Patient Flow Management in Emergency Departments

Junfei Huang

With Boaz Carmeli and Prof. Avishai Mandelbaum

Outline

- Motivation
- Modeling ED Operations
- Two ED Models
    - Queue Length Model
    - Sojourn Time Model
- Case Study
- Intuition and Technical Ideas
- Summary and Contributions
- Future Directions

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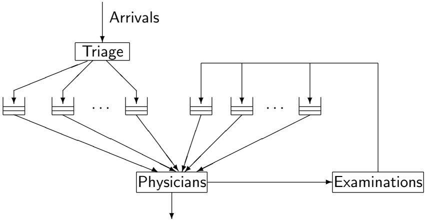
Emergency Department (ED) in Rambam Hospital

- Rambam Hospital is the largest hospital in northern Israel;
- The ED has 40 beds; 245 patients arriving daily;
- Patients: New vs. Work In Process (WIP);
- Canadian triage system:
  - New (triage) patients are classified into 5 **clinical** classes:
    - Resuscitation; Emergent; Urgent; Less Urgent; Non-urgent;
  - Result in several classes of WIP patients;
- Existing scheduling policy (static priority);



- ED is blocked, long sojourn time (with mean 4.5 hours, 10% over 6 hours);

Research Questions

- Objective of the project:
  - Design new scheduling policy within present triage system;
    - New: Pre-specified requirements on *time till first examination* – **Clinical**;
    - WIP: Push them out as soon as possible – **Operational**;
  - How do we make the tradeoff between **clinical** and **operational** considerations?
    - **Minimize the congestion**;
    - **Subject to the deadline constraints**;
- Achievement:
  - A **"nearly" optimal** and **implementable** dynamic scheduling policy;

A Good Hospital in China

Emergency Departments in China

卫生部拟将急诊分三区 病人按病情分四级

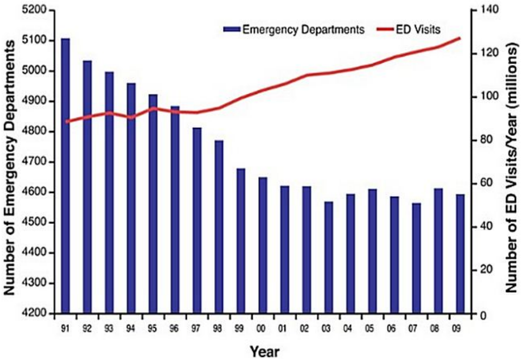
2011年09月07日 08:55:01 来源： 人民日报 【字号 大小】【收藏】【打印】【关闭】 分享到新华微博

卫生部日前公布《急诊病人病情分级试点指导原则（征求意见稿）》。卫生部拟将急诊科从功能结构上分为红黄绿“三区”，将病人的病情分为“四级”，从而提高急诊病人分诊准确率，保障急诊病人医疗安全。

征求意见稿提出，急诊病人病情的严重程度决定病人就诊及处置的优先次序。急诊病人病情分级不仅仅是给病人排序，而且要分流病人，使病人在合适的时间去合适的区域获得恰当的诊疗。

- In 2011, the MOH of China proposed to introduce a triage system to manage EDs;
  - Improve the quality of care (safety of patients);
- Patients are classified into 4 levels according to their severity;
- A natural problem is how to schedule the patients;

Emergency Departments in the United States



- ED environment has become more crowded (waiting time increased by 25% (46.5 to 58.1 mins), from 2003 to 2009);
- The need for the **tradeoff** becomes more pronounced;

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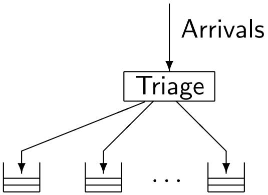
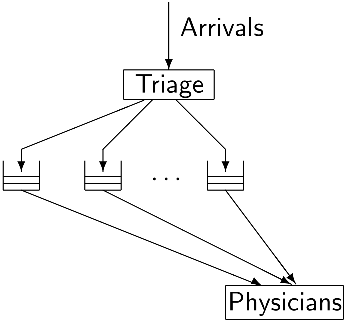
Patient Flow in Emergency Departments (EDs)

Arrivals

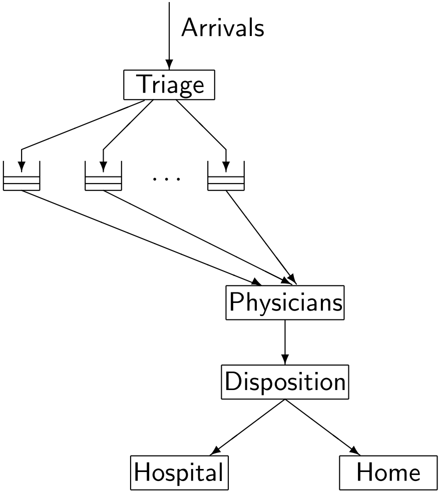
Patient Flow in Emergency Departments (EDs)



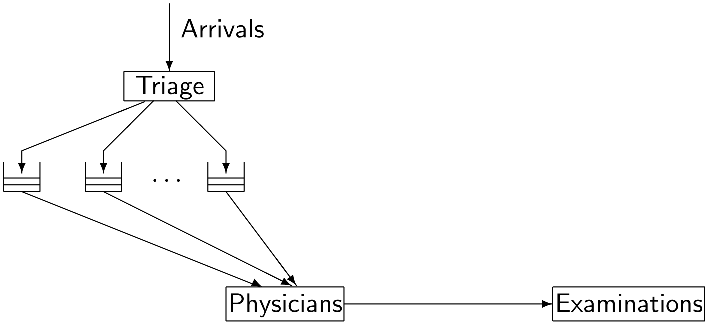
Patient Flow in Emergency Departments (EDs)



Patient Flow in Emergency Departments (EDs)



Patient Flow in Emergency Departments (EDs)



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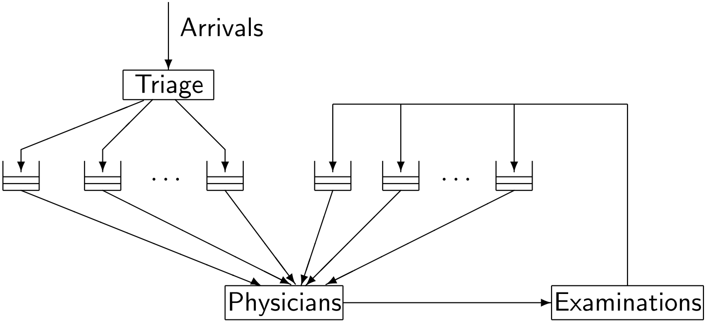

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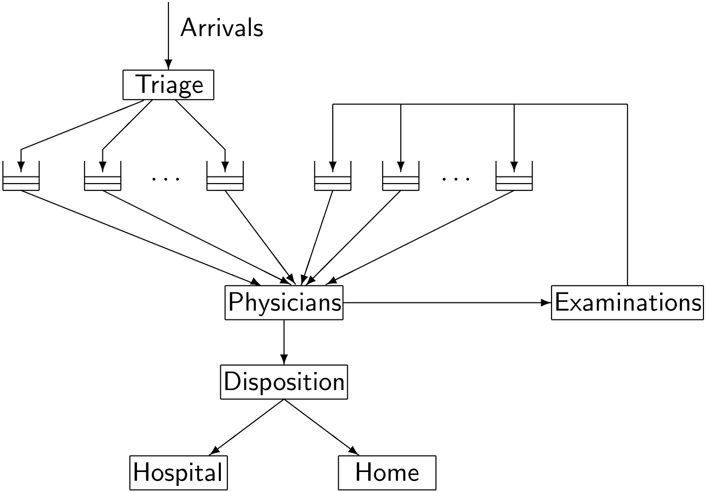

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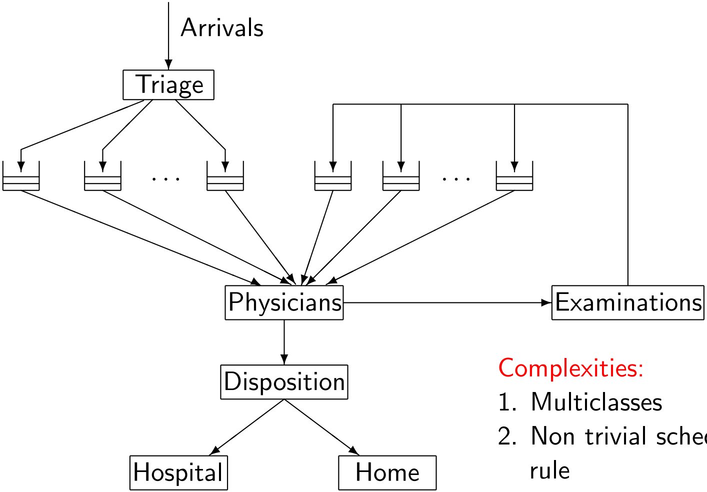


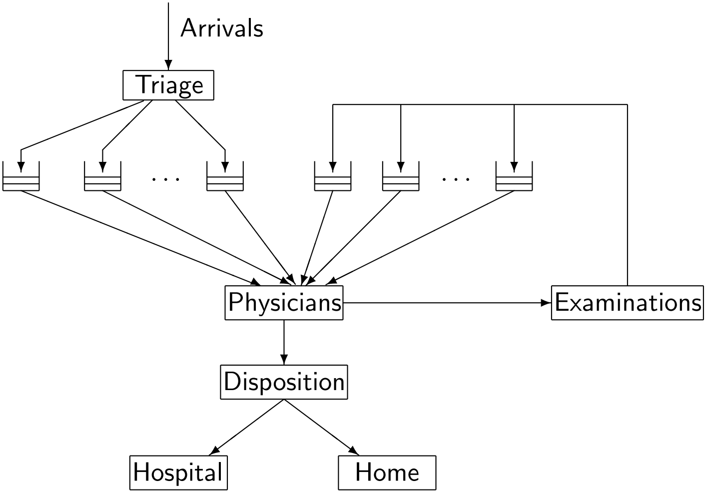
Complexities:

1. Multiclasses
2. Non trivial scheduling rule
3. ...

Complexities and the State of the Arts

- Many **simulation**-based studies;
- Few analytical models;
- An analytical model:
  - Manage patient flow in an ED, from a **queueing-theory** perspective;
  - Capture the **most important** features (what are they?);

What Are the Most Important Features?



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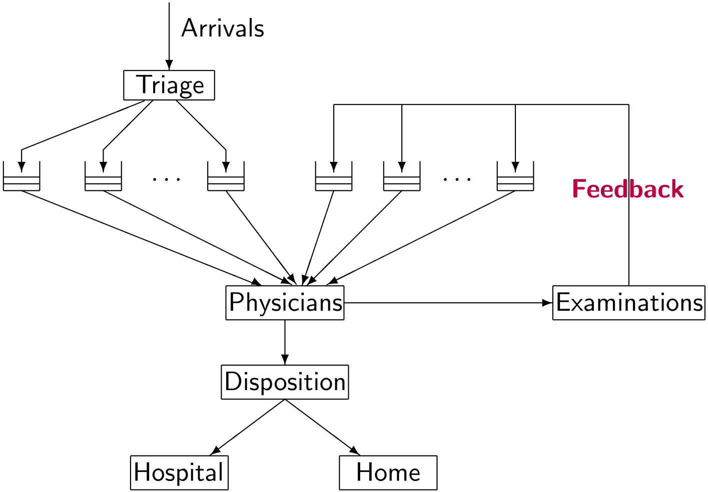

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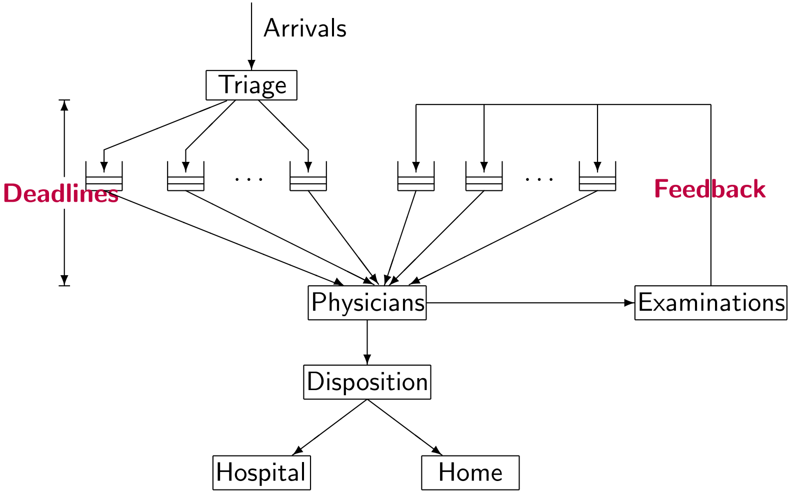

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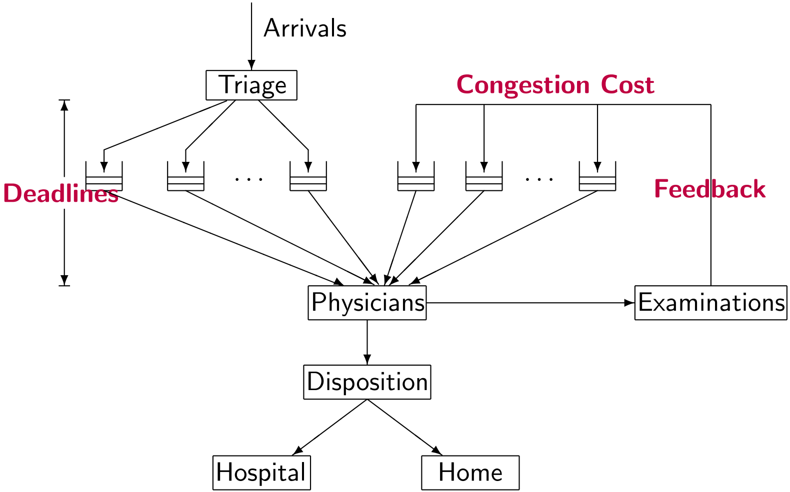

Example: The ED in Rambam Hospital

- **Feedback**
  - Empirical analysis shows, on average, each patient visits the physician for *at least 3* times;
- **Deadlines** on *time-till-first-treatment*:
  - Canadian Triage System – patients are classified into 5 levels:
    - Resuscitation (Immediate);
    - Emergent (15 mins); Urgent (30 mins); Less Urgent (60 mins); Non-urgent (120 mins);
- **Congestion** costs:
  - Waiting costs; clinical costs; emotional costs; psychological costs; others (long waits increase the probability of disaster);

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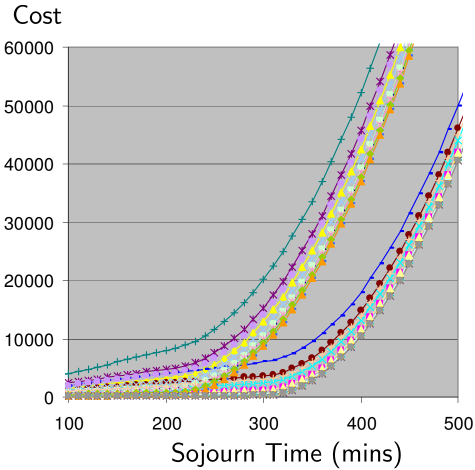
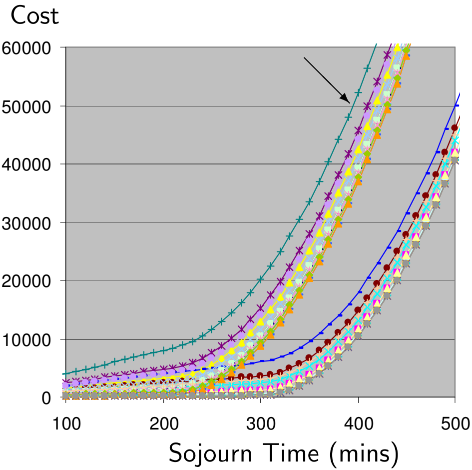


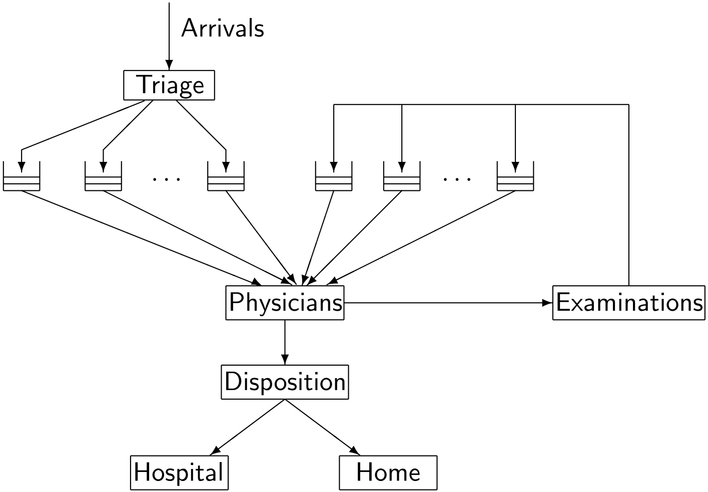
- Triage class (3); age (4); decision after treatments (2);

Example: The ED in Rambam Hospital

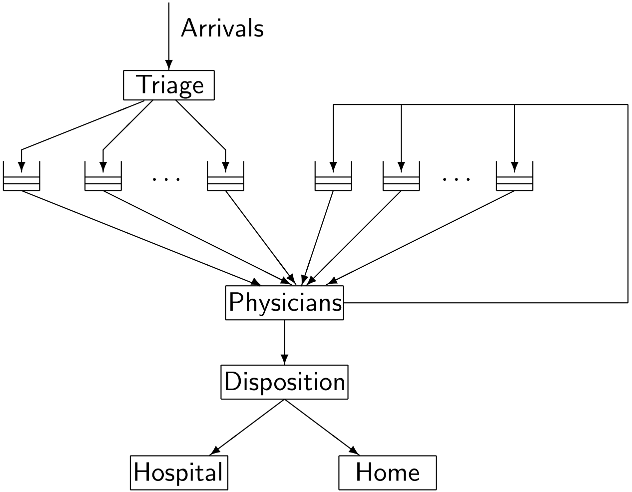


- Triage class (3); age (4); decision after treatments (2);
- Triage class 3 (urgent), Age>75, Discharged;

Modeling


Arrivals
Triage
. . .
. . .
Physicians
Examinations
Disposition
Hospital
Home

Modeling



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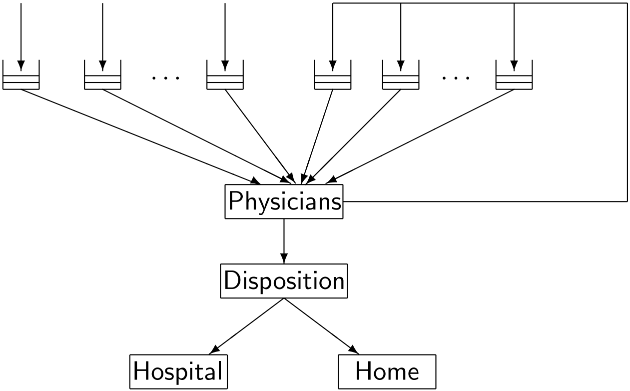

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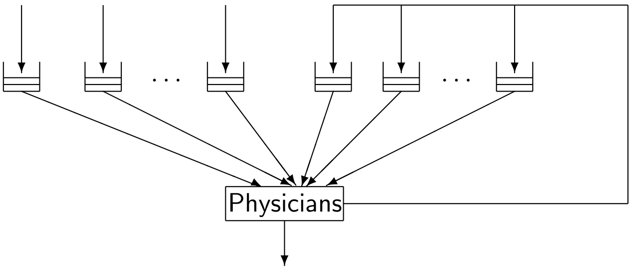

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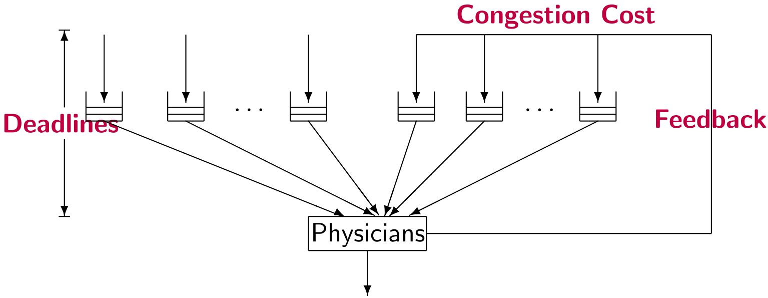

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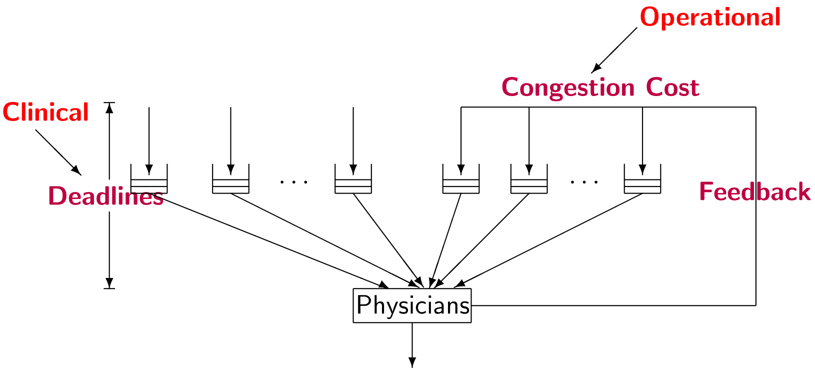
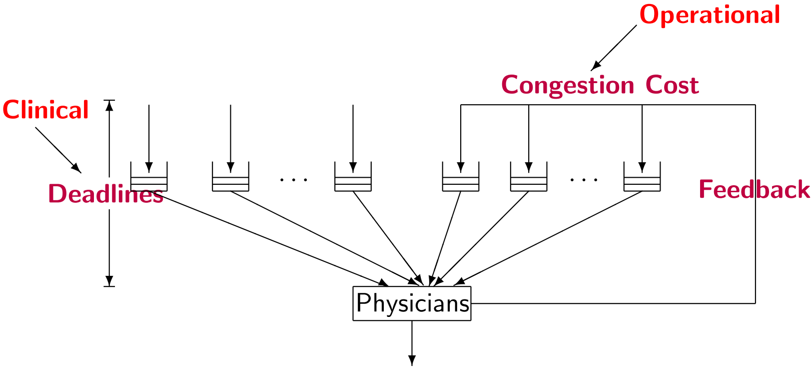


Operational
Congestion Cost
Clinical
Deadlines
Feedback
Physicians

Modeling



Clinical v.s. Operational – How to Balance?

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Two ED Models

	Queue Length Model	Sojourn Time Model
Congestion Cost	Queue Length	Sojourn Time
WIP Transition	Markovian	Deterministic

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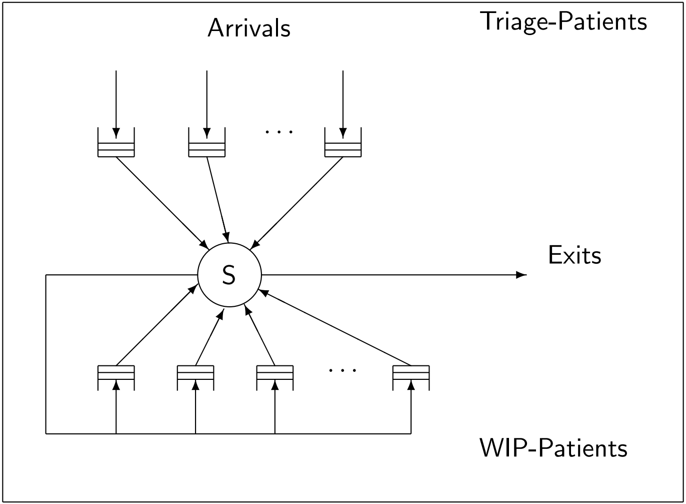
Queue Length Model – Structure



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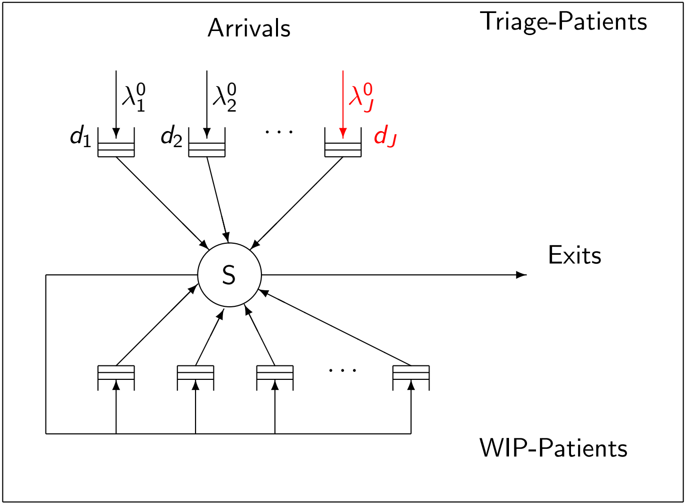
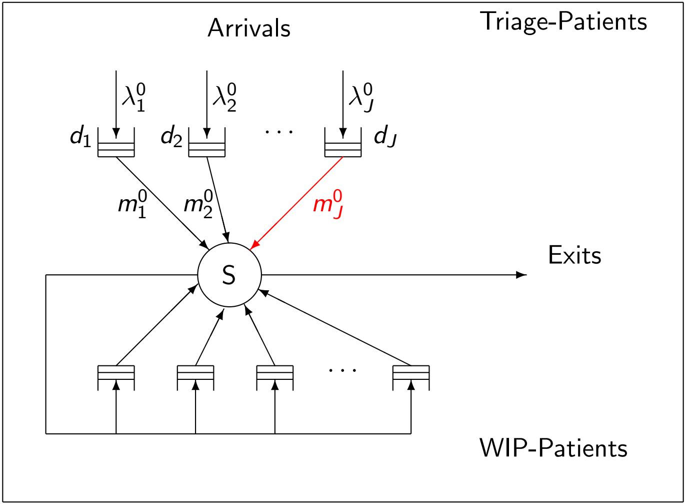


Age (in system) of the head-of-the-line patient: $\tau_j(t)$, then deadline constraints: $\tau_j(t) \leq d_j, j \in \mathcal{J}$.

Queue Length Model – Structure



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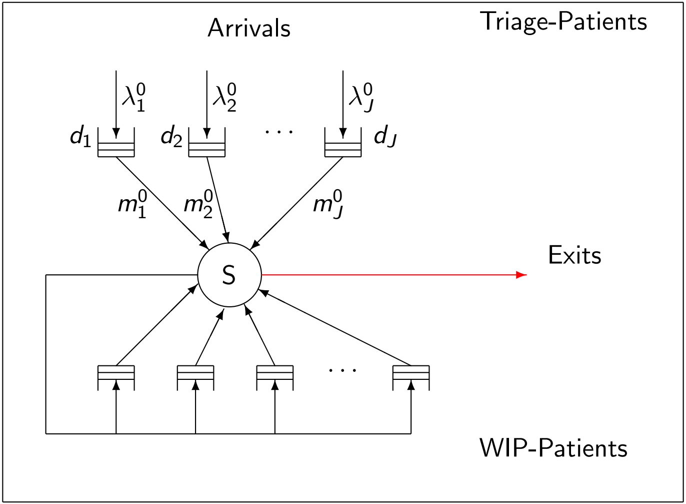

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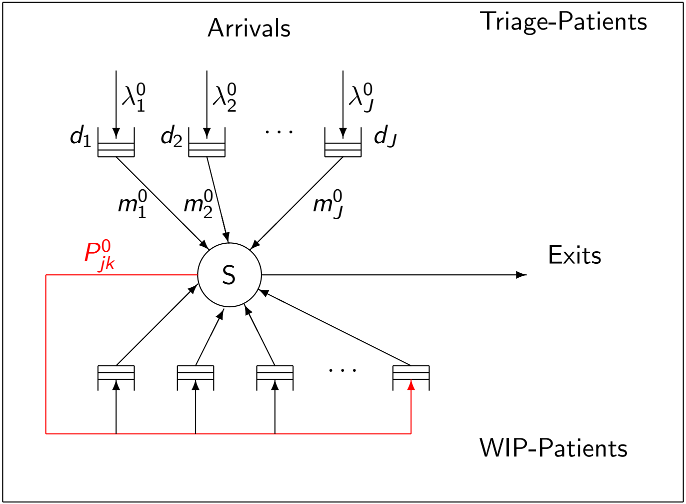

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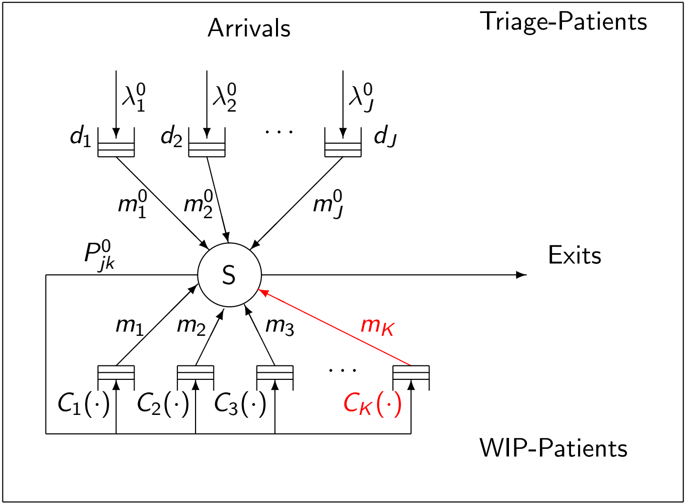
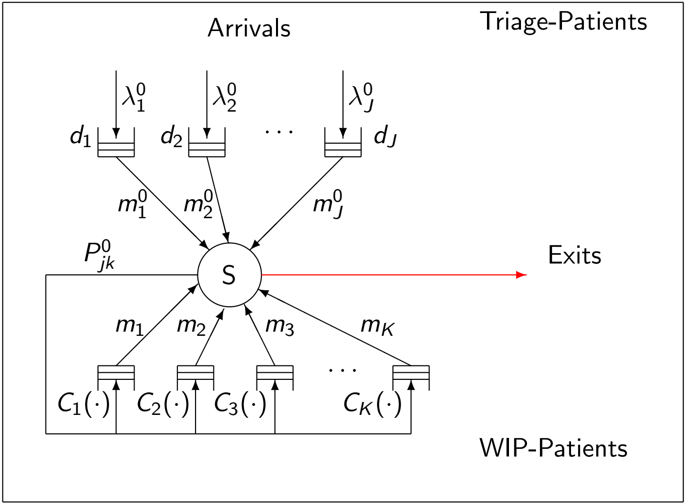


Queue length at time t: $Q_k(t)$, incur queueing (holding) cost at rate $C_k(Q_k(t))$; the total cost rate: $\sum_{k \in \mathcal{K}} C_k(Q_k(t))$.

Queue Length Model – Structure



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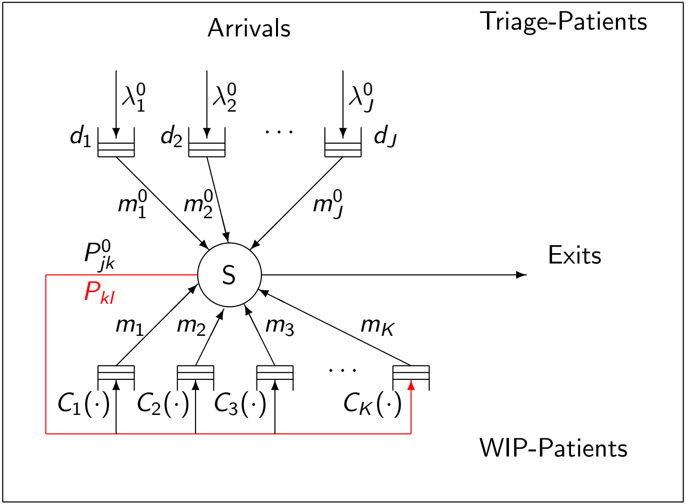


Problem Formulation:

- **Constrained optimization problem**: for any $T > 0$,

$$\min_{\pi \in \Pi} \int_0^T \sum_{k \in \mathcal{K}} C_k(Q_k^\pi(s)) \mathrm{d}s$$

$$\text{s.t.} \quad \tau_j^\pi(t) \le d_j, \quad \forall j \in \mathcal{J} \quad \text{and} \quad 0 \le t \le T.$$

- Example:
  - Linear holding cost:

$$C_1(x) = 2x \Rightarrow \int_0^T C_1(Q_1(t))dt = 2\int_0^T Q_1(t)dt;$$

  - $\tau_3(t) \le 30$, $\tau_4(t) \le 60$, $\tau_5(t) \le 120$;
- Infeasibility: $\tau_j, j \in \mathcal{J}$ random, d_j deterministic;
- **Asymptotic framework:**
  - relax: "feasibility" $\to$ "asymptotical compliance (feasibility)";
  - relax: "optimality" $\to$ "asymptotical optimality".

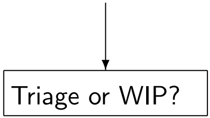
Main Result

Main Result

Identified a scheduling policy that is *"nearly"* optimal when the system is heavily loaded.

- The rigorous meaning of *"nearly optimal"* will be defined in the asymptotic framework;
- Heavily loaded system:
  - Intuitive: Arrival rate $\approx$ service capacity;
  - Rigorous: Traffic intensity $\rho = \frac{1}{S} \sum_{j \in \mathcal{J}} \lambda_j m_j^e \approx 1$.
  - Realistic: Crowded ED environment;
- What is the structure of the scheduling policy?

Queue Length Model – Scheduling Policy



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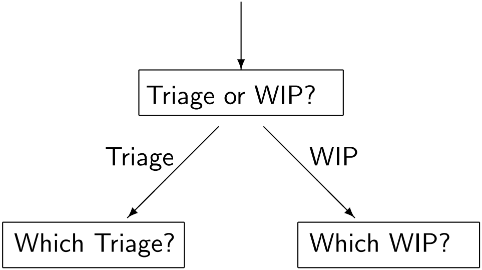

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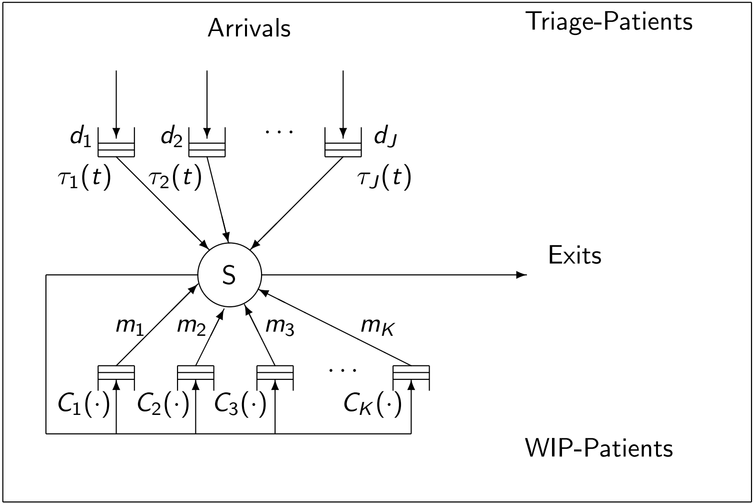

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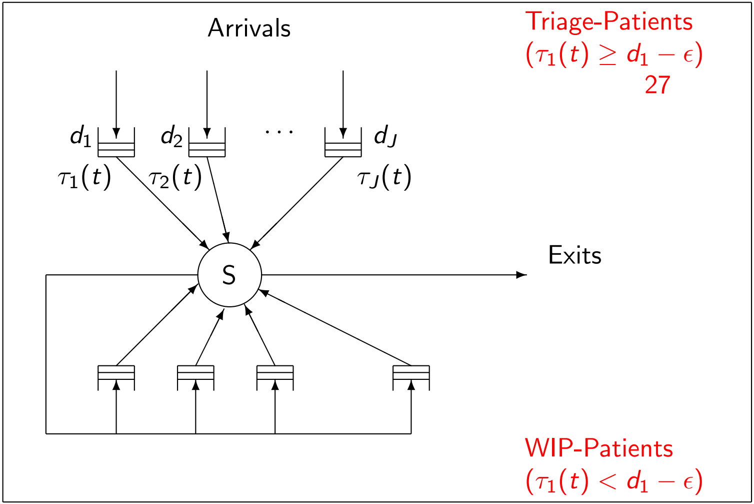

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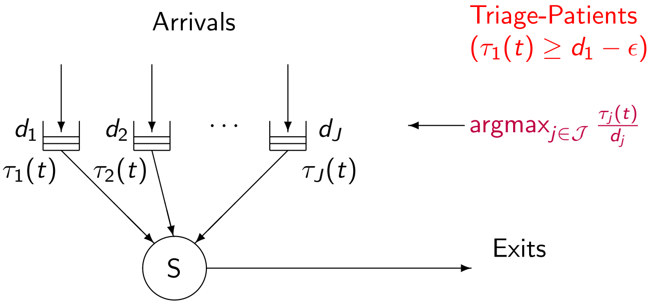
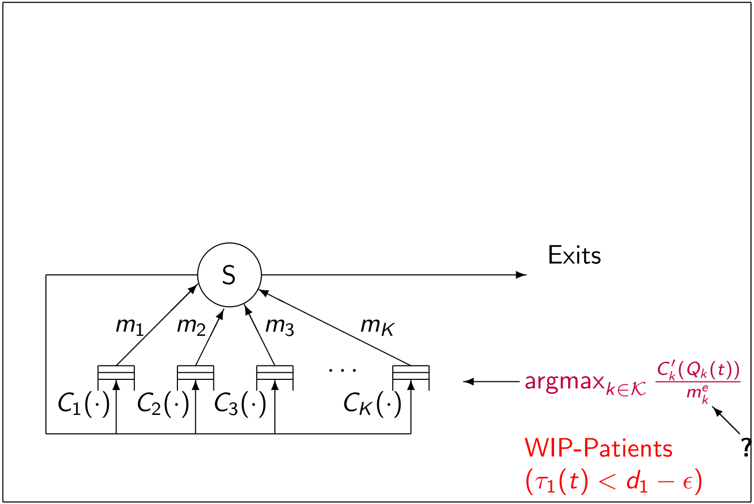


An Example:

Deadlines (d_j)	30	60	120
Ages (τ_j)	15	20	50
τ_j/d_j	0.5	0.33	0.42

Queue Length Model – Scheduling Policy



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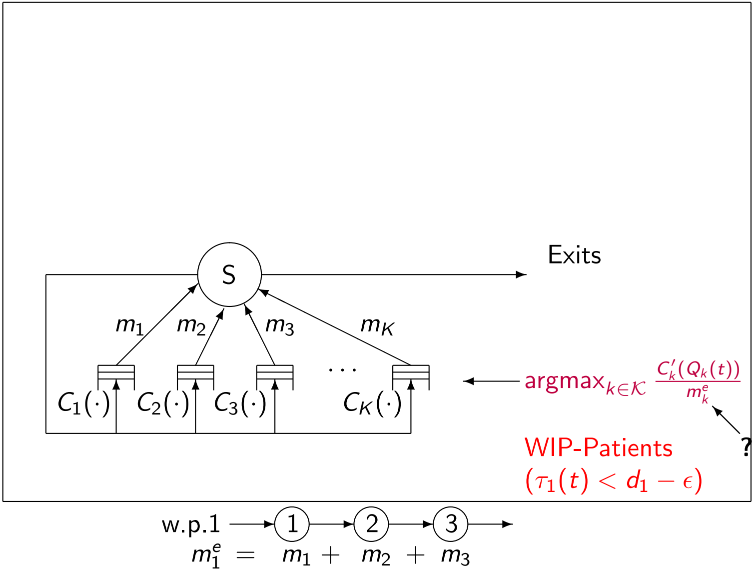

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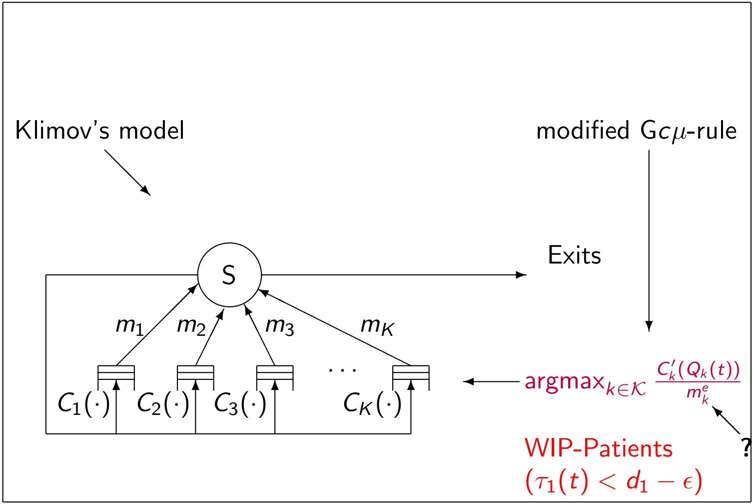
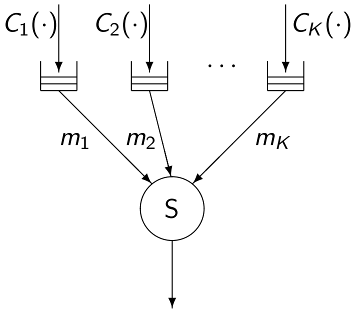


Surprising: a "$Gc\mu$"-rule for Klimov's model!

Queue Length Model – Scheduling Policy

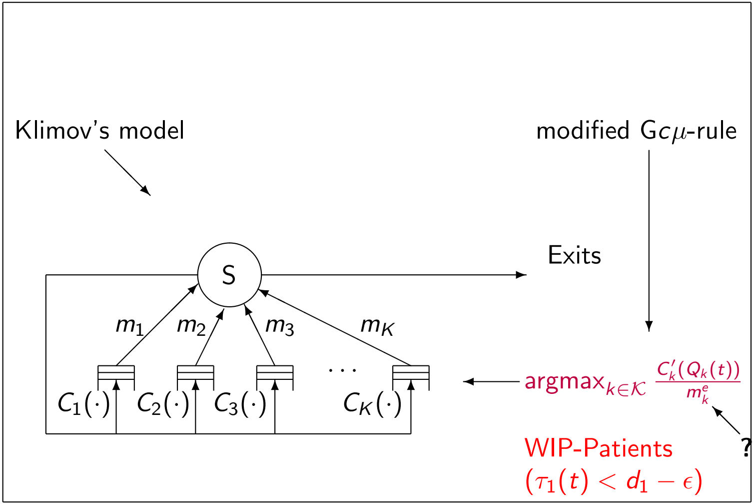


$Gc\mu$-rule:

argmax $C_k'(Q_k(t))\mu_k$

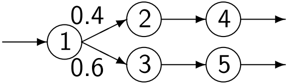
$(\mu_k = \frac{1}{m_k})$

Queue Length Model – Scheduling Policy



Effective Mean Service Time

- **Effective mean service time** of a patient is the expectation of the *total* potential service requirement of that patient after entering into that class;
- An example:



$$m_1^e \;=\; m_1 + 0.4m_2^e + 0.6m_3^e$$

- $M^e = (m_k^e)_{k\in\mathcal{K}}$ of WIP patients[1]:

$$M^e = M + PM^e \quad \Rightarrow \quad M^e = [I-P]^{-1}M;$$

- $M^e_{\mathcal{J}} = (m_j^e)_{j\in\mathcal{J}}$ of Triage patients:

$$M^e_{\mathcal{J}} = M_{\mathcal{J}} + P_{\mathcal{J}\mathcal{K}}M^e;$$

[1] $M_{\mathcal{J}} = (m_j)_{j\in\mathcal{J}}, P_{\mathcal{J}\mathcal{K}} = [P^0_{jk}], M = (m_k)_{k\in\mathcal{K}}, P = [P_{kl}];$

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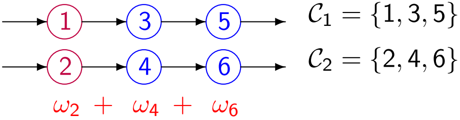
Sojourn-Time Model

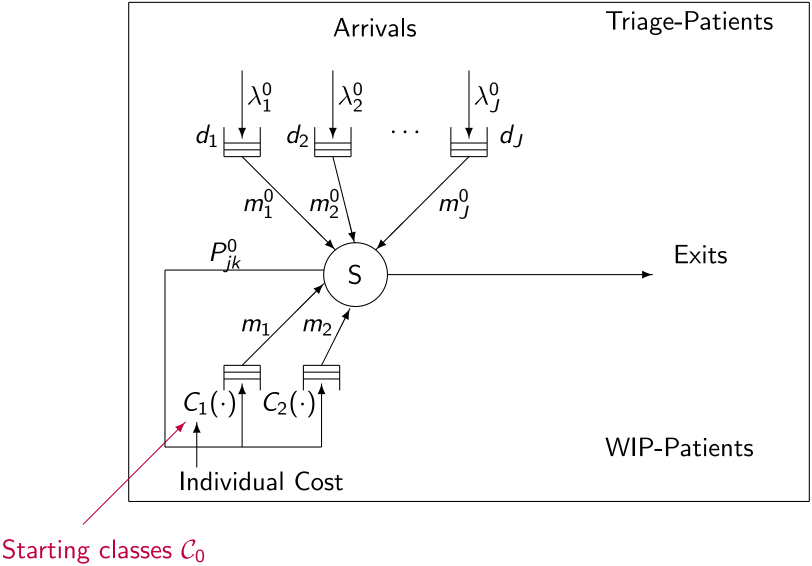
- Deadline constraints on triage patients do not change;
- For WIP visits, each patient has a deterministic routing vector;
  - **Realistic**: blood test → X-ray → ECG → ...;
- $\mathcal{C}_0$: the *starting classes* of any route;
- $\mathcal{C}_k$: all classes on a route starting with $k \in \mathcal{C}_0$;
- $\bigcup_{k \in \mathcal{C}_0} \mathcal{C}_k \backslash \{k\}$: *subsequent classes*;



- Congestion cost:

$$\mathcal{S}(t) := \sum_{k \in \mathcal{C}_0} \sum_{i=1}^{E_k(t)} C_k \left(\sum_{k' \in \mathcal{C}_k} \omega_{k'}(i) \right);$$

Sojourn Time Model – Structure



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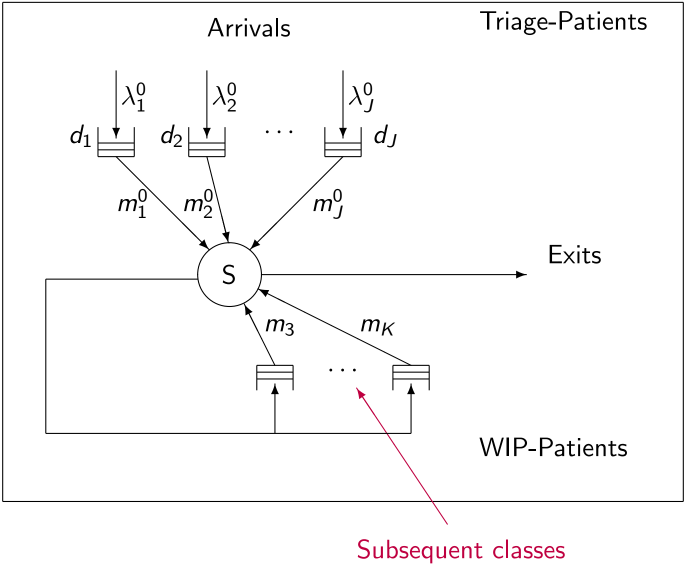
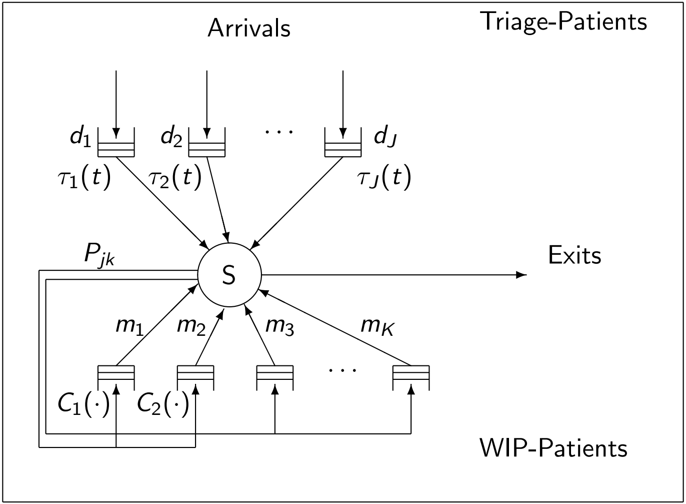


Sojourn Time Model – Scheduling Policy



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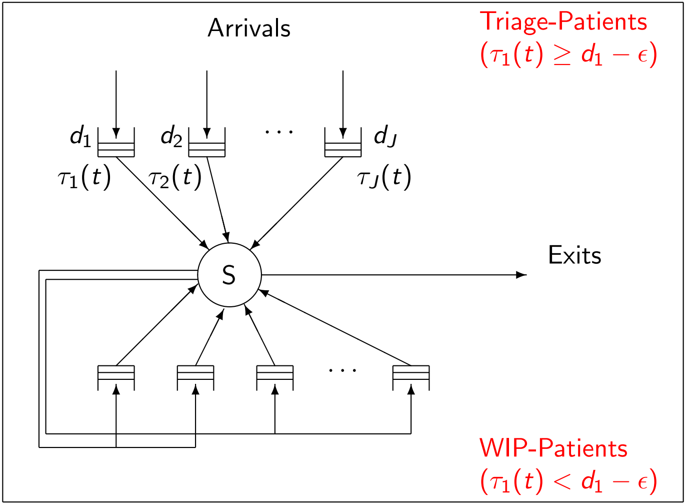

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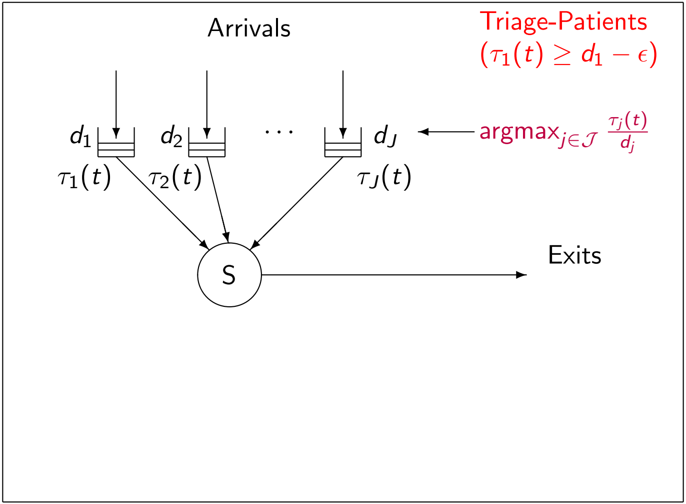
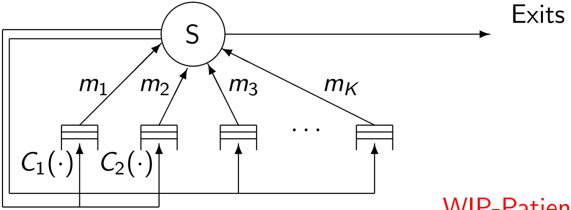
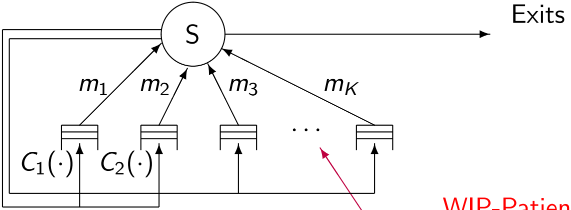


Sojourn Time Model – Scheduling Policy



WIP-Patients
$(\tau_1(t) < d_1 - \epsilon)$

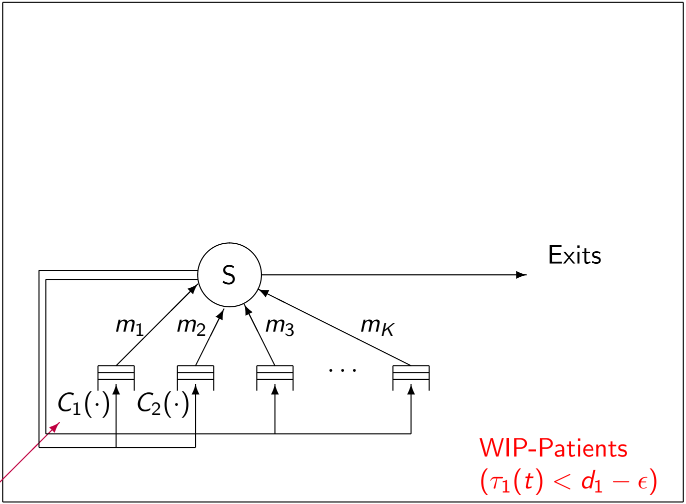
Sojourn Time Model – Scheduling Policy



WIP-Patients
$(\tau_1(t) < d_1 - \epsilon)$

Subsequent classes

Sojourn Time Model – Scheduling Policy



Starting classes, $\text{argmax}_{k \in \mathcal{C}_0} \frac{C'_k(\tau_k(t))}{m_k^e}$

Interim Summary

- Analyze the emergency departments;
  - Capture the tradeoff between triage- vs. WIP-patients;
- Build two queueing models;
  - Queue Length Model;
  - Sojourn Time Model;
- Provide good and implementable scheduling policies;

Table : Comparison of Two Models

	Queue Length Model	Sojourn Time Model
Congestion Cost	Queue Length	Sojourn Time
WIP Transition	Markovian	Deterministic
WIP Policy	Queue Length	Age

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An ED Case Study: Value of Information

- Data is from ED in Rambam; **sojourn time model**;

# WIP visits	1	2	3	4	5
Proportion	0.28	0.30	0.28	0.11	0.03

A & D Status	Admitted	Discharged
Proportion/Cost function	0.40, t^2	0.60, $2t^2$

- Is it worthy to estimate these two kinds of information upon a patient's arrival?
- Patients are classified into different classes according to the availability of these two kinds of information:

	Case 1	Case 2	Case 3
# WIP visits	N	Y	Y
A & D Status	N	N	Y
Congestion Cost	Benchmark	⇓18.01%	⇓26.8%

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- **Good news:** A well trained nurse can estimate both kinds of information very accurately!

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Intuition for the Queue Length Model

- $A(t)$: total potential service requirement brought into the ED;
- $T(t)$: amount of service requirement has been served;
- $W(t) = A(t) - T(t)$: total potential service requirement left:
  - minimized by work-conserving policy;
  - **invariant** to any work-conserving policy;
  - conditional on the queue length processes,

$$\boxed{W(t) \approx \sum_{j \in \mathcal{J}} m_j^e \times Q_j(t) + \sum_{k \in \mathcal{K}} m_k^e \times Q_k(t)}$$

- As a result, $\sum_{j \in \mathcal{J}} m_j^e \times Q_j(t) + \sum_{k \in \mathcal{K}} m_k^e \times Q_k(t)$ is minimized and invariant to any work-conserving policy;

Intuition on the Policy

$$\begin{aligned} \text{min} \quad & \textstyle\sum_{k \in \mathcal{K}} C_k(Q_k(t)) \\ \text{s.t.} \quad & \tau_j(t) \le d_j, \quad j \in \mathcal{J}; \end{aligned}$$

Myopic

Intuition on the Policy

$$\min \quad \sum_{k\in\mathcal{K}} C_k(Q_k(t))$$

$$\text{s.t.} \quad \tau_j(t) \le d_j, \quad j \in \mathcal{J};$$

$$\sum_{k\in\mathcal{K}} m_k^e \times Q_k(t) + \sum_{j\in\mathcal{J}} m_j^e \times Q_j(t) \approx W(t);$$

Intuition on the Policy

$$\begin{aligned} \min \quad & \sum_{k\in\mathcal{K}} C_k(Q_k(t)) \\ \text{s.t.} \quad & \tau_j(t) \le d_j, \quad j \in \mathcal{J}; \\ & \sum_{k\in\mathcal{K}} m_k^e \times Q_k(t) \approx W(t) - \sum_{j\in\mathcal{J}} m_j^e \times Q_j(t); \end{aligned}$$

Intuition on the Policy

$$\min \quad \sum_{k \in \mathcal{K}} C_k(Q_k(t))$$

s.t.

$$\sum_{k \in \mathcal{K}} m_k^e \times Q_k(t) \approx (W(t) - \sum_{j \in \mathcal{J}} \lambda_j d_j m_j^e)^+$$

Intuition on the Policy

min $\quad \sum_{k\in\mathcal{K}} C_k(Q_k(t))$

s.t.

$$\sum_{k\in\mathcal{K}} m_k^e \times Q_k(t) \approx (W(t) - \sum_{j\in\mathcal{J}} \lambda_j d_j m_j^e)^+$$

$$\Big\downarrow \text{KKT}$$

$$\frac{C_k'(Q_k(t))}{m_k^e} = \frac{C_{k'}'(Q_{k'}(t))}{m_{k'}^e}$$

"$Gc\mu$"-rule

Analysis – Asymptotic Framework

What is “nearly optimal”?

Asymptotic Framework

- A sequence of systems, indexed by $r \uparrow \infty$:
- Arrival rate for class j triage patients $\lambda_j^r \to \lambda_j > 0, \quad j \in \mathcal{J}$;
- Service requirement & routing behavior do not change;
- *Traffic intensity*: $\rho^r = \sum_{j \in \mathcal{J}} \lambda_j^r m_j^e$;
- (Conventional) *heavy traffic condition:* there exists a $\beta \in \mathbb{R}$,

$$r(\rho^r - 1) \to \beta, \quad \text{as} \quad r \to \infty.$$

- A family of control policies $\{\pi^r\}$ is called *asymptotically compliant (feasible)* if for any fixed $T > 0$, as $r \to \infty$,

$$\sup_{0 \le t \le T} \left[\widehat{\tau}_j^r(t) - \frac{d_j^r}{r} \right]^+ \Rightarrow 0, \quad j \in \mathcal{J}.$$

$\widehat{\tau}_j^r(t) := \frac{1}{r} \tau_j^r(r^2 t)$ – diffusion scaled age processes w.r.t $\{\pi^r\}$.

Asymptotic Optimization

- Diffusion scaled queue length processes:

$$\widehat{Q}_k^r(t) = \frac{1}{r} Q_k^r(r^2 t), \quad k \in \mathcal{K}.$$

- Cumulative queueing cost:

$$\mathcal{U}^r(t) := \int_0^t \sum_{k \in \mathcal{K}} C_k \left(\widehat{Q}_k^r(s) \right) \mathrm{d}s.$$

- A family of control policies $\{\pi_*^r\}$ is said to be *asymptotically optimal* if
  - it is asymptotically compliant and
  - it stochastically minimizes the cumulative cost:

$$\limsup_{r \to \infty} \mathcal{U}_*^r(t) \leq_{s.t.} \liminf_{r \to \infty} \mathcal{U}^r(t),$$

$\{\mathcal{U}_*^r\}$ – queueing cost corresponding to $\{\pi_*^r\}$;

$\{\mathcal{U}^r\}$ – corresponding to any asymptotically compliant policies.

The proposed family of control policies is asymptotically optimal.

A Roadmap for the Proofs

- There is a **lower bound** for any asymptotically compliant family of policies;
- The proposed family of scheduling policies **achieves** the lower bound;
  - State-Space-Collapse (SSC);

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Conclusions – Contributions

- **Practical:** model and analyze the control of patient flow in EDs:
  - Give rise to insightful and **implementable** scheduling policies;
  - Capture the **tradeoff** between triage- vs. WIP-patients;
  - Enable analysis of the value of **information** in a real ED setup.
- **Theoretical:** analyze multiclass queueing systems with feedback:
  - Prove the **conjecture** in Mandelbaum and Stolyar (OR, 2004), improve upon it with simpler asymptotically optimal policies;
  - **G**$c\mu$ **rule** for Klimov's model with convex costs (queue length, individual waiting times and cumulative sojourn times);
  - Analyze multiclass queueing systems with feedback, under **any** work-conserving policy;

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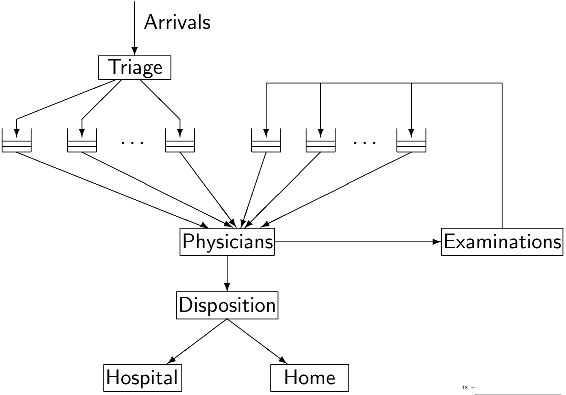
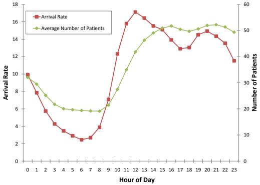
Future Directions



- Other features:
  - Time varying arrival rate;
  - Adding delays between transfers;
  - Adding global constraint on sojourn times;
  - Adding abandonment (LWBS, LAMA);



Thank You!

Questions?

Intuition on the Policy

$$\min \quad \sum_{k\in\mathcal{K}} C_k(Q_k(t))$$

$$\text{s.t.} \quad \tau_j(t) \le d_j, \quad j \in \mathcal{J};$$

Intuition on the Policy

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\end{aligned}
$$

Intuition on the Policy

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& \textstyle\sum_{k \in \mathcal{K}} m_k^e \times Q_k(t) \approx W(t) - \sum_{j \in \mathcal{J}} m_j^e \times Q_j(t);
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↓ Minimize

$$(W(t) - \sum_{j\in\mathcal{J}} \lambda_j d_j m_j^e)^+$$

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↓ How?

A threshold policy

$$\sum_{j\in\mathcal{J}} \lambda_j m_j^e \tau_j(t) \text{ vs. } \sum_{j\in\mathcal{J}} \lambda_j m_j^e d_j$$

Intuition on the Policy

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$$\sum_{j\in\mathcal{J}} \lambda_j m_j^e \tau_j(t) \text{ vs. } \sum_{j\in\mathcal{J}} \lambda_j m_j^e d_j$$

$\xleftarrow{\frac{\tau_j(t)}{d_j} \approx \frac{\tau_{j'}(t)}{d_{j'}}}$

$$\tau_1(t) \text{ vs. } d_1$$

Intuition on the Policy

min $\quad \sum_{k\in\mathcal{K}} C_k(Q_k(t))$

s.t.

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