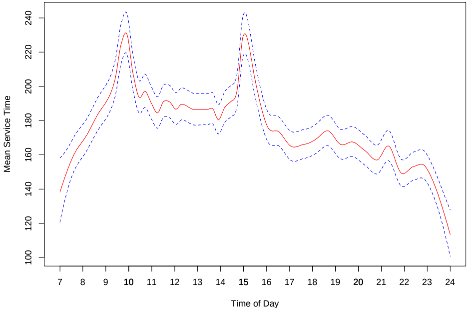


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**Explanations:**

- Common: Service protocol different (longer) during peak times.
- Operational: The needy abandon less during peak times; hence the **VIP remain** on line, with their **long service** times.

# Erlang-A: Practical Relevance?

**Experience:**

- Arrival process **not pure Poisson** (time-varying, $\sigma^2$ too large)
- Service times **not Exponential** (typically close to LogNormal)
- Patience times **not Exponential** (various patterns observed).

- Building Blocks need **not be independent** (eg. long wait possibly implies long service)
- Customers and Servers **not homogeneous** (classes, skills)
- Customers return for service (after busy, abandonment)
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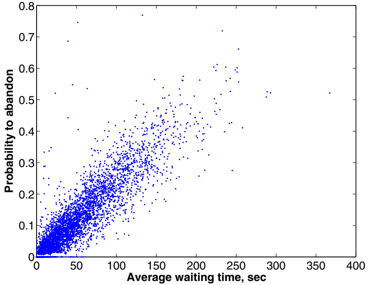
**Question**: **Is Erlang-A Practically Relevant?**

# Estimating (Im)Patience: via P{Ab} $\propto$ E[$W_q$]

Assume **Exp($\theta$)** (im)patience. Then, $\boxed{\mathrm{P}\{\mathrm{Ab}\} \;=\; \theta \cdot \mathrm{E}[W_q]}$.

## Israeli Bank: Yearly Data

Hourly Data

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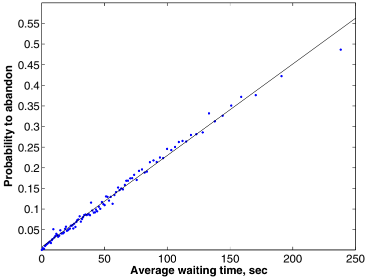


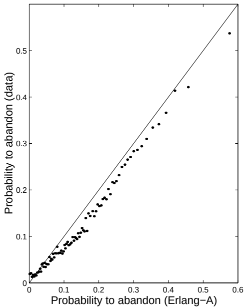

Graphs based on 4158 hour intervals.

Estimate of mean (im)patience: 250/0.55 $\approx$ **450 seconds**.

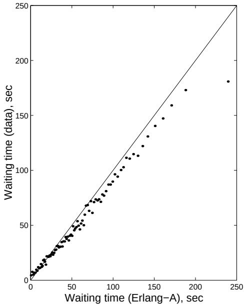
# Erlang-A: Fitting a Simple Model to a Complex Reality

- **Small Israeli Banking Call-Center** (10 agents)
- (Im)Patience ($\theta$) estimated via P{Ab} / E[$W_q$]
- Graphs: **Hourly Performance vs. Erlang-A Predictions**, during 1 year (aggregating groups with 40 similar hours).

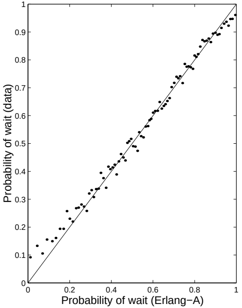
**P{Ab}**




**E[$W_q$]**




**$P\{W_q > 0\}$**



# Erlang-A: Simple, but Not Too Simple

**Further Natural Questions:**

1. Why does Erlang-A practically work? justify **robustness**.
2. When does it fail? chart **boundaries**.
3. Generalize: time-variation, SBR, networks, uncertainty , . . .

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**Answers** via **Asymptotic Analysis**, as load- and staffing-levels increase, which reveals model-essentials:

- **E**fficiency-**D**riven (**ED**) regime: Fluid models (deterministic)
- **Q**uality- and **E**fficiency-**D**riven (**QED**): Diffusion refinements.

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**Motivation**: Moderate-to-large service systems (**100's - 1000's** servers), notably **call-centers**.

Results turn out **accurate** enough to also cover **10-20** servers. Important – relevant to **hospitals** (nurse-staffing: de Véricourt & Jennings, 2006), ...

# Operational Regimes: Conceptual Framework

Assume: **Offered Load** $R = \frac{\lambda}{\mu}$ $(= \lambda \times \mathrm{E}[S])$ not too small.

**QD Regime:** $\boxed{N \approx R + \delta R}$ $[(N - R)/R \to \delta$, as $N, \lambda \uparrow \infty]$

- Essentially **no** delays: $[P\{W_q > 0\} \to 0]$.

**ED Regime:** $\boxed{N \approx R - \gamma R}$

- Garnett, M. & Reiman 2003
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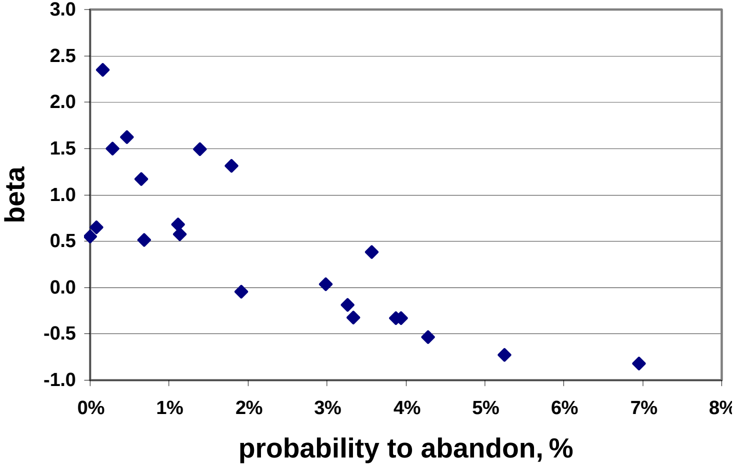
**QED+ED:** $N \approx (1 - \gamma)R + \beta\sqrt{R}$

- Zeltyn & M. 2006
- QED refining ED to accommodate "timely-delays": $P\{W_q > T\}$.

# QED: Practical Support

**QOS parameter $\beta = (N - R)/\sqrt{R}$ vs. %Abandonment**



# QED Theory (Erlang '13; Halfin-Whitt '81; Garnett MSc; Zeltyn PhD)

Consider a sequence of M/M/**N**+G models, **N=1,2,3,…**

Then the following **points of view** are equivalent:

- **QED** $\quad \%\{\text{Wait} > 0\} \approx \alpha, \qquad 0 < \alpha < 1$ ;
- **Customers** $\quad \%\{\text{Abandon}\} \approx \dfrac{\gamma}{\sqrt{N}}, \qquad 0 < \gamma$ ;
- **Agents** $\quad \text{OCC} \approx 1 - \dfrac{\beta + \gamma}{\sqrt{N}} \qquad -\infty < \beta < \infty$ ;
- **Managers** $\quad N \approx R + \beta\sqrt{R}, \qquad R = \lambda \times \text{E(S)}$ not small;

QED performance (ASA, ...) is easily computable, all in terms of $\beta$ (the square-root safety staffing level) – see later.

# QED Approximations (Zeltyn, M. '06)

$G$ – patience distribution,

$g_0$ – patience density at origin $\quad (g_0 = \theta,\ \text{if } \exp(\theta))$.

$$N = \frac{\lambda}{\mu} + \beta\sqrt{\frac{\lambda}{\mu}} + o(\sqrt{\lambda})\,, \qquad -\infty < \beta < \infty\,.$$

$$\mathrm{P}\{\mathrm{Ab}\} \approx \frac{1}{\sqrt{N}} \cdot \left[h(\hat\beta) - \hat\beta\right] \cdot \left[\sqrt{\frac{\mu}{g_0}} + \frac{h(\hat\beta)}{h(-\beta)}\right]^{-1},$$

$$\mathrm{P}\left\{W > \frac{T}{\sqrt{N}}\right\} \approx \left[1 + \sqrt{\frac{g_0}{\mu}} \cdot \frac{h(\hat\beta)}{h(-\beta)}\right]^{-1} \cdot \frac{\bar\Phi\left(\hat\beta + \sqrt{g_0\mu}\cdot T\right)}{\bar\Phi(\hat\beta)},$$

$$\mathrm{P}\left\{\mathrm{Ab}\,\middle|\, W > \frac{T}{\sqrt{N}}\right\} \approx \frac{1}{\sqrt{N}} \cdot \sqrt{\frac{g_0}{\mu}} \cdot \left[h\left(\hat\beta + \sqrt{g_0\mu}\cdot T\right) - \hat\beta\right].$$

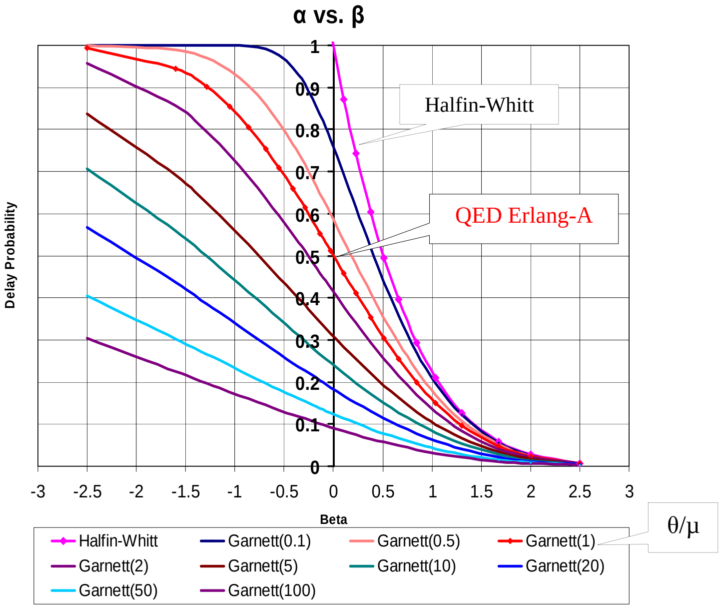
Here

$$\hat\beta = \beta\sqrt{\frac{\mu}{g_0}}$$

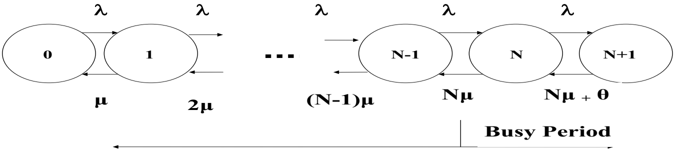
$$\bar\Phi(x) = 1 - \Phi(x)\,,$$

$$h(x) = \phi(x)/\bar\Phi(x)\,, \quad \text{hazard rate of } N(0,1).$$

# Garnett / Halfin-Whitt Functions: $P\{W_q > 0\}$



# QED Intuition via Excursions: Busy/Idle Periods




$Q(0) = N$: all servers busy, no queue.

Let $T_{N,N-1} =$ Busy Period (down-crossing $N \downarrow N-1$ )

$T_{N-1,N} =$ Idle Period (up-crossing $N-1 \uparrow N$ )

Then $$P(Wait > 0) = \frac{T_{N,N-1}}{T_{N,N-1} + T_{N-1,N}} = \left[1 + \frac{T_{N-1,N}}{T_{N,N-1}}\right]^{-1}$$

# QED Intuition via Excursions: Asymptotics

Calculate $$T_{N-1,N} = \frac{1}{\lambda_N E_{1,N-1}} \sim \frac{1}{N\mu \times h(-\beta)/\sqrt{N}} \sim \frac{1}{\sqrt{N}} \cdot \frac{1/\mu}{h(-\beta)}$$

$$T_{N,N-1} = \frac{1}{N\mu\pi_+(0)} \sim \frac{1}{\sqrt{N}} \cdot \frac{\beta/\mu}{h(\delta)\,/\delta}, \qquad \delta = \beta\sqrt{\mu/\theta}$$

Both apply as $\sqrt{N}\,(1-\rho_N) \to \beta,\ -\infty < \beta < \infty.$

Hence, $$P(Wait > 0) \sim \left[1 + \frac{h(\delta)/\delta}{h(-\beta)/\beta}\right]^{-1}.$$

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Special case: $\mu = \theta$ (Impatient):

Then $\boldsymbol{Q} \stackrel{d}{=} \boldsymbol{M/M/\infty}$, since sojourn-time is $\exp(\mu = \theta)$.

If also $\beta = \mathbf{0}$ (Prevalent): $\boldsymbol{P\{Wait > 0\} \approx 1/2}$.

# Process Limits (Queueing, Waiting)

- $\hat{Q}_N = \{\hat{Q}_N(t), t \geq 0\}$ : stochastic process obtained by centering and rescaling:

$$\hat{Q}_N = \frac{Q_N - N}{\sqrt{N}}$$

- $\hat{Q}_N(\infty)$ : stationary distribution of $\hat{Q}_N$

- $\hat{Q} = \{\hat{Q}(t), t \geq 0\}$ : process defined by: $\hat{Q}_N(t) \xrightarrow{d} \hat{Q}(t)$.

$$\begin{tikzcd}
\hat{Q}_N(t) \arrow[r, "t \to \infty"] \arrow[d, "N \to \infty"'] & \hat{Q}_N(\infty) \arrow[d, "N \to \infty"] \\
\hat{Q}(t) \arrow[r, "t \to \infty"'] & Q(\infty)
\end{tikzcd}$$

Approximating (Virtual) Waiting Time

$$\hat{V}_N = \sqrt{N}\, V_N \Rightarrow \hat{V} = \left[\frac{1}{\mu}\,\hat{Q}\right]^+ \qquad \text{(Puhalskii, 1994)}$$

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   - **Constraint Satisfaction**: eg. **Min. N , s.t. QOS constraints** .
3. Robustness depends:
   - **Without Abandonment: QED** covers all, at amazing accuracy.
   - **With Abandonment: ED, QED, ED+QED** all have a role.

# Operational Regimes: Rules-of-Thumb

| Constraint | P{Ab} | | E[$W$] | | P{$W > T$} | |
|---|---|---|---|---|---|---|
| Offered Load | Tight<br>1-10% | Loose<br>$\geq$ 10% | Tight<br>$\leq$ 10%E[$\tau$] | Loose<br>$\geq$ 10%E[$\tau$] | Tight<br>$0 \leq T \leq$ 10%E[$\tau$]<br>$5\% \leq \alpha \leq 50\%$ | Loose<br>$T \geq$ 10%E[$\tau$]<br>$5\% \leq \alpha \leq 50\%$ |
| Small (10's) | QED | QED | QED | QED | QED | QED |
| Moderate-to-Large (100's-1000's) | QED | ED, QED | QED | ED, QED if $\tau \stackrel{d}{=}$ exp | QED | ED+QED |

**ED:** $N \approx R - \gamma R \quad (0.1 \leq \gamma \leq 0.25)$.

**QD:** $N \approx R + \delta R \quad (0.1 \leq \delta \leq 0.25)$.

**QED:** $N \approx R + \beta\sqrt{R} \quad (-1 \leq \beta \leq 1)$.

**ED+QED:** $N \approx (1-\gamma)R + \beta\sqrt{R} \quad (\gamma, \beta$ as above$)$.

# ED

DO NOT forget to insert

# ED+QED

## Back to “Why does Erlang-A Work?"

**Theoretical** Answer: $M_t^J/G/N_t + G \overset{d}{\approx} (M/M/N+M)_t\ , \quad t \geq 0.$

- **General Patience**: Behavior at the origin is all that matters.
- **General Services**: Empirical insensitivity beyond the mean.
- **Time-Varying Arrivals**: Modified Offered-Load approximations.
- **Heterogeneous Customers**: 1-D state collapse.

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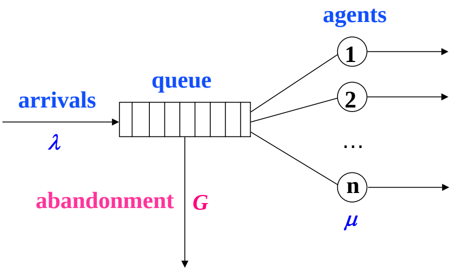
**Practically**: Why do (stochastic-ignorant) Call Centers work?

**"The right answer for the wrong reason"**

## “Why does Erlang-A Work?" General Patience




(Im)Patience times **Generally Distributed**: M/M/$n$+G

**Exact** analysis in steady-state (Baccelli & Hebuterne, 1981): solve Kolmogorov’s PDE’s (semi-Markov) for the offered-wait $V$; Generalized by Brandt & Brandt in late 90’s.

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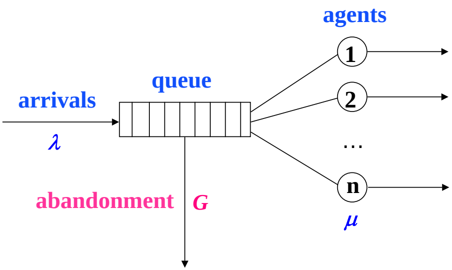


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**QED** analysis (w/ Zeltyn, 2006): $\boldsymbol{n} \approx \boldsymbol{R} + \beta\sqrt{\boldsymbol{R}}$.

- Assume (Im)Patience density $\boldsymbol{g(0) > 0}$.
- $\boldsymbol{V}$ asymptotics ($\lambda \uparrow \infty$): Laplace Method, leading to
- **QED Approximations: Use Erlang-A** as is, with $\theta \leftrightarrow g(0)$.

# General Patience: Fitting Erlang-A

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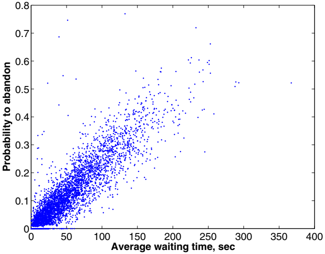


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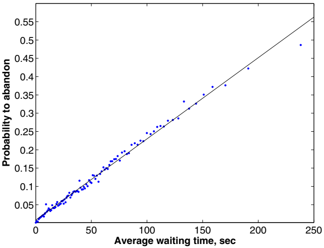


**Theory:**

**Erlang-A:** P{Ab} $= \theta \cdot$ E[$W_q$]; **M/M/N+G:** P{Ab} $\approx g(0) \cdot$ E[$W_q$].

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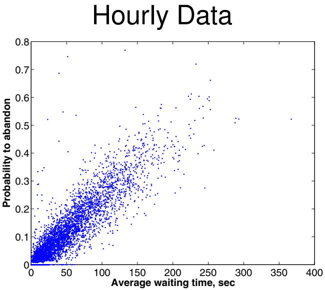


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**Recipe:**

In both cases, use Erlang-A, with $\hat{\theta} = \widehat{\mathrm{P}\{\mathrm{Ab}\}} / \widehat{\mathrm{E}[W_q]}$ (slope above).

## Why Does Erlang-A Work? General Services

Established: M/M/$N$+G $\approx$ M/M/$N$+M ($\theta = g(0)$).

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**Numerical Experiments:** Whitt (2004), Rosenshmidt (2006) demonstrate a **useful fit** for typical call-center parameters.

Lognormal (CV=1) vs. Exponential Service Times, QED Regime;
100 agents, average patience = average service

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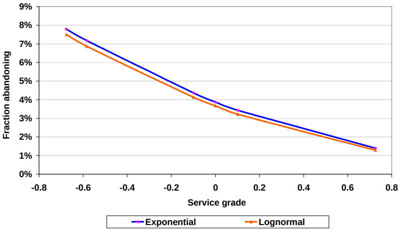


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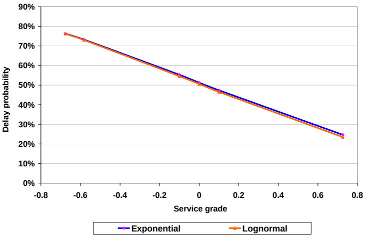

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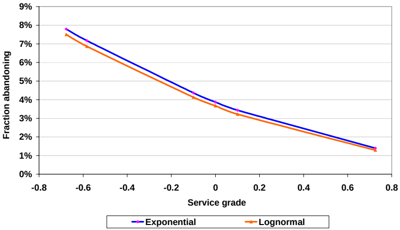


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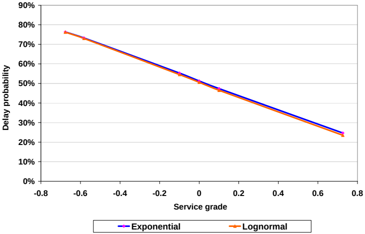


**QED G-Services**: G/$D_K$/N+G (w/ Momčilović, ongoing).

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1. Modified Offered-Load: $\lambda$
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**Serendipity: Time-stable** performance, supported by **ISA** = Iterative Staffing Algorithm, and QED diffusion limits ($M_t/M/N+M$, $\mu = \theta$).

# Stable Performance of Time-Varying Q's

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- Consider $M_t^J/G/N_t + G$ with arrival rates $\lambda_j(t), t \geq 0$.
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**Serendipity: Multi-Class Multi-Skill**, w/ **class-dependent** services.
Support: ISA, QED diffusion limits (Atar, M. & Shaikhet, 2007).

# Heterogeneous Customers (SBR): V-Model

# Additional Simple (QED) Models of Complex Realities: Exponential Services; i.i.d. Customers, i.i.d. Servers

- **Performance Analysis**:
  - Khudiakova, Feigin, M. (Semi-Open): Call-Center + IVR/VRU;
  - De Véricourt, Jennings (Closed + Delay), then w/ Yom-Tov (Semi-Open): **Nurse staffing** (ratios), bed sizing;
  - Randhawa, Kumar (Closed + Loss): Subscriber queues.
- **Optimal Staffing**: Accurate to **within 1**, even with very small $n$'s, for both **constraint-satisfaction** and cost/revenue **optimization** (staffing, abandonment and waiting costs).
  - Armony, Maglaras: ($M_x$/M/N) Delay information (Equilibrium);
  - Borst, M., Reiman (M/M/N): Asymptotic framework;
  - Zeltyn, M. (M/M/N+G): Optimization still ongoing.
- **Time-Varying Queues**, via 2 approaches:
  - Jennings, M., Massey, Whitt, then w/ Feldman: **Time-Stable Performance** (ISA, leading to Modified Offered Load);
  - M., Massey, Reiman, Rider, Stolyar: Unavoidable **Time-Varying Performance** (Fluid & Diffusion models, via Uniform Acceleration).

# Less-Simple (QED) Models: General Service-Times

The Challenge: Must keep track of the state of ***n* individual servers, as *n*** $\uparrow \infty$. (Recall Kiefer & Wolfowitz).

- Shwartz, M. (M/G/N), Rosenshmidt, M. (M/G/N+G): Simulations; **LogNormal better then Exp**, 2-valued same as D.
- Whitt (GI/M+0/N): Covering $CV \geq 1$;
- Puhalskii, Reiman (GI/PH/N): Markovian process-limits (no steady-state); also priorities;
- Jelencović, M., Momčilović (GI/D/N): steady-state (via round-robin); then M., Momčilović (G/$D_K$/N): process-limits, via “Lindley-Trees"; G/$D_K$/N+G ongoing.
- Kaspi, Ramanan (G/G/N): Fluid, next Diffusion (measure-valued ages, following Kiefer & Wolfowitz);
- Reed (GI/GI/N): Fluid, Diffusion (**Skorohod-Like Mapping**).

# Complex (QED) Models: Skills-Based Routing (Heterogeneous Customers or/and Servers - Theory)

- **V-Model**: Harrison, Zeevi; Atar, M., Reiman; Gurvich, M., Armony;
  then **Class-dependent** services: Atar, M., Shaikhet;
- **Reversed-V**: Armony, M.;
  then **Pool-dependent** services: Dai, Tezcan; Gurvich, Whitt (**G-$c\mu$**); Atar, M., Shaikhet (Abandonment);
- **General**: Atar, then w/ Shaikhet (Null-controllability, Throughput-suboptimality); Gurvich, Whitt (FQR);
- **Distributed Networks**: Tezcan;
- **Random Service Rates**: Atar (Fastest or longest-idle server).

# The Technion SEE Center / Laboratory

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