Empirical Analysis of Call Center Traffic

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Presentation for Call Center Forum
Wharton Financial Institutions Center
May 8, 2003

Joint work with
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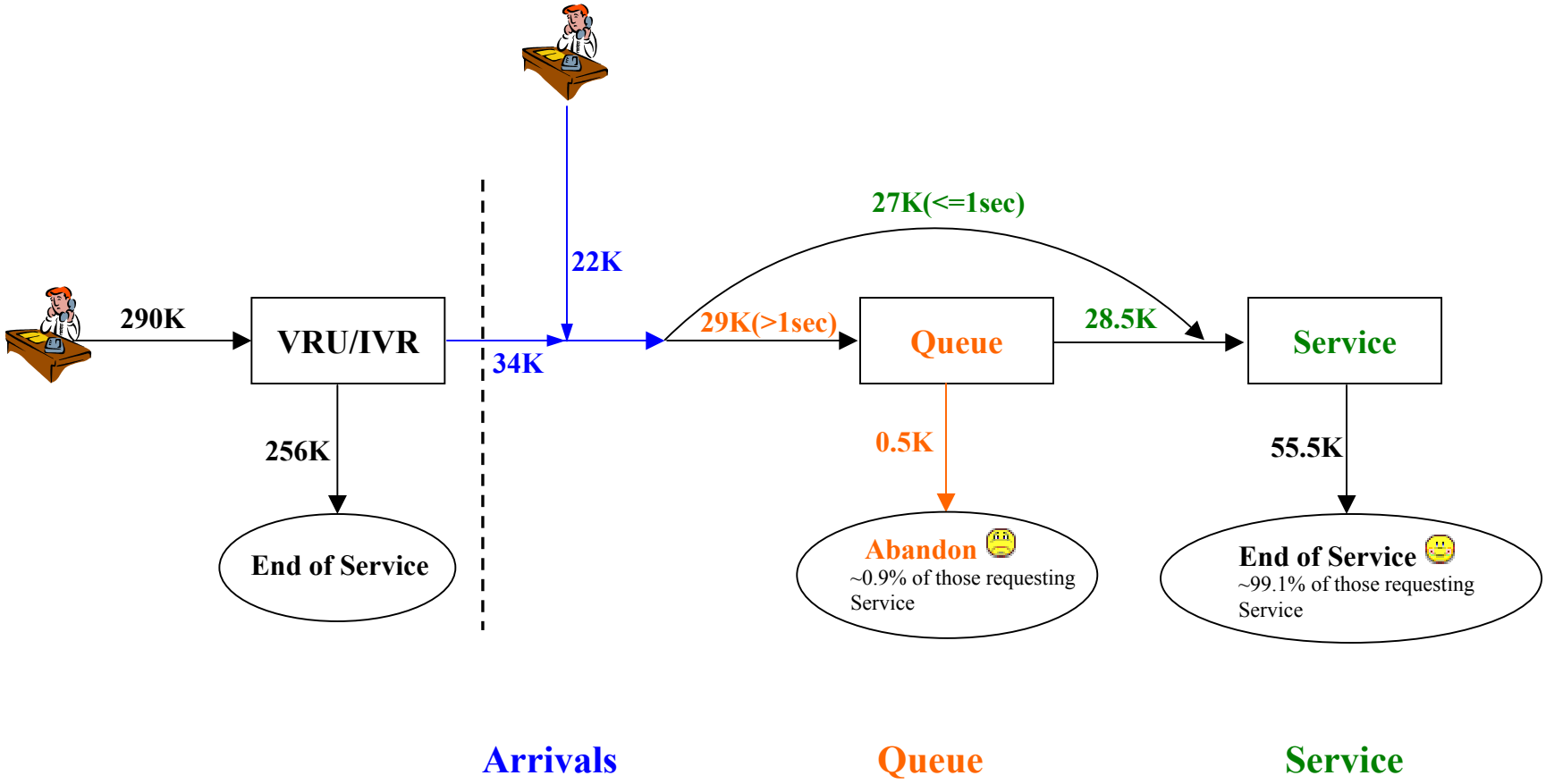
Also based on
"Statistical analysis of a Telephone Call Center: A Queueing Science Perspective", by *Lawrence Brown, Noah Gans, Avishai Mandelbaum, Anat Sakov, Haipeng Shen, Sergey Zeltyn and Linda Zhao*
available at http://fic.wharton.upenn.edu/fic/papers/03.html.

Two Data Resources:

1. An Israeli Bank Call Center
   a. Gathered in 1999
   b. Relatively small size (10-15 agents)
   c. Analyzed in depth in "Statistical analysis of a Telephone Call Center: A Queueing Science Perspective"
2. A Northeastern U.S. Bank Call Center
   a. Gathered mid 2001 to date
   b. Relatively large size (~600 agents in 3 locations)
   c. Preliminary analysis of data in progress

Today's talk mostly uses data from the US bank, *but*
In analyses motivated by those developed for the Israeli bank data.

Schematic Diagram of a Call



Numbers are average call volumes on an ordinary weekday at US Center.

(Ave.wait for all those (=56K) entering "Service Q" is ~12 sec.)

Structure of Data

For each Call:

Date and Time (in seconds) of

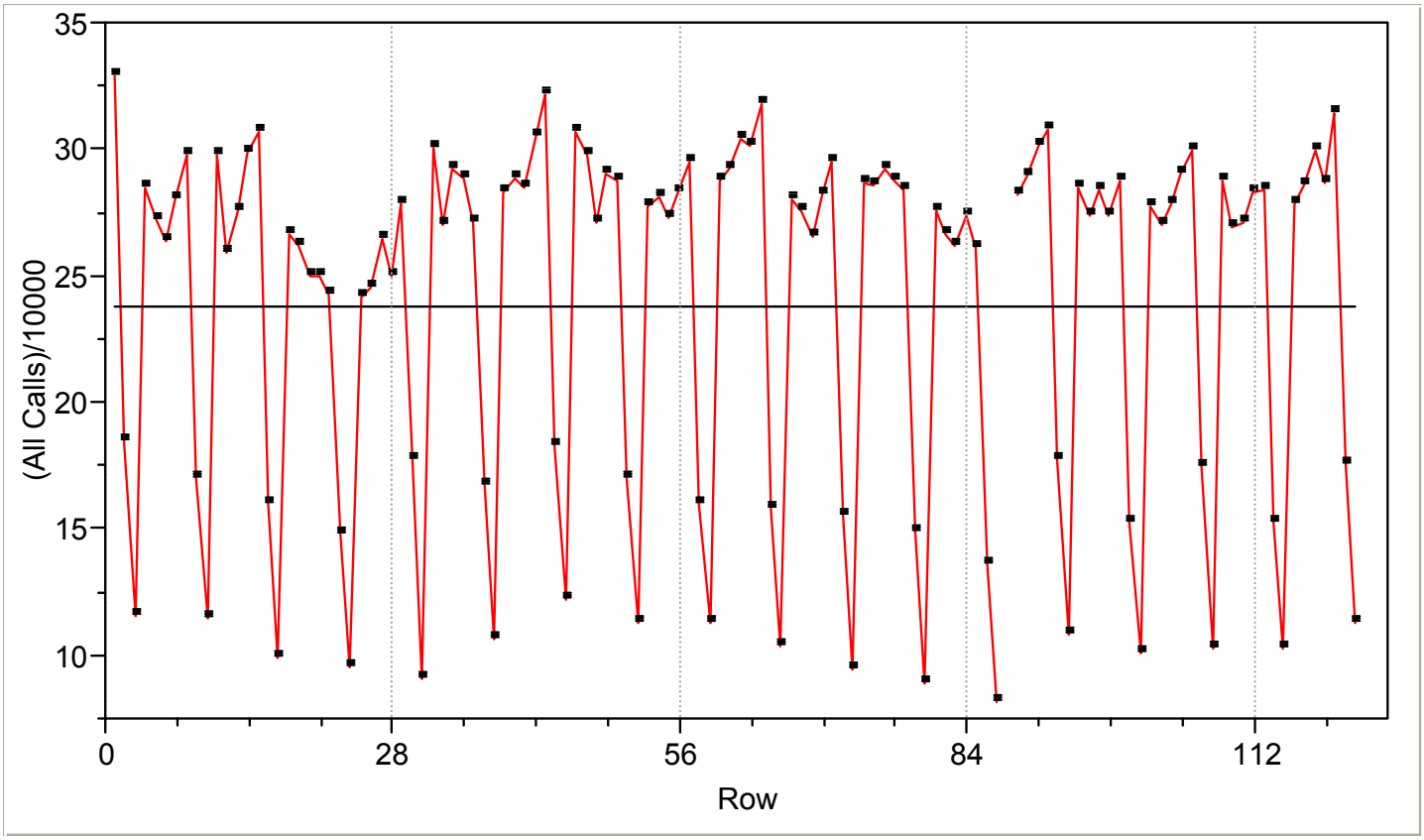
- Arrival to VRU
- Completion of VRU activity
- Arrival to Service-Q (if so)
- "Abandonment" by customer or of Answer by Agent
- Completion of Service

Other Information

- Type of Agent service ("Regular", "Premier", etc.)
- Identity of Agent(s*) (coded), if any involved
- Locations of Call arrival and service*
- Other time and some covariate information*

* = information not used in current analyses

Time Series: All Calls
Segment =March through June, 2002



Day #1 = March 1, 2002
Day #122 = June 30, 2002

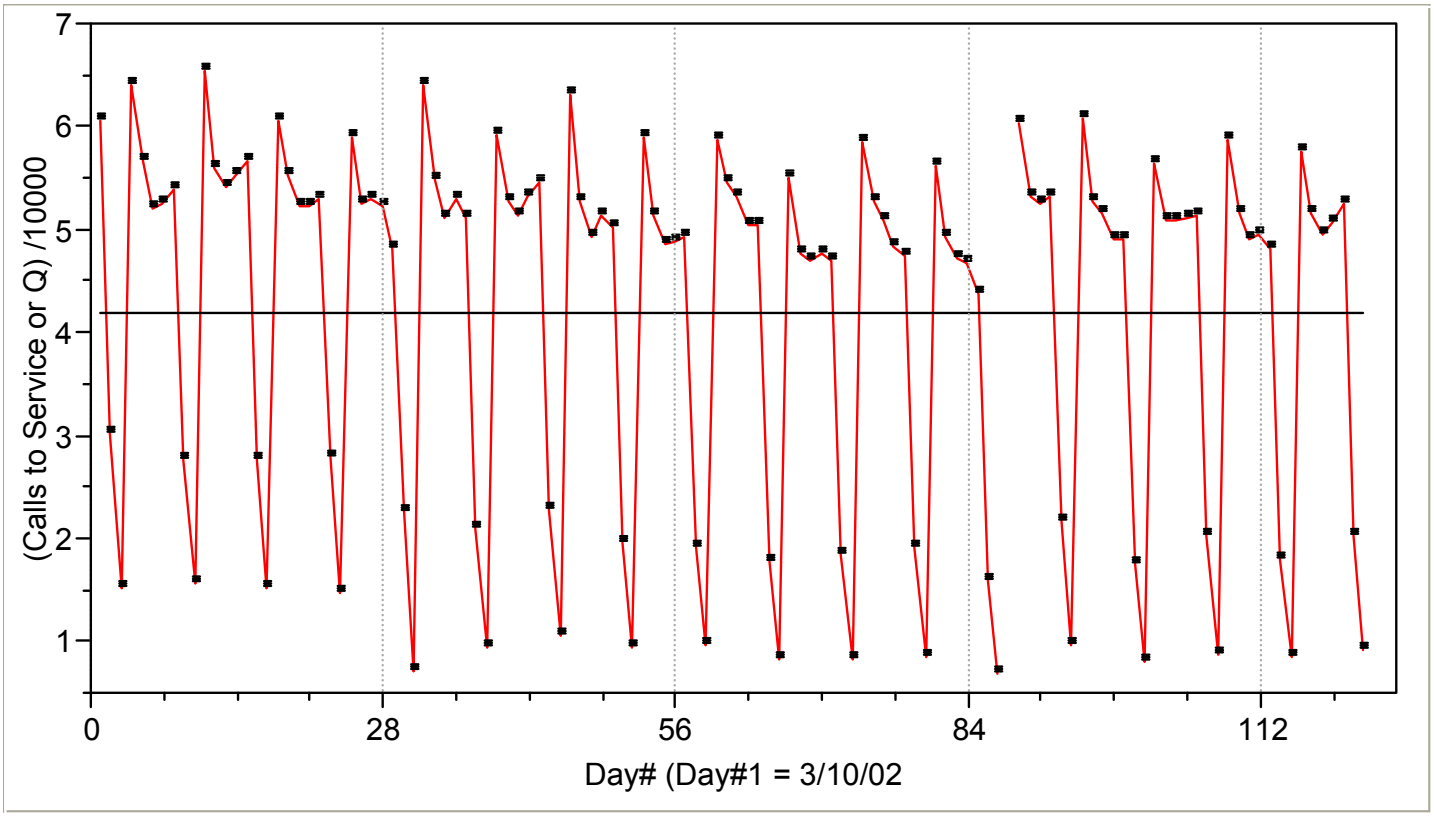
Note Missing Day (= Memorial Day Holiday)

Pattern:

Decreased volumes on Sat. and Sun.
Fluctuations during week, but no special pattern

Time Series: Calls to “Service Q”

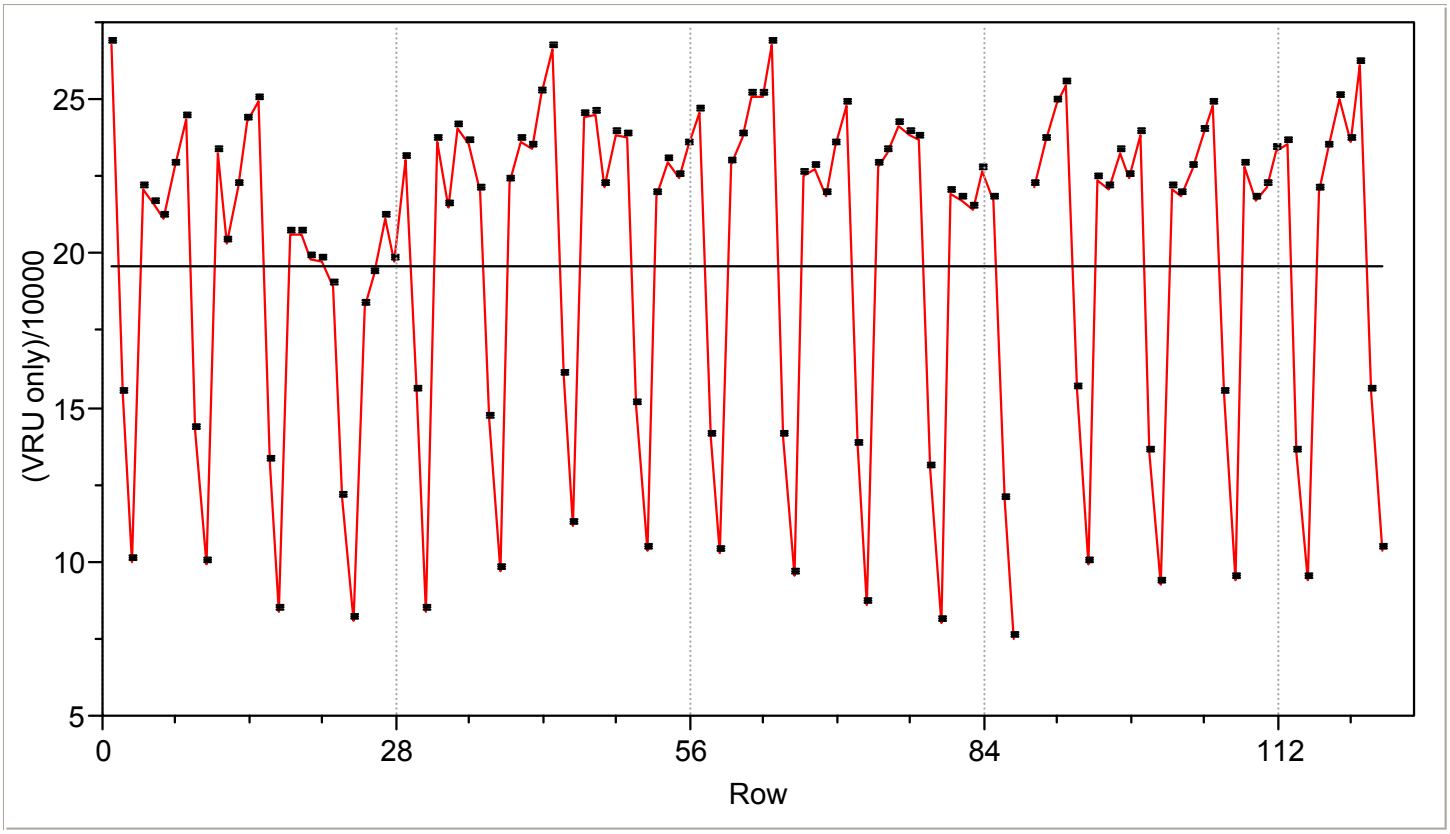
(“Agent” + “Hang” Calls)



Day #1 = March 1, 2002 Day #122 = June 30, 2002

Note Daily Pattern (Decreasing volume as week progresses)

Time Series: (VRU only)/10000



Day #1 = March 1, 2002 Day #122 = June 30, 2002

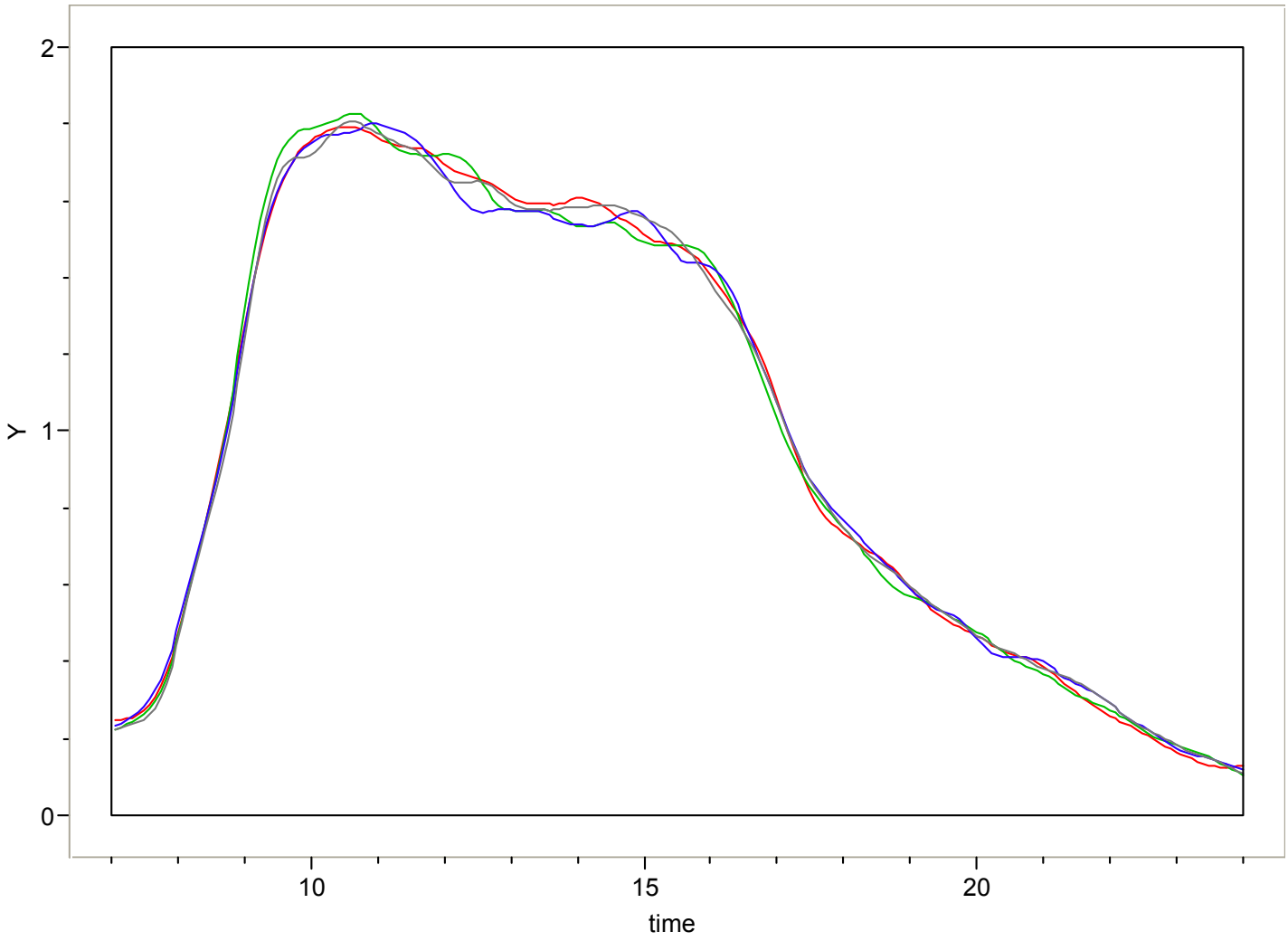
Note Daily Pattern (Generally mildly increasing from Mon. to Fri.)

VRU is important. BUT
Remainder of talk is about Q & Agent component.

Service-Q Arrivals
by Time-of-Day & Day-of-Week
MONDAY

Plot shows (spline-smooth of)

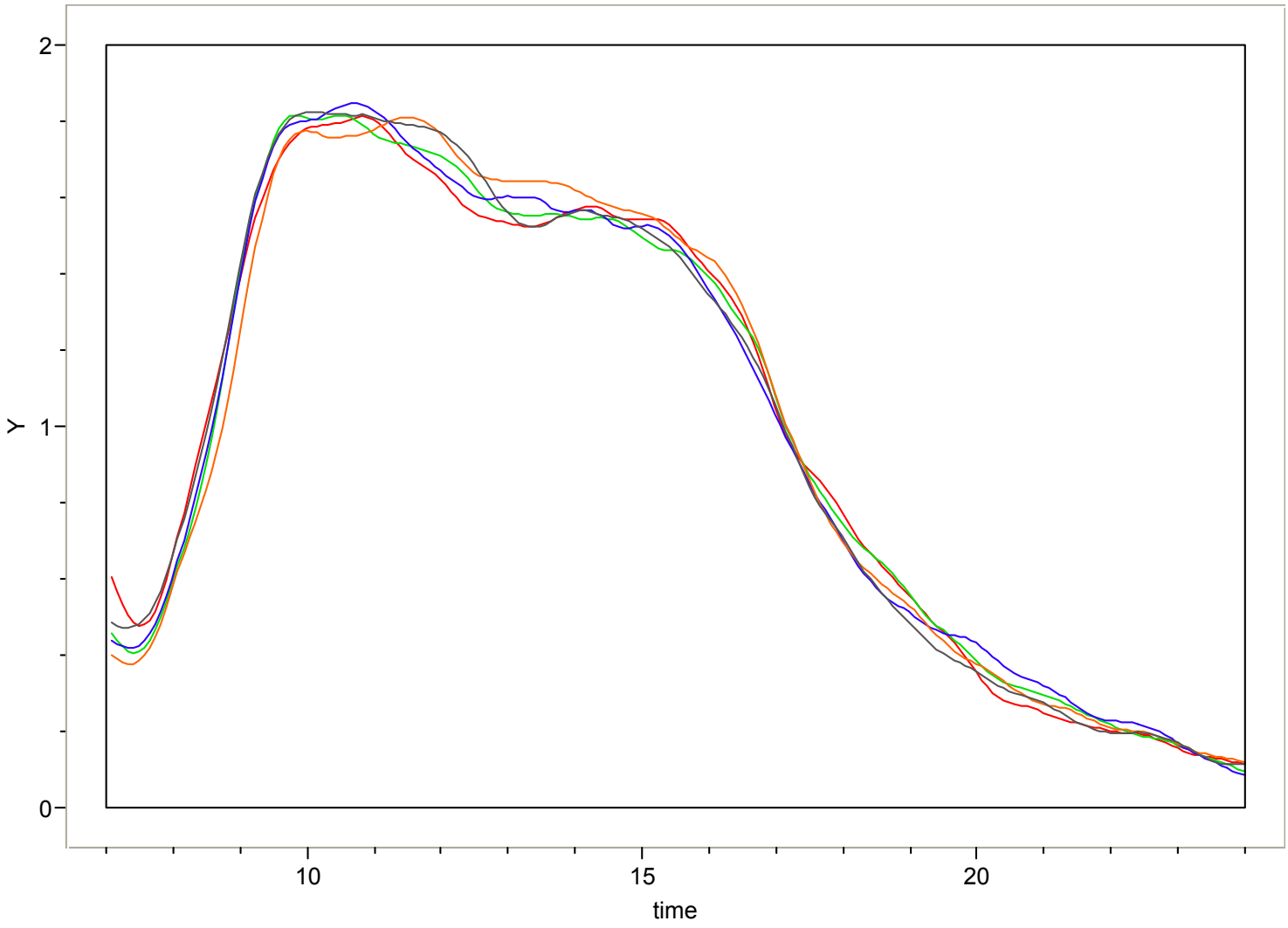
$$\text{Normalized Arrivals}_{\text{given day}} = \frac{\text{Arrivals per Hour}_{\text{that day}}}{\text{Average}_{\text{that day}}\ \text{(arriv's. per hour)}}$$



The variable **"time"** is on a 24 hour clock.

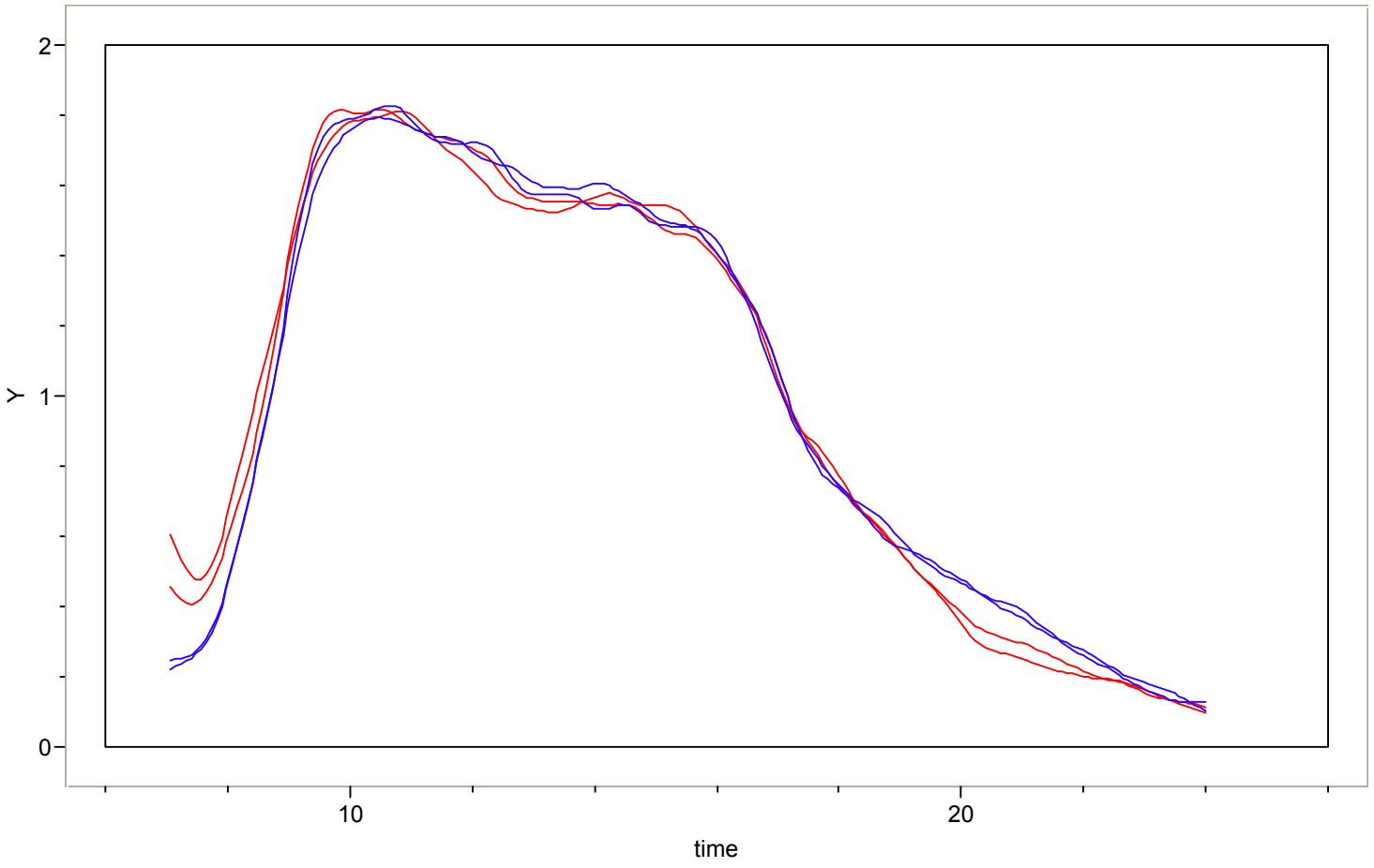
_____ = 8/05/02
_____ = 8/12/02
_____ = 8/19/02
_____ = 8/26/02

Service-Q Arrivals
Friday



Note Dip between 7 & 7:30 am

Service-Q Arrivals
COMPARE Monday andFriday



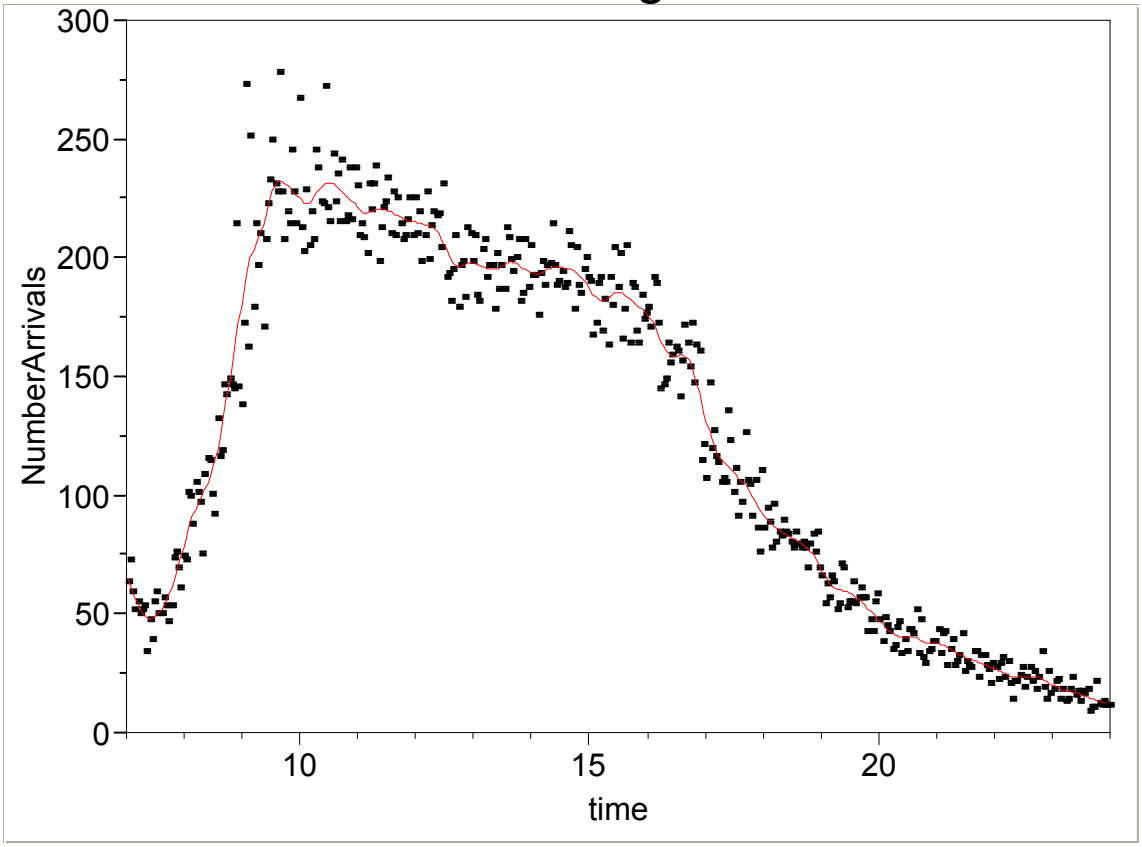
Friday

Has characteristic beginning dip, AND

Daily volume shifted to (slightly) earlier in the day

Y
Norm'ized Arr's 8/2/02
Norm'ized Arr's 8/9/02
Norm'ized Pred for 8/5/02
Norm'ize Pred for 8/12/02

Details of Fitting Process



Plot for Aug 9 (Fri.)

- Divide day (7am to midnight) into time intervals of 150 seconds (=2½ minutes)
- Count number arrivals in each interval, and make scatterplot
- Fit using a nonparametric regression smoothing technique. (Automated bandwidth/smoothing-parameter techniques are possible, but weren't used.)

Nonparametric density estimation could also be used here to get similar looking plots, but that method doesn't extend to create types of plots later in the analysis.

Queueing Structure

- **Staffing**: Number of Agents *On-Active-Duty*
- Call **Load**:

(Number of incoming calls during any time period)**×**
(*Potential* duration of call)

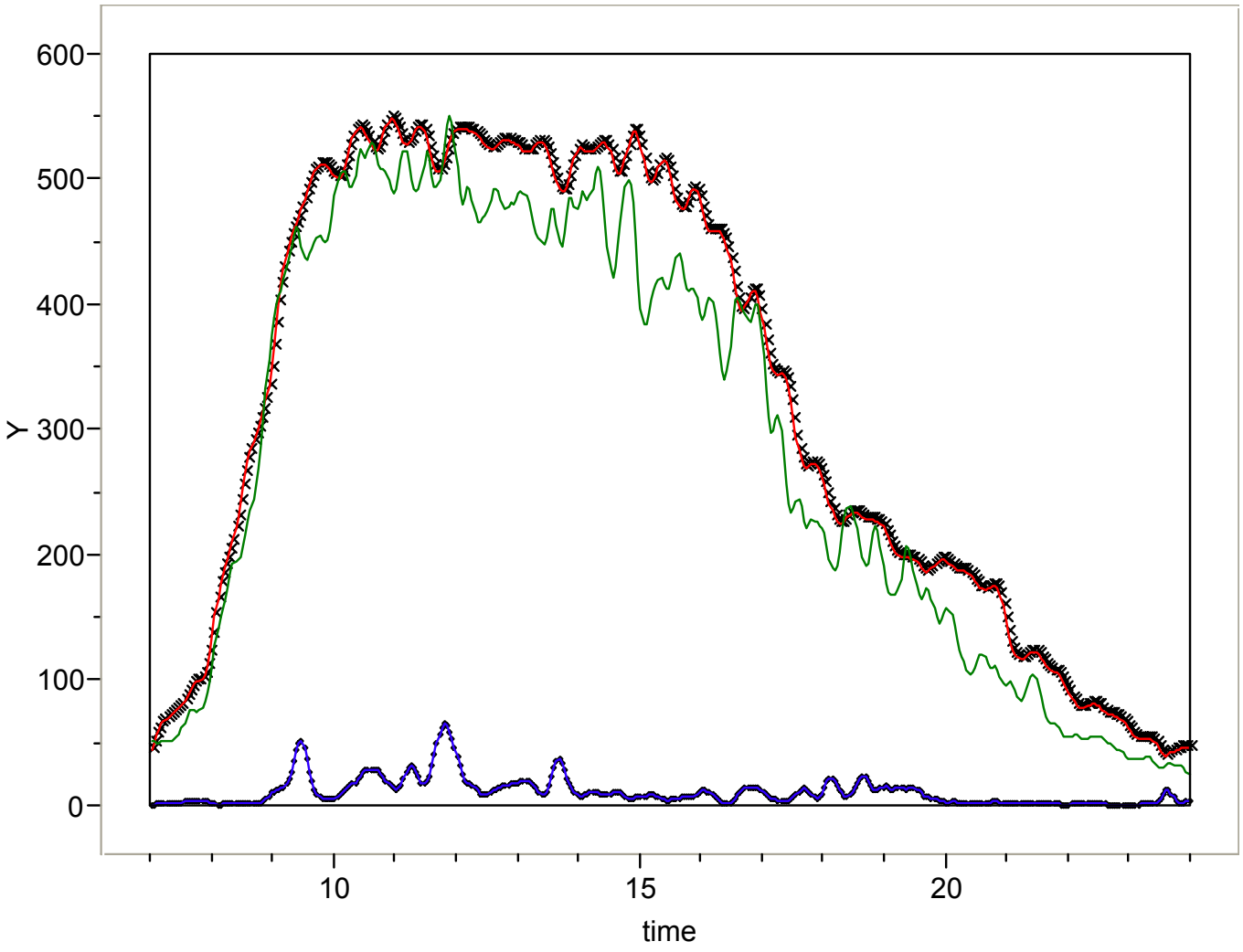
This is the minimum number of person-hours of on-duty agents needed to handle all calls in a perfectly lubricated system (if customers time their calls just right and/or are willing to wait as long as needed).

$$\text{System Utilization} = \frac{\text{Load}}{\text{Staffing}}$$

- **Consequences** of Load & Staffing:
  - **Average wait**
  - Customer **abandonment**
    (depends on waiting times and on **customer patience)**

Efficiency Plots

Showing Load and Staffing

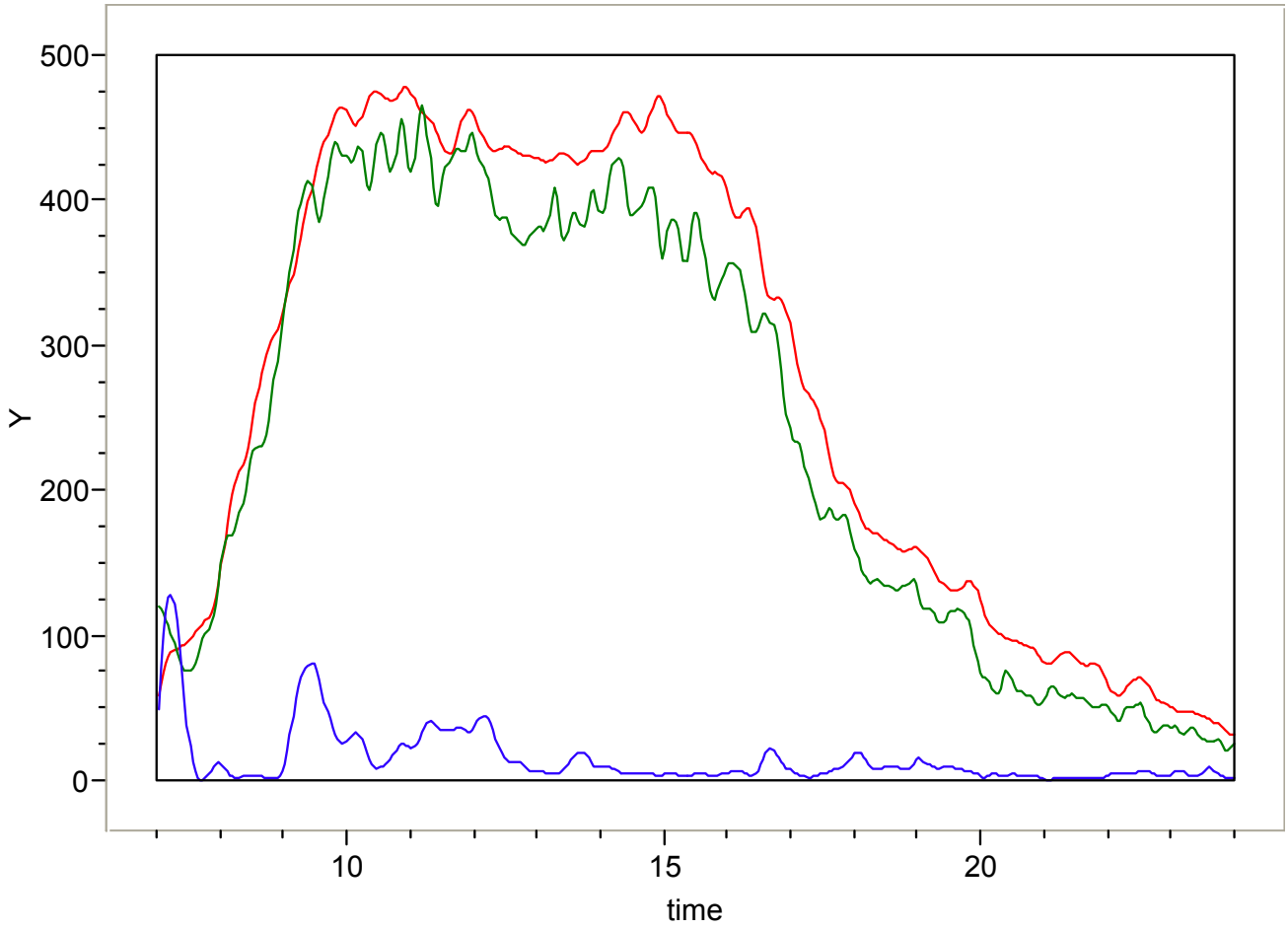


Plot is for Monday 8/05/02

Y — NumberAgents (s)
load (s)
AvgQueueWaitAll (s)

"Agents" = *Estimate* of number of agents on-duty at that time.
[In each 150 second interval an agent is estimated to be on-active-duty for the entire interval if (s)he is on the phone sometime in that interval.]

Efficiency Plots, cont

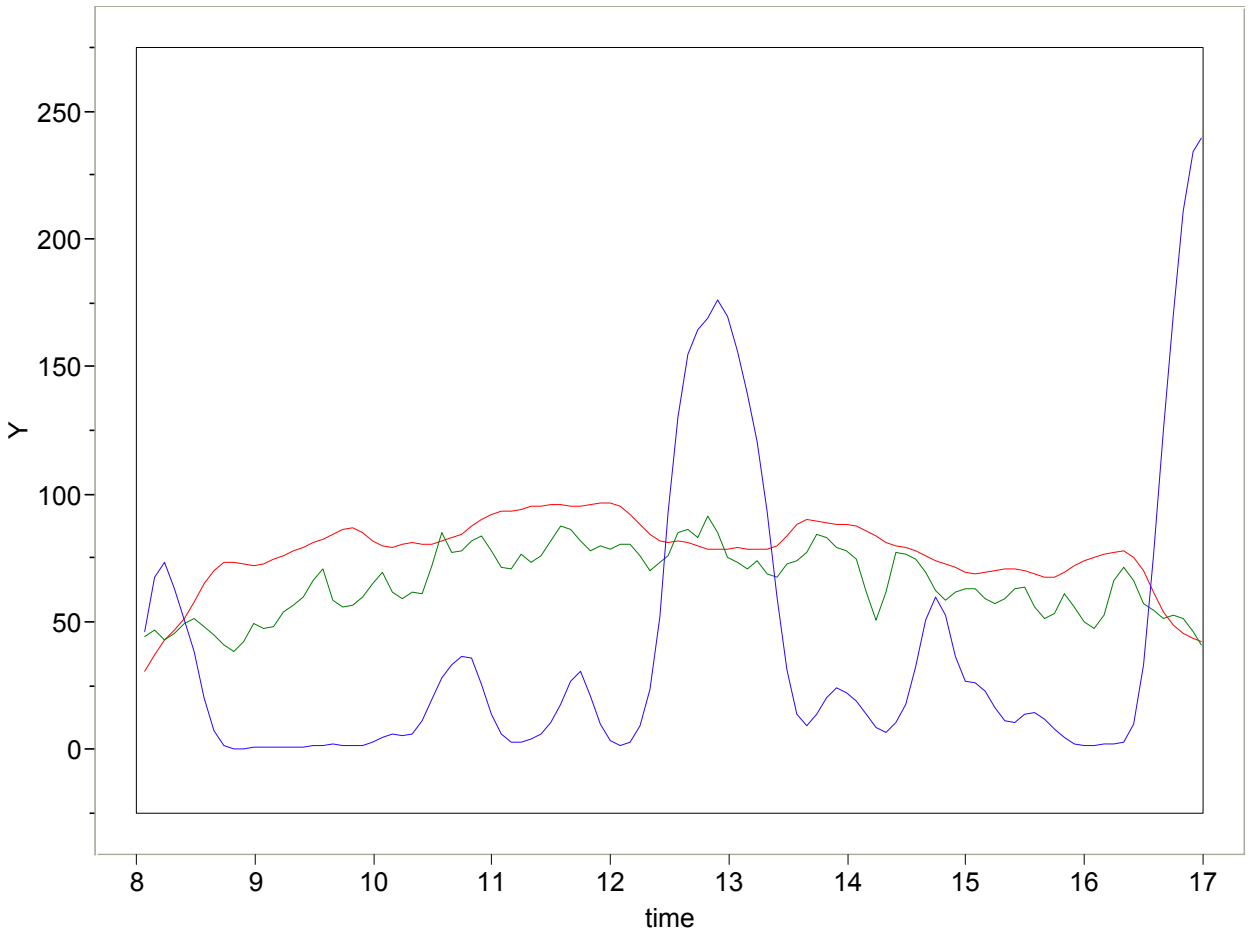


Plot is for Friday 8/02/02

Note increased usage from 7-7:30 am (typical of Fridays).
Note increased average Queue-Wait during this time.
(Accompanied by a rise in abandonments to about 10%.)

Overall Utilization: 8/02/02 = 88%
8/05/02 = 89%

Weekends are Different!



Sunday 8/18/02

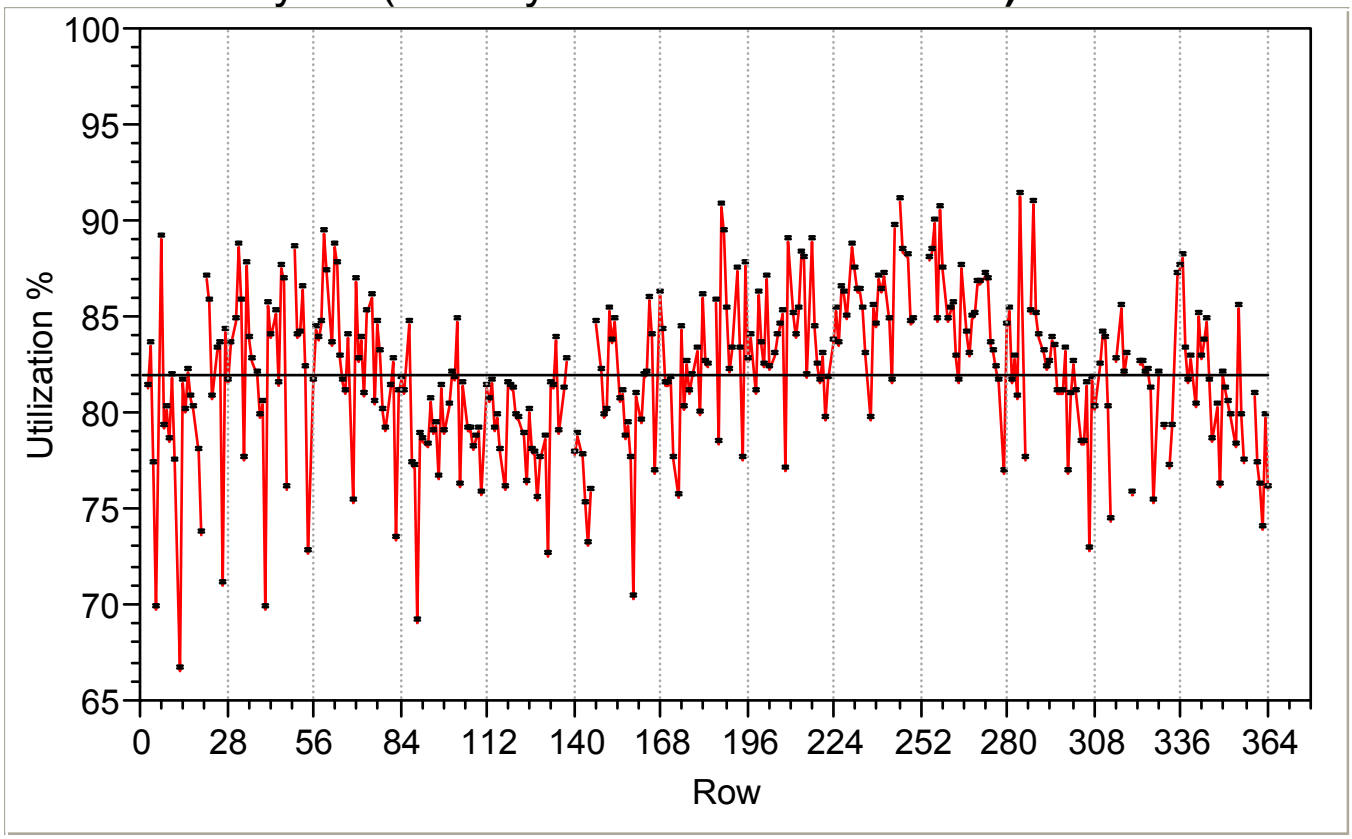
Y — NumberAgents (s)
— load (s)
— AvgQueueWaitAll (s)

Utilization = 89%
Abandonment Rate peaks at just over 20% (at ~1pm)

Utilization Fluctuates around ~80%

Time Series: Utilization % each day

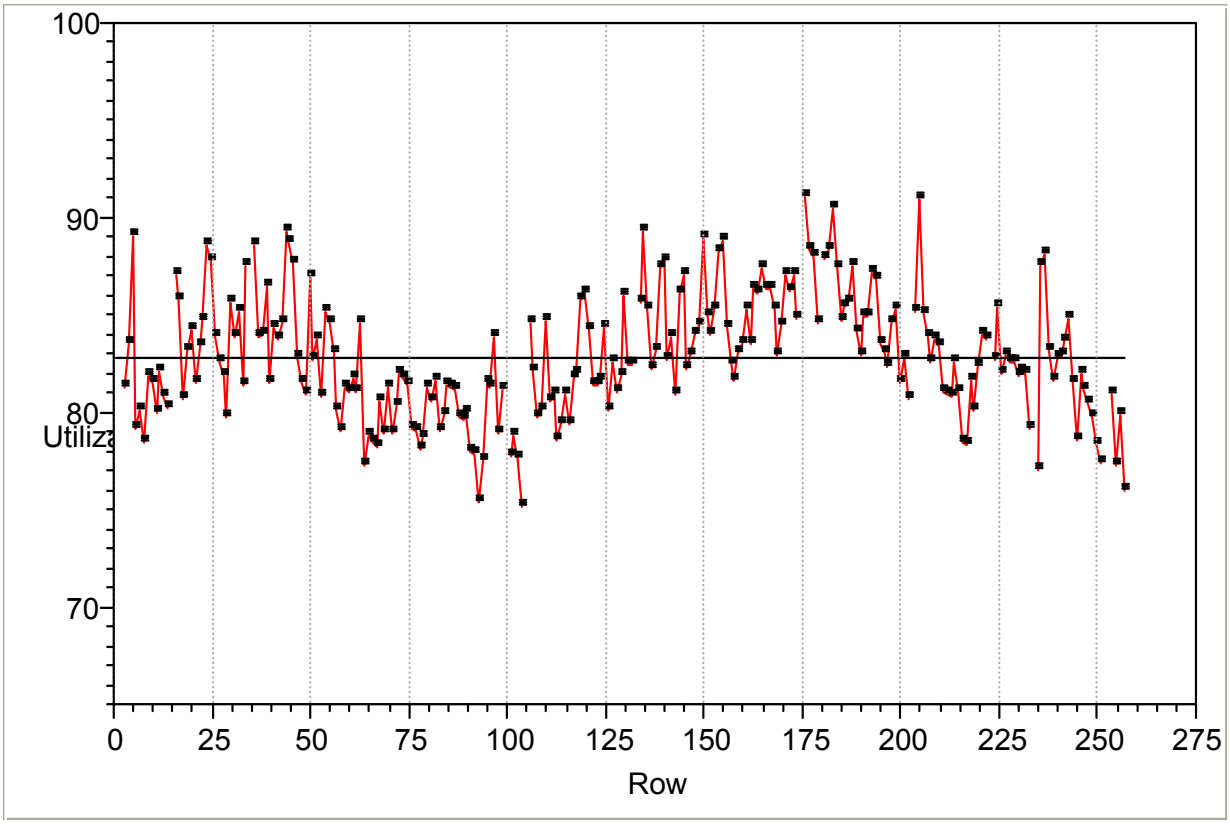
entire year (holidays and outliers omitted)



Mean 82.04

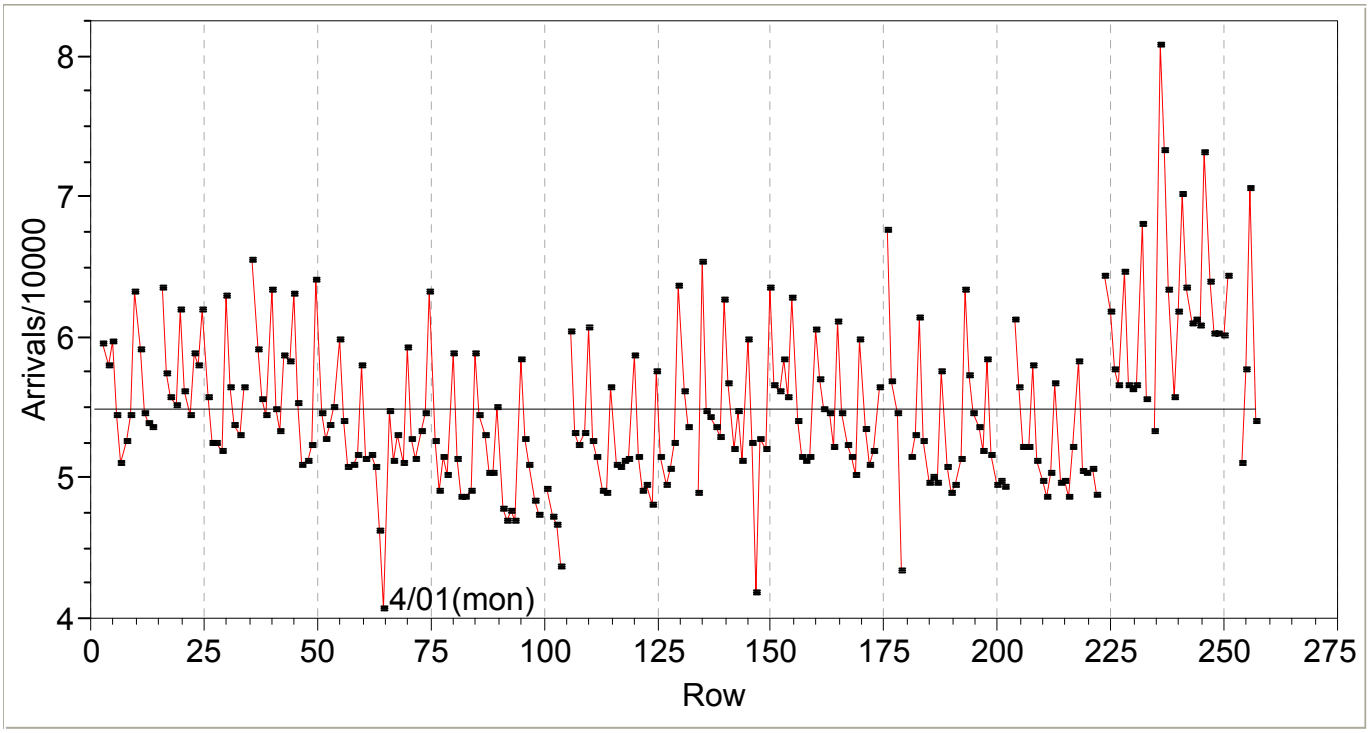
Time Series: Utilization %

Regular weekdays only (holidays and outliers omitted)



Mean 82.89

For Comparison
Daily Call Volumes (weekdays)



Prediction of Arrivals

Regularity of arrival density during the day -given day-of-week - has been noted.

Thus, let

L_{ij}=# Arrivals on day i during (150 second) interval j.

$d(i)$=Day-of-week of day i.

π_{dj}=theoretical proportion of arrival volume at time j on a day-of-week d.

MODEL: Given L_{i+},

$$L_{ij} = Multin(L_{i+}, \pi_{d(i)j}).$$

Approximate restatement:

(1) $$\left\{ \sqrt{L_{ij} + 1/4} \,\middle|\, L_{i+} \right\} \sim \mathbb{N}\left(\sqrt{L_{i+}\pi_{d(i)j}}, \frac{1}{4} \right)$$

Confirmation of (1)

Use the data, including the observed values of L_{ij}, and least squares (=maximum likelihood) to fit the model

$$\sqrt{L_{ij} + 1/4} \sim \mathbb{N}\left(\sqrt{L_{i+}\pi_{d(i)j}}, \frac{1}{4} \right).$$

IF THE MODEL IS CORRECT THEN MSE SHOULD BE APPROXIMATELY $\frac{1}{4}$, AS PREDICTED BY (1).

This was done for August 2002, and we found the following for each *d(i)*:

Confirmation (cont.)

The values of MSE for each day are given in the following table

week day	Ave Arr Num	MSE
Sun.	45.90	0.228
Mon.	299.60	0.263
Tues.	269.01	0.261
Wed.	257.27	0.281
Thur.	262.67	0.350
Fri.	261.01	0.310
Sat.	100.02	0.293

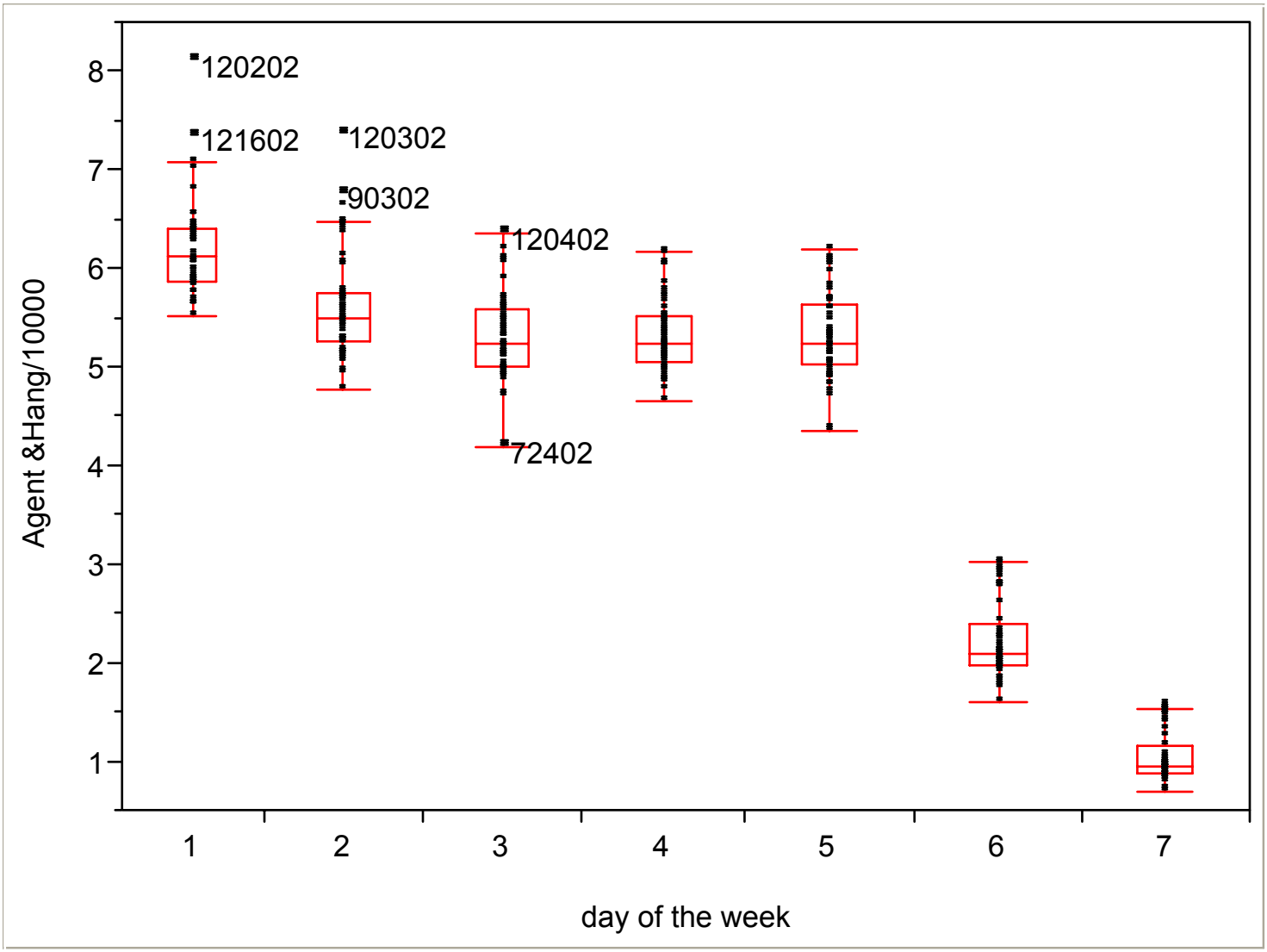
The relatively minor excess of MSE over **0.250** is due to (randomness and) to variability of the day-of-week densities. *(If arrivals were not inhomogeneous Poisson this could also contribute to excess; but we've ruled out such a possibility on the Israeli data, and it thus seems unreasonable here too.)*

Predicting the Daily Volume, L_{i+}

Empirically, this is the tough part!

Knowing the day-of-week of course helps:

Calls to Service-Q by Day-of-Week



Calls to Service-Q by Day-of-Week

Means and Std Errors

Day	Number	Mean	Std Err Mean	Lower 95%	Upper 95%
Mon	44	**6.23**	0.074	6.08	6.37
Tue	49	**5.63**	0.077	5.47	5.78
Wed	49	**5.33**	0.060	5.21	5.44
Thurs	50	**5.33**	0.053	5.23	5.44
Fri	52	**5.31**	0.059	5.20	5.43
Sat	52	**2.22**	0.053	2.12	2.33
Sun	51	**1.05**	0.035	0.98	1.12

ANOVA followed by a Tukey-Kramer-Hayter Multiple Comparison Test (at level $\alpha = 0.05$) yields as significant comparisons:

WED

MON > TUE > THU > SAT > SUN

FRI

Such a Model Still Isn't Very Good!

IF the L_{i+} were well predicted by a day-of week effect, **THEN** they would be Poisson (λ_d). Correspondingly, (approximately)

$$\sqrt{L_{i+} + 1/4} \sim \mathbb{N}\left(\sqrt{\lambda_{d(i)}}, \frac{1}{4}\right).$$

An ANOVA of $\sqrt{L_{i+} + 1/4}$ on *d(i)* yields

MSE = 110.3

This is

Much Bigger Than

the target value of MSE = 1/4.

Conclusion: Fluctuations in daily volume are **not** well explained by only the day-of-week and random Poisson fluctuation.

Suggestions

A prediction of tomorrow's $L_{tomorrow,+}$ should derive from a prediction of $W_{tomorrow} \triangleq \sqrt{L_{tomorrow,+}}$.
This should involve $d(tomorrow)$.
We suggest (as a first try) an autoregressive model of the form

$$W_j = \alpha_{d(j)} + \sum_{k=1}^{K} \rho_k \left(W_{j-k} - \alpha_{d(j-k)} \right) + \varepsilon_j$$

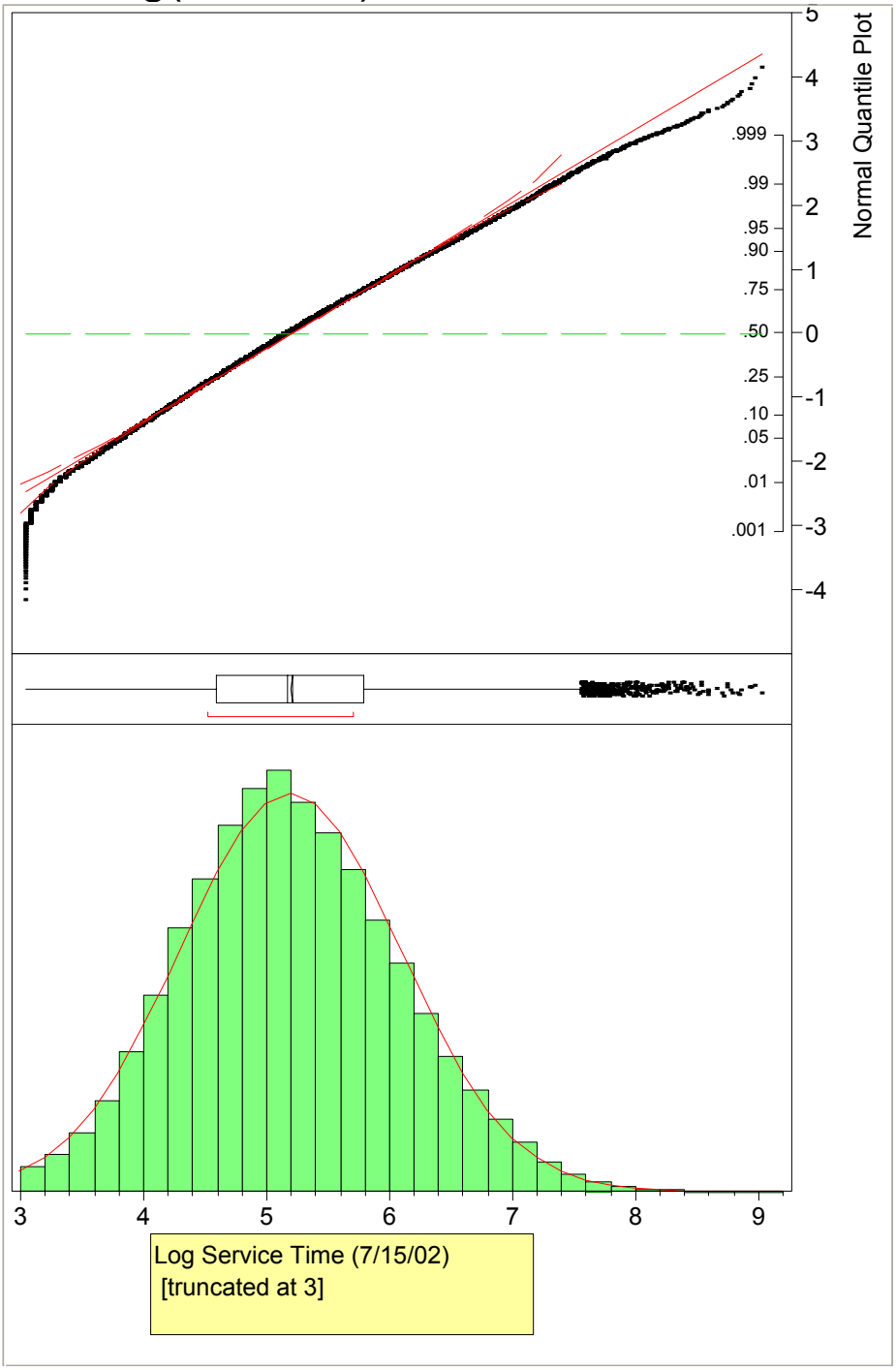
We fit such a model to the data from 2002.

- AIC/BIC suggested a choice of K=7.
- The significant coefficients were those for lags **1** and 7.
- The MSE was reduced to 46.3. This is a valuable reduction by $R^2 = 57\%$ compared to the model without the AR component.
- The variability in W_j is still **very large**.
- Suggestions.
    - Use other covariates or subjective information
    - Use volume in the morning (up to – say - 10am) to help predict volume for the remainder of the day

Service Times

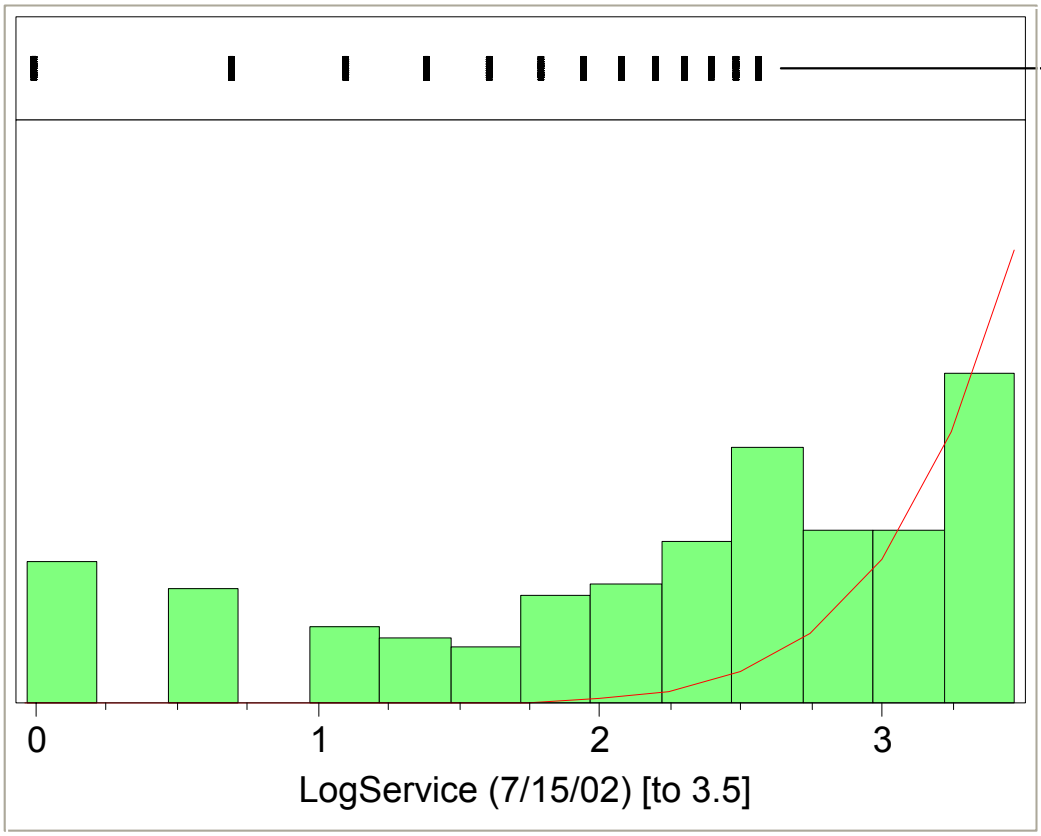
Service Times are (mostly) LogNormal

Log(SerTime) Plot for all times >20



Fit is a Normal(μ=5.2,μ=0.9) density, based on median and interquartile distance of the truncated log(Sertime) distribution.

Here is the plot for the smaller service times



The red curve is the normal density plot from the preceding fit

About 5% of the service times on 7/15/02 were ≤ 20 (*ie*, Log <3).

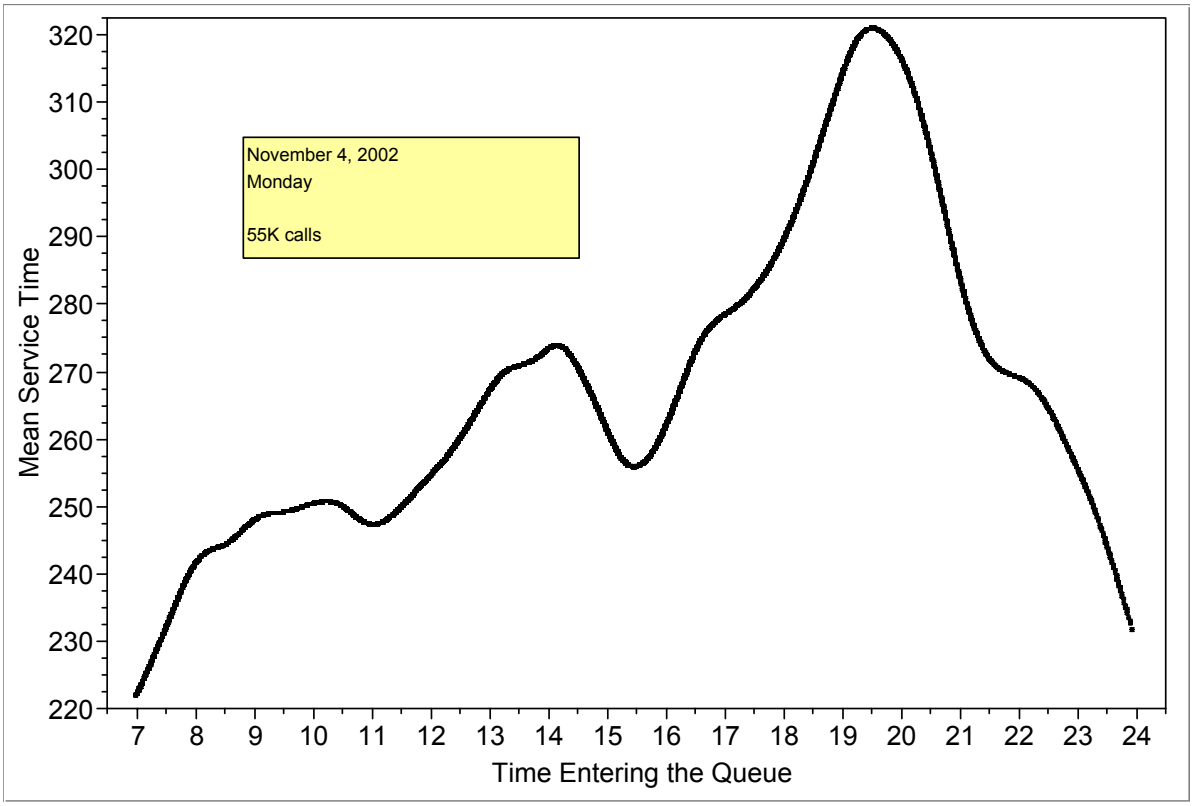
Note that the curve doesn't fit well in this part of the distribution, so the distribution of small service times does NOT follow the lognormal pattern.

CONCLUSION: Maybe Service times here should really be modeled as a mixture of 95% of a lognormal distribution and 5% of some special distribution involving only small times.

To Do: This general pattern works for 7/16 – 18 as well; but with minor – though statistically significant – changes in the parameters. Need to investigate all other days.

Mean Service Times Vary with Time of Day

Plot of Estimated **Mean** Service Time



Method for estimating the mean takes into account the fact that Service Times are (approximately) Lognormal.

(Other days we've looked at have qualitatively similar patterns.)