Data-Based Processing Networks

or Inference, Design & Control of Service Systems

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http://ie.technion.ac.il/serveng

DSC/e Launch Symposium, December 2013

- **Lecture will be downloadable from my Technion website**

Research Goals

- Reality: Service (Processing) Networks
  e.g. Hospitals, Call Centers, Websites, . . .

- Models = **ServNets**

  Simple models at the service of complex realities:

  Q-Nets, Sim-Nets; F-Nets, D-Nets, . . .

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- Research: **Data-based creation**, **analysis** and **validation**
- Goals: (Reproducible) research & teaching (that impact practice)

$\Rightarrow$ Data & analysis of ServNets: accessible, useful

$\Rightarrow$ **Creation & Validation** of ServNets: Automatic, online
(Starting **w/ A. Gal, A. Senderovic, M. Weidlich)**

Why be Optimistic?

- Research in **OR/QS/IE** + Technologies, at Technion / SEELab:
  - Need Process Mining to overcome "curse of simplicity": “my models too simple to be credible"
    – Validate my “simple" models against complex PM “realities"
    – Enrich them when desired (research) or needed (practice)
    – Create a Science for SEELab technologies/heuristics
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  - PM: Create SimNet, QNet, FNet, DNet from hospital data
  - OR: Compare FNet against QNet – **Accuracy**
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  - OR: Refine FNet with DNet – improve accuracy
  - **OR+PM: Validate FNet+DNet against SimNet $\approx$ Reality** – **Value**
    Note: No need for QNets

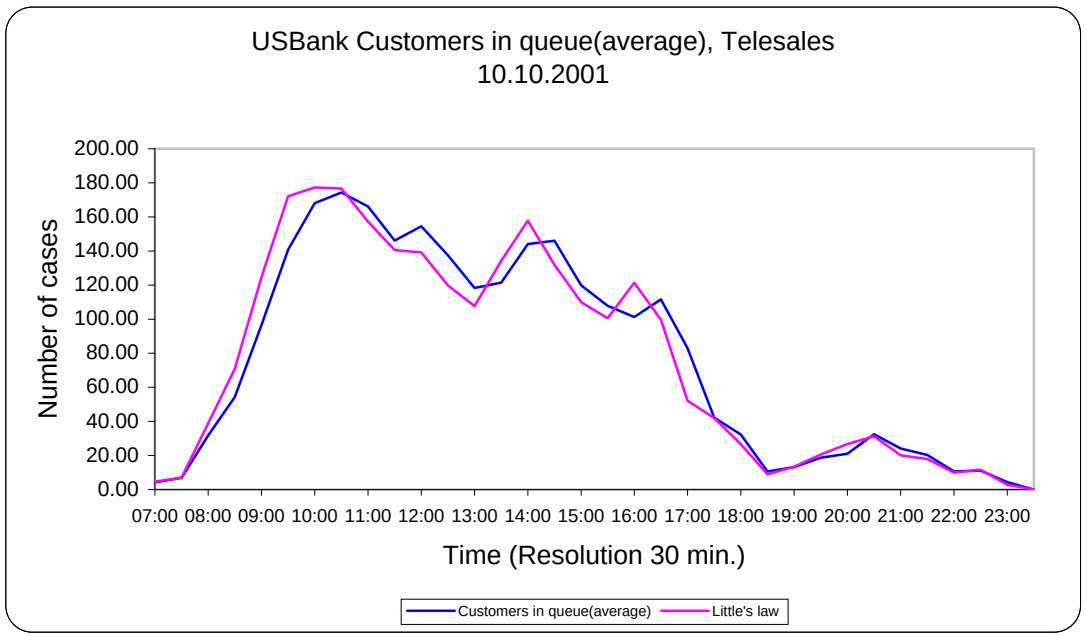
Contents

1. **2 simple models**
   - Emergency Department: **Time-Varying**
     - Mass Casualty Event: QNet and FNet (2 hours) - **performance**
     - Normal: QNet, DNet and SimNet (over 1 day) - **staffing**
   - Call Center: **Stationary**
     - Q-Net and D-Net (piecewise stationary) - **congestion laws**

2. Empirical adventures at the Technion IE&M **SEELab**: **Mining operational building blocks of ServNets**
   - Primitives
   - Structure
   - Protocols

Little's Law $L = \lambda \times W$, in a Time-Varying Environment

Time-Gap: **# in System** lags behind **Little / 30 min**



Call Center:

$\Rightarrow$ **Piecewise Steady-State**

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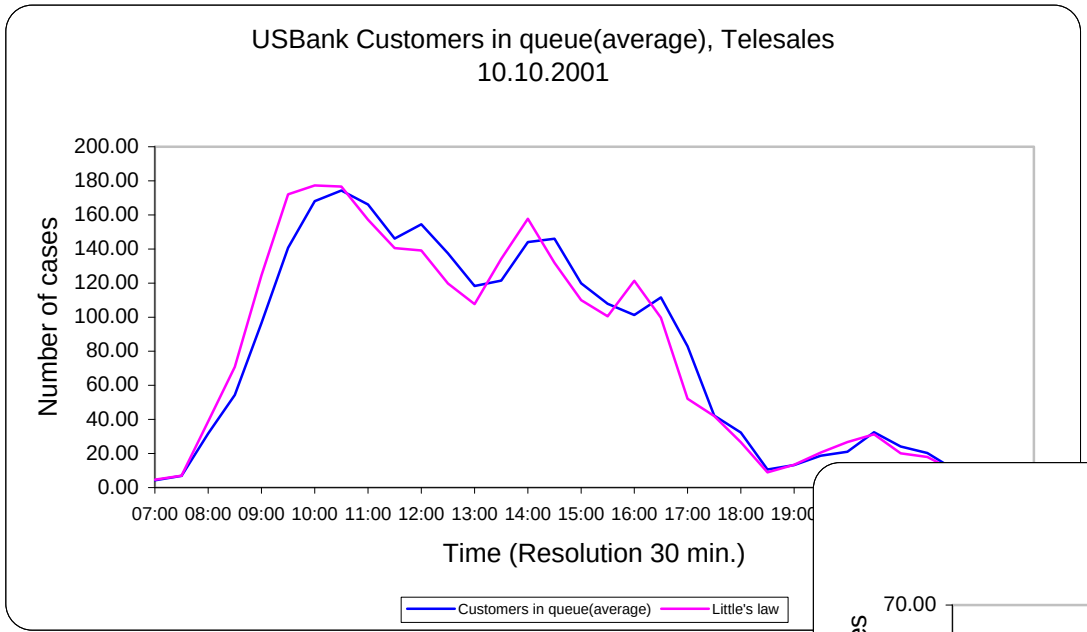


Call Center:

$\Rightarrow$ **Piecewise Steady-State**

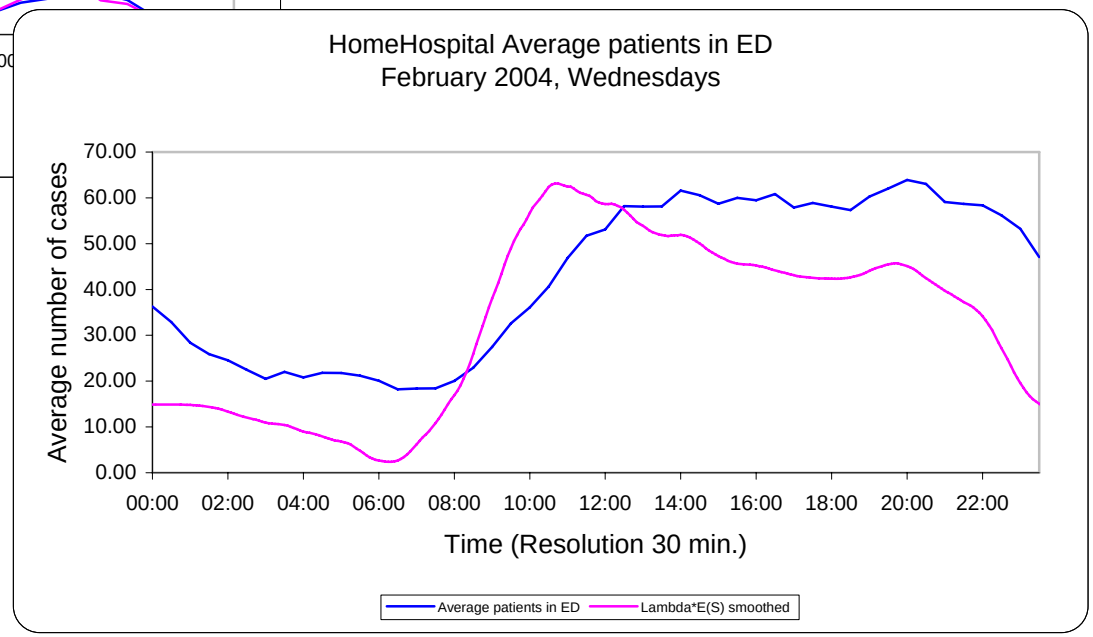
Emergency Dept:

$\Rightarrow$ **Time-Varying Transient**

$$EL(t) = \tilde{\lambda}(t) \times EW,$$

$$\tilde{\lambda}(t) = E\lambda(t - W_e).$$

(Bertsimas, Mourtzinou; Fralix, Riano, Serfozo)



ER / ED Environment: Service Network

Acute (Internal, Trauma)

Walking

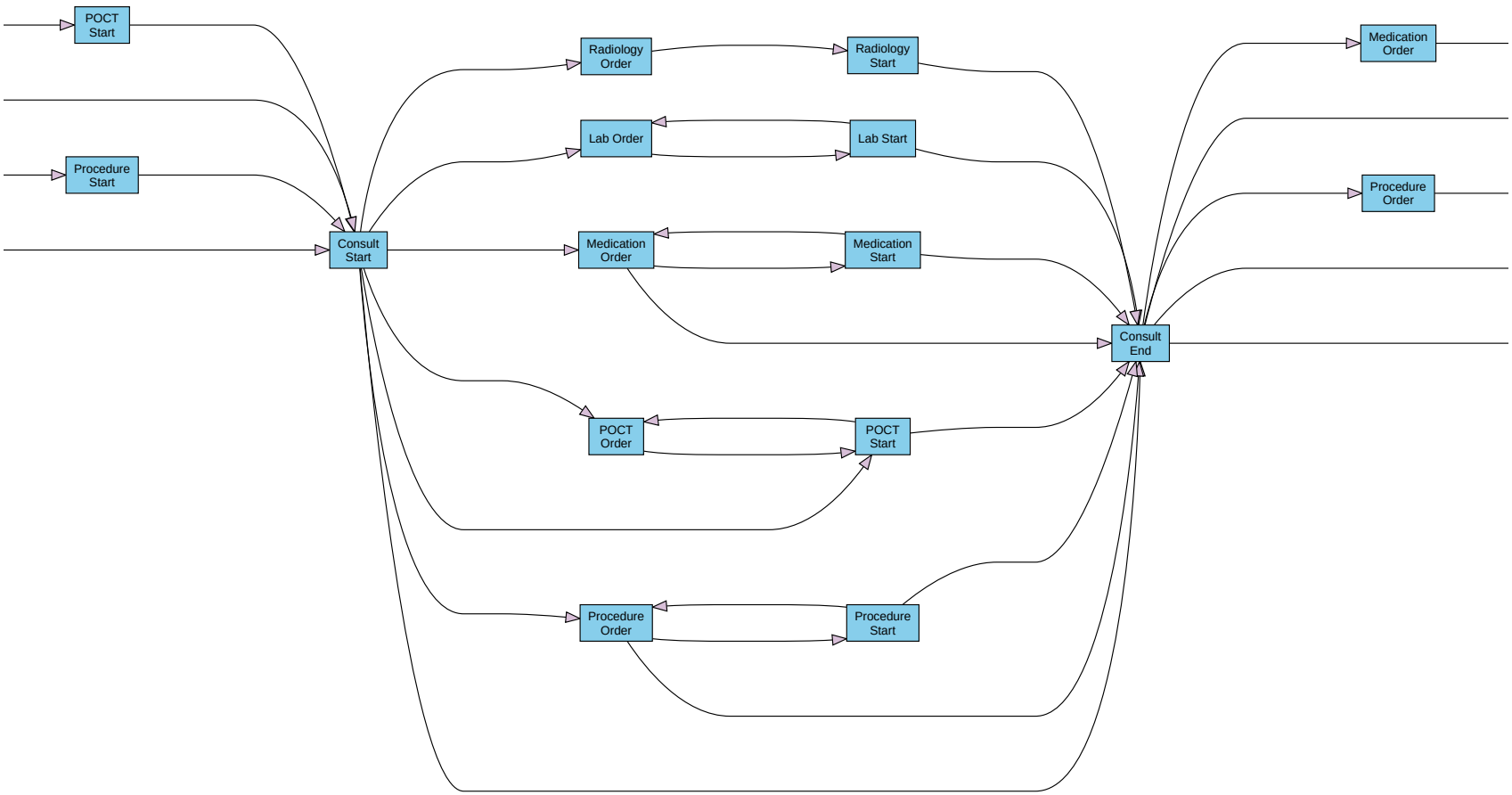
Multi-Trauma

ED-Environment in Israel

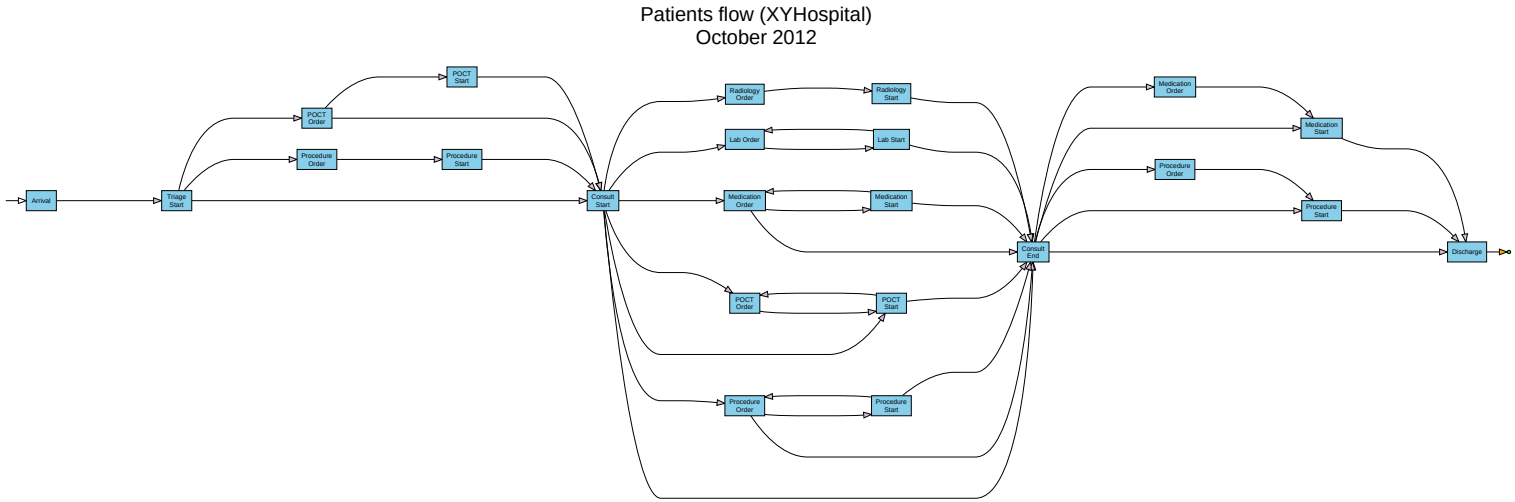
Simple Models at the Service of Complex Realities: FNets

1. ED in Normal days (Time-Varying Periodic): Personnel Staffing (offline)
2. ED in **Mass Casualty Event** (**Transient**): Forecasting, Staffing (online)

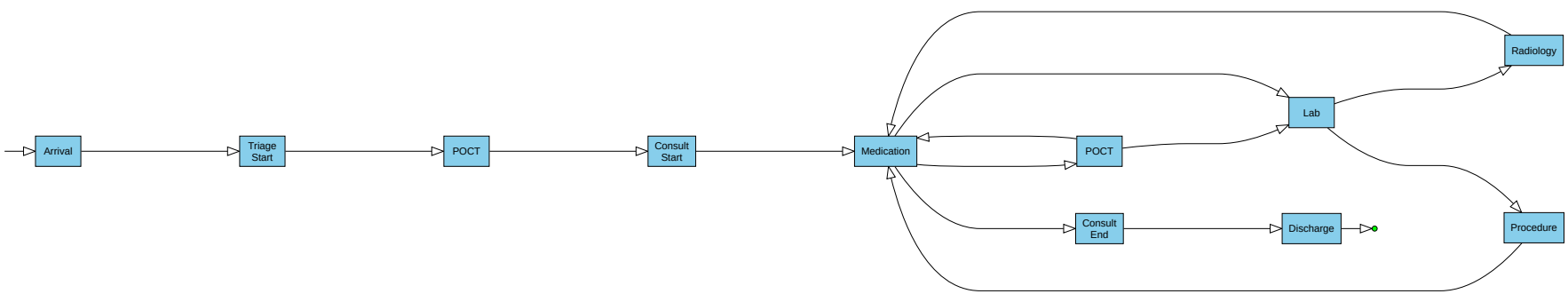
Emergency Department in XYHospital, October 2012



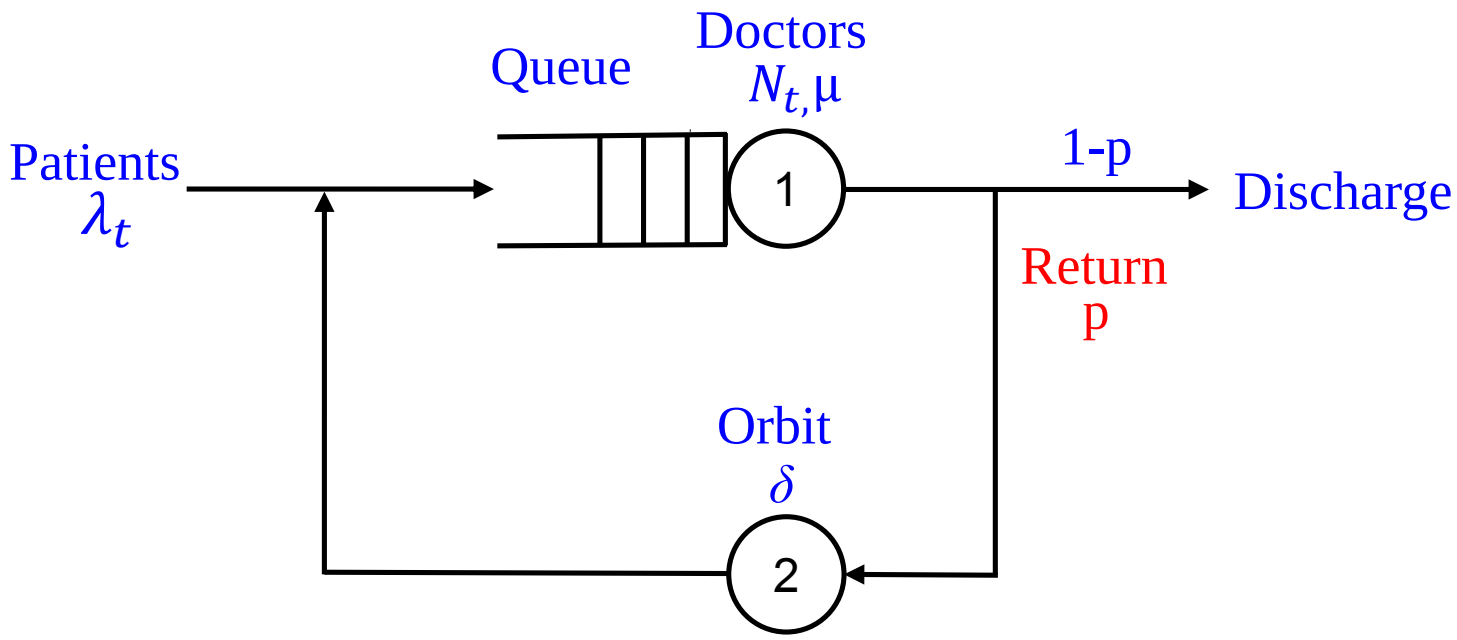
Recurrent Service Process in the ED



Capture **Recurrent** nature of service process: **Multiple doctor visits**



The Basic Service-Network Model: Erlang-R



w/ G. Yom-Tov

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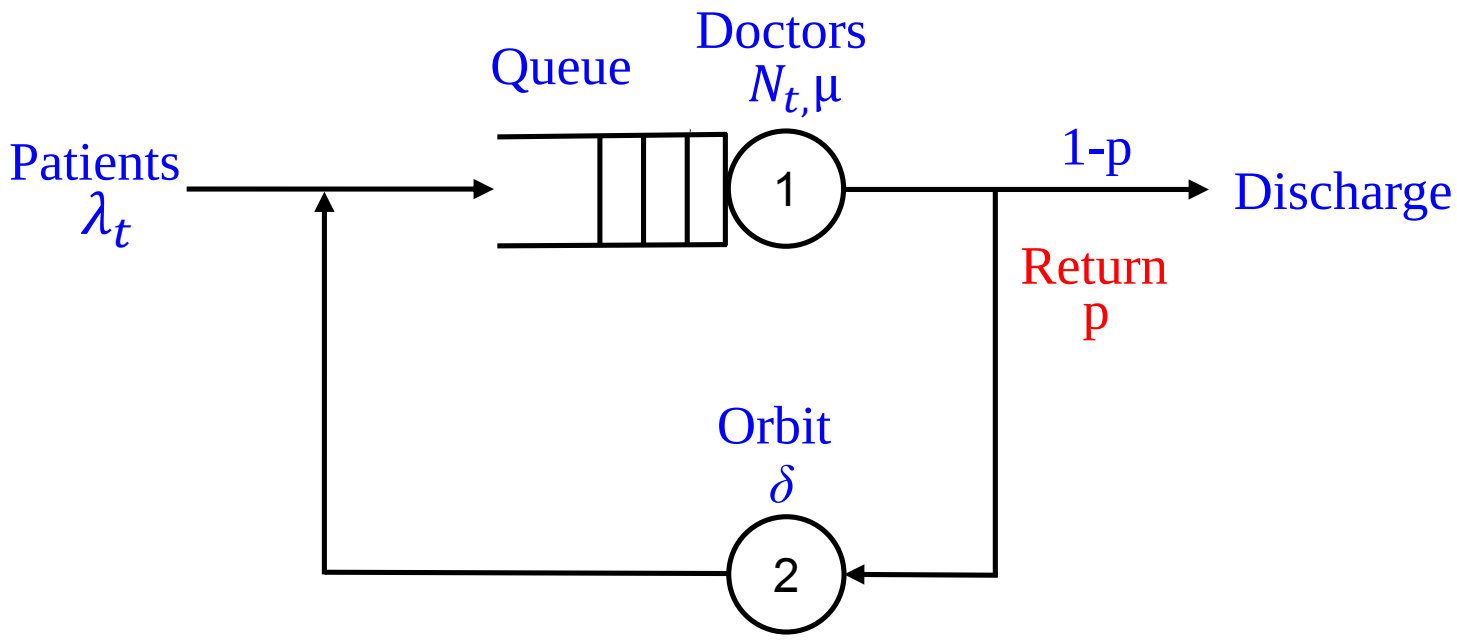


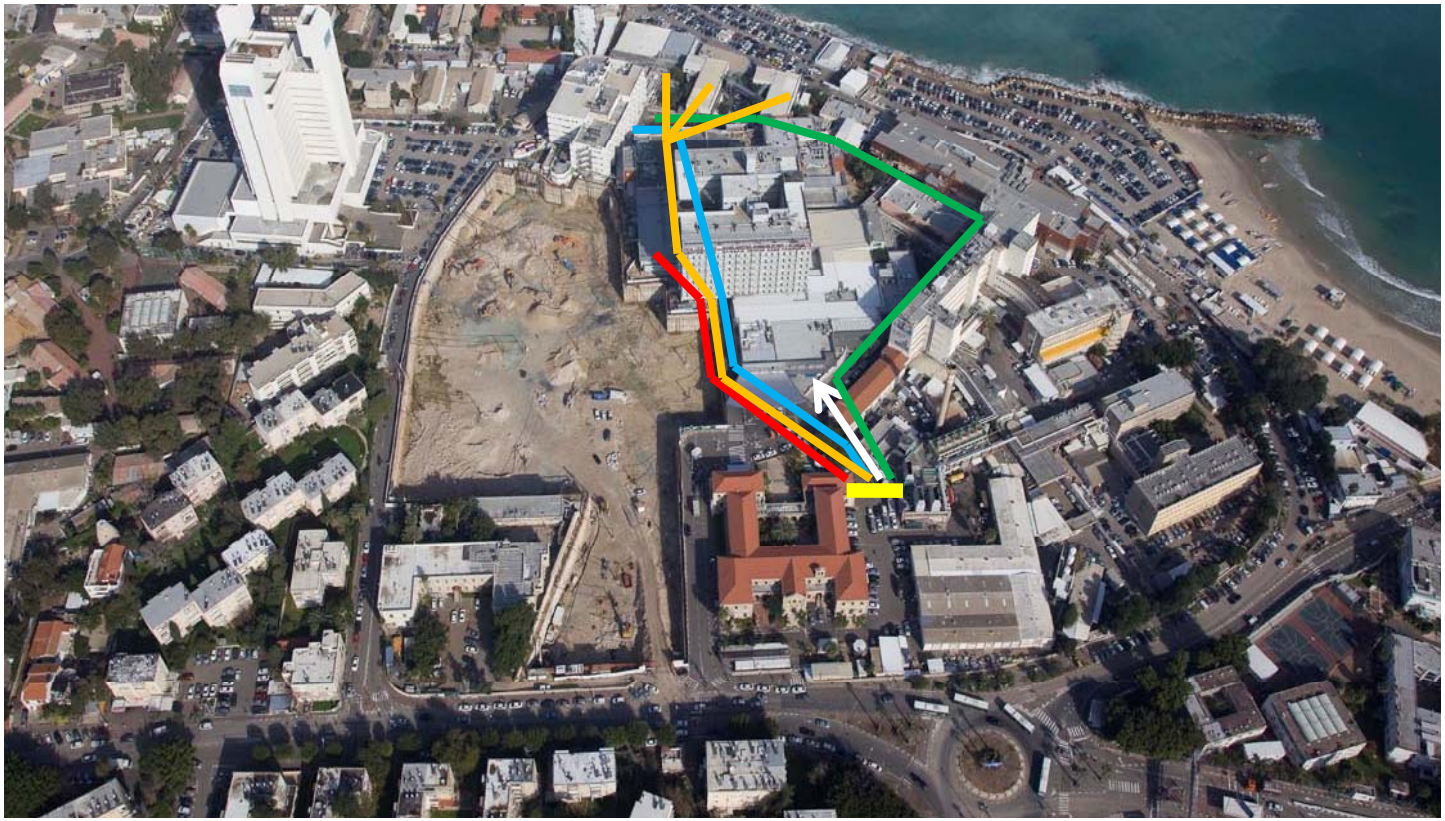
w/ G. Yom-Tov

2-station "Jackson" Network = (M/M/S, M/M/∞) :

- λ_t – **Time-Varying** **Arrival** rate
- N_t – Number of **Servers** (Physicians, or Nurses)
- μ – **Service** rate (E[Service] $= \frac{1}{\mu}$)
- p – **Return** (ReEntrant) fraction
- δ – **Orbit-to-Queue** rate (E[Delay]$_{19} = \frac{1}{\delta}$)

RFID-Based Data in Mass Casualty Event (Drill)

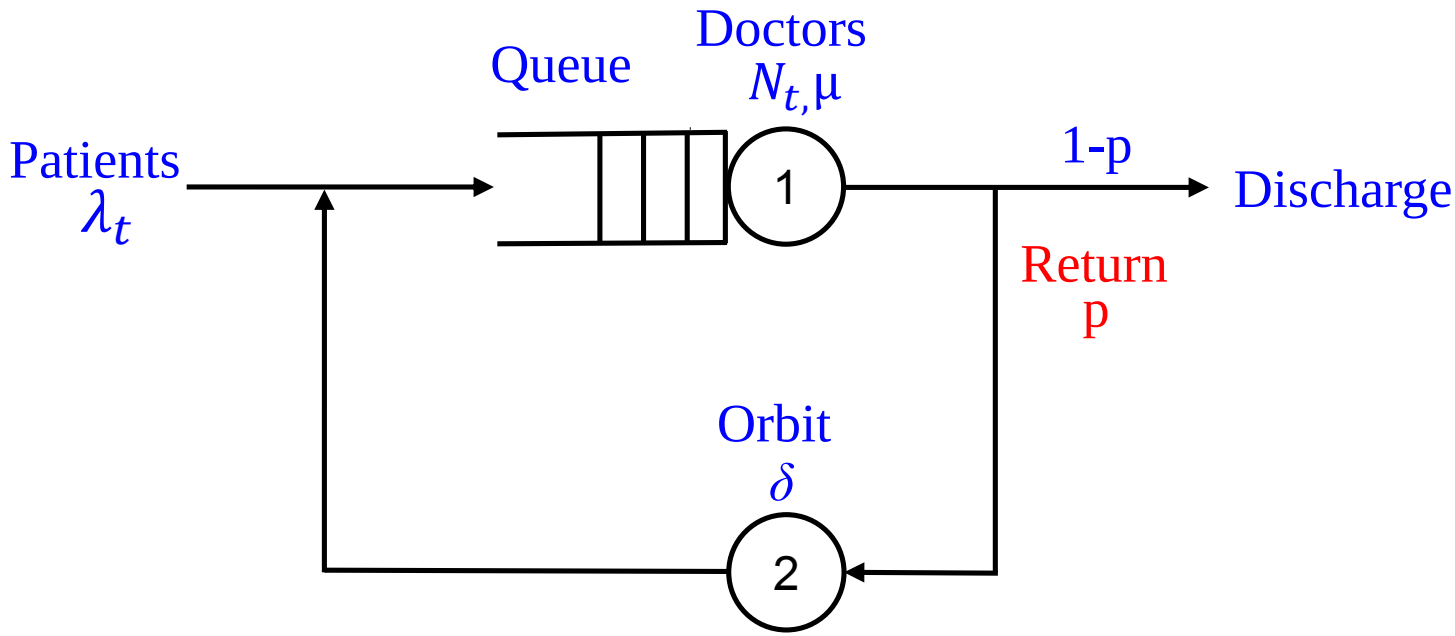
Chemical MCE, Rambam Hospital (May 2010, 11:00-13:00)



Fluid Model:

- **Predictable** Variability $\Rightarrow$ **Time-Varying**
- **Stochastic** Individualism averaged-out $\Rightarrow$ **Deterministic**

Fluid Model $\leftrightarrow$ (Time-Varying) Erlang-R System



Functional Strong Law of Large Numbers, for a 2-station QNet. BUT

FNet = ODE: derived **directly** (no QNet), spreadsheet "solution"

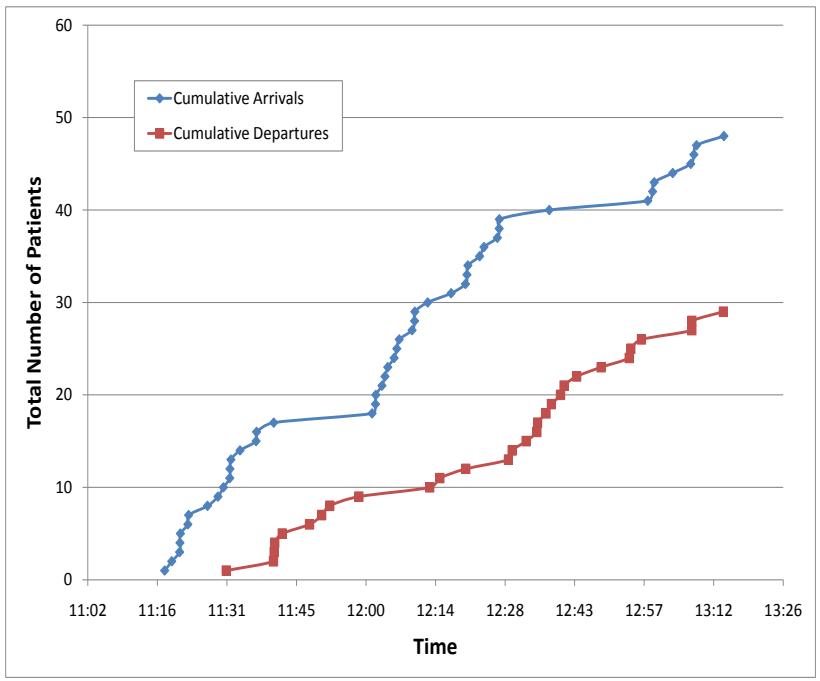
$$\frac{d}{dt} q_t^1 = \lambda_t - \mu \cdot (q_t^1 \wedge N_t) + \delta \cdot q_t^2$$

$$\frac{d}{dt} q_t^2 = p \cdot \mu \cdot (q_t^1 \wedge N_t) - \delta \cdot q_t^2$$

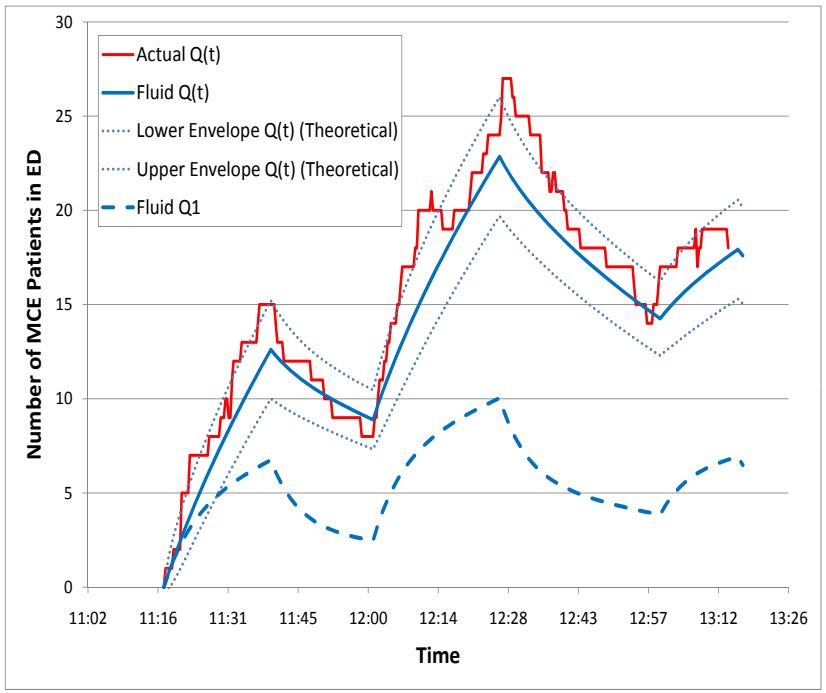
Erlang-R Value: FNet vs. Data

Chemical MCE Drill (Israel, May 2010, 11:00-13:00)

Arrivals & **Departures** (RFID)



Erlang-R (Fluid , Diffusion)



- **Recurrent/Repeated** services in Chemical MCE: injection every 15/30/60 min

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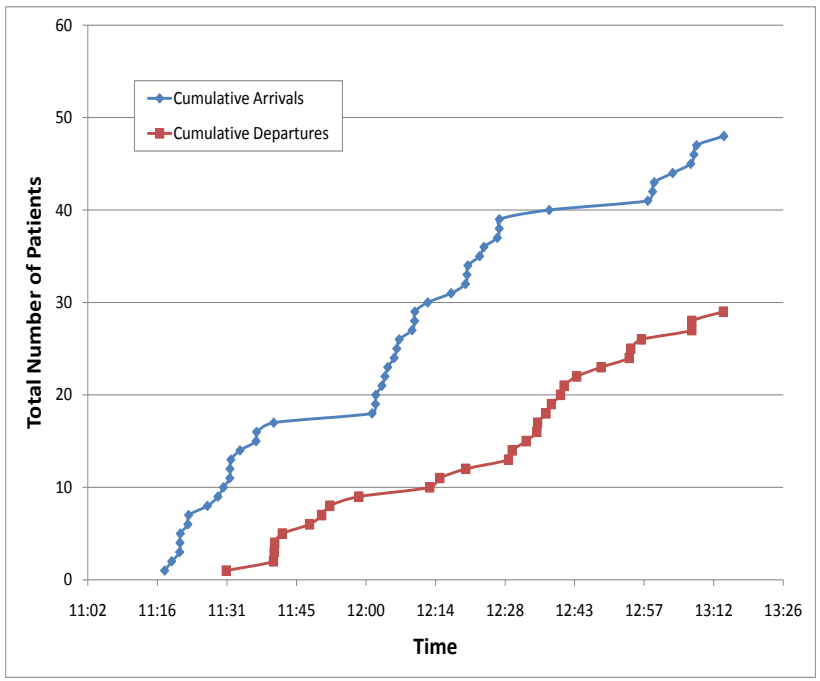

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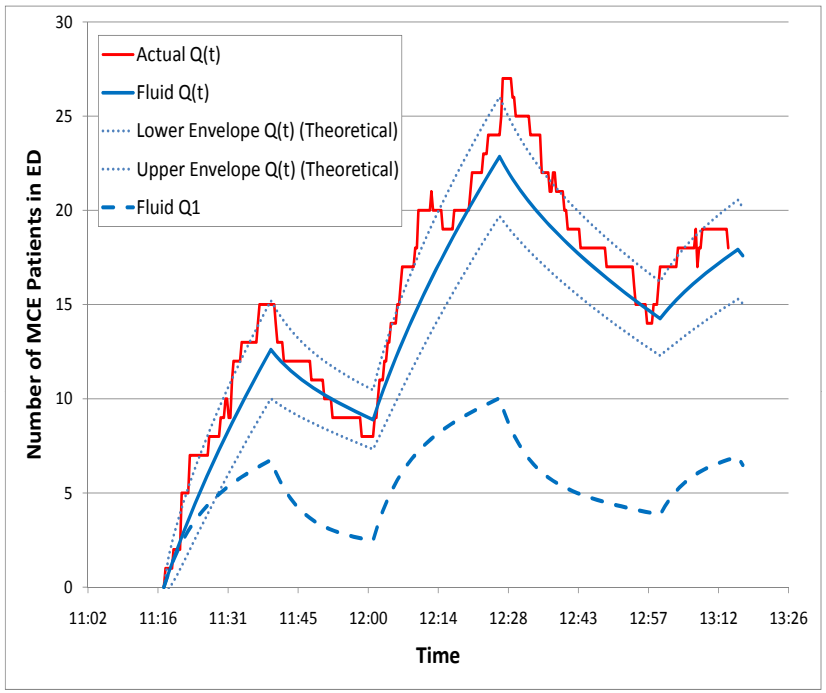

- **Recurrent/Repeated** services in Chemical MCE: injection every 15/30/60 min
- **Fluid** = ODE
- **Diffusion** (confidence band), via F. Central Limit Theorem: Usefully narrow

A Data-Based Framework, or “Erlang-R in the ED"

System = e.g. Emergency Department

- **QNet** = Erlang-R (time-varying 2-station Jackson; w/ Yom-Tov)
- **FNets** = 2-dim dynamical system (Massey & Whitt)
- **DNets** = 2-dim Markovian Service Net (w/ Massey and Reiman)
- **SimNet** = Customized ED-Simulator (Marmor & Sinreich)

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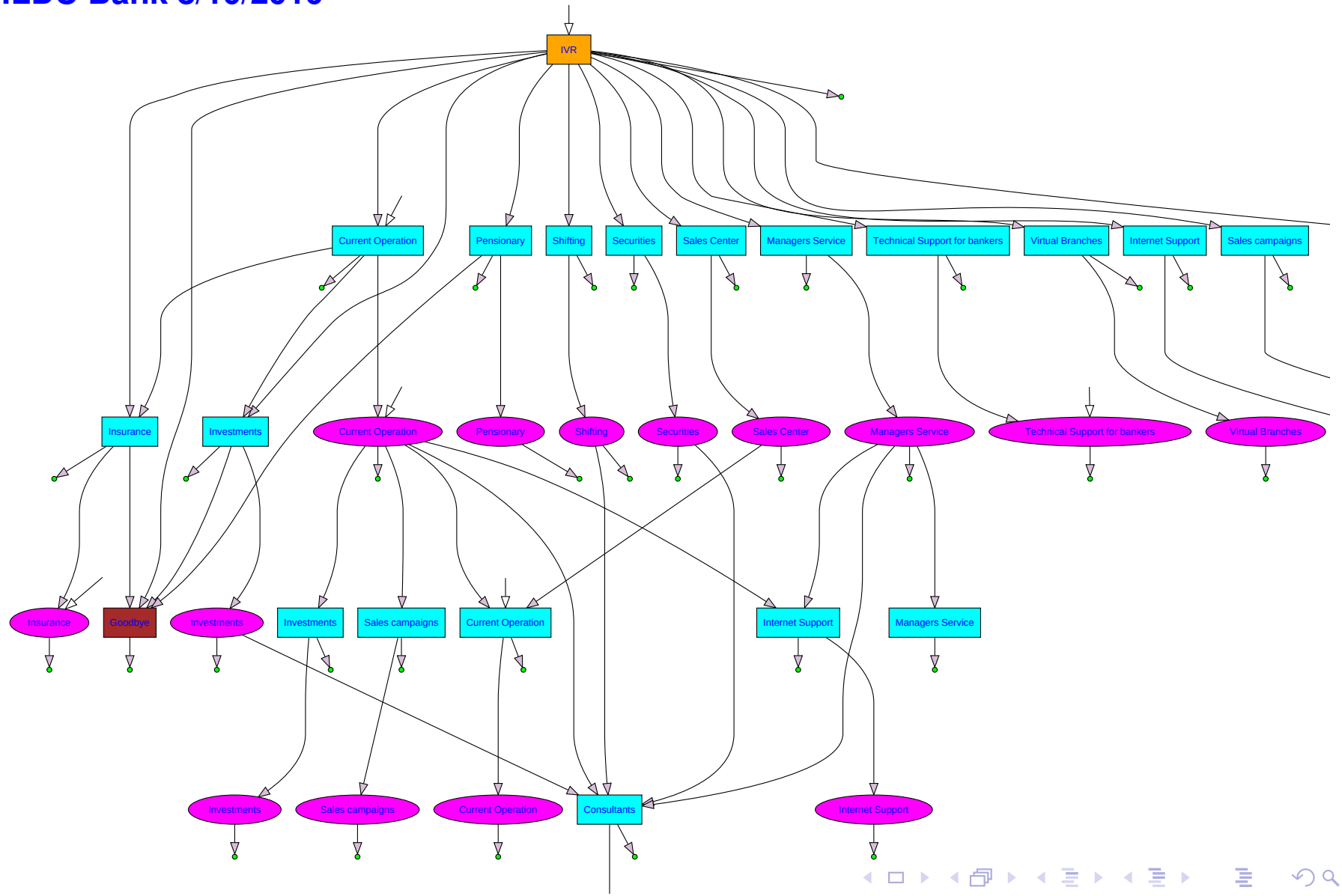
Framework: Mining (all) ServNets from Data

- **MCE ED**: FNet $\Rightarrow$ Census, DNet = Confidence band
  Performance Analysis, Prediction
  Validated against Data
- **Normal ED**: FNet $\Rightarrow$ Physician offered-load $\Rightarrow$ $\sqrt{}$-Staffing
  Staffing to stabilize operational performance
  Validated against SimNet

Call-Center Environment: Service Network

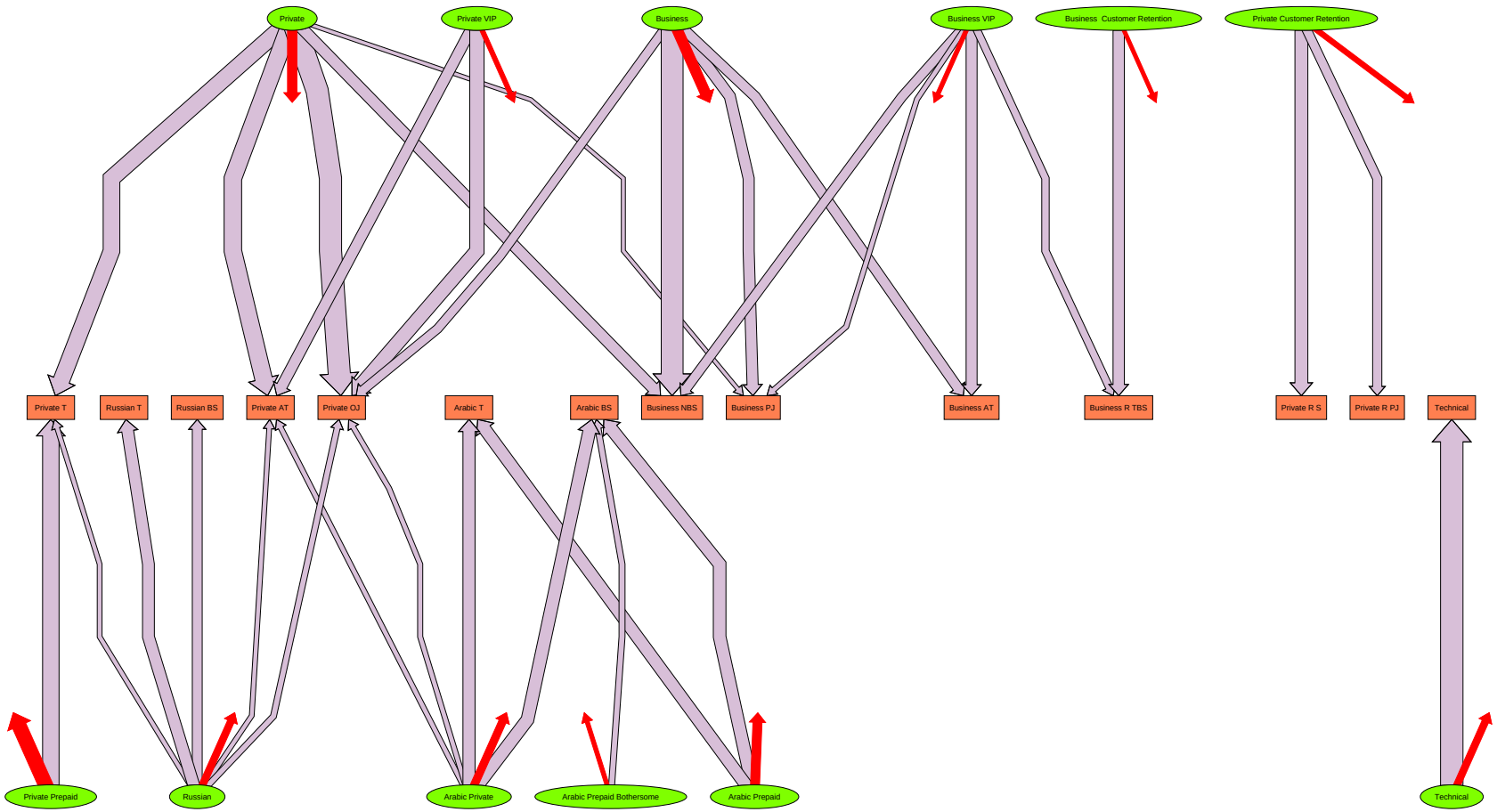
Customer Flow in Call Centers

ILDU Bank 8/10/2010



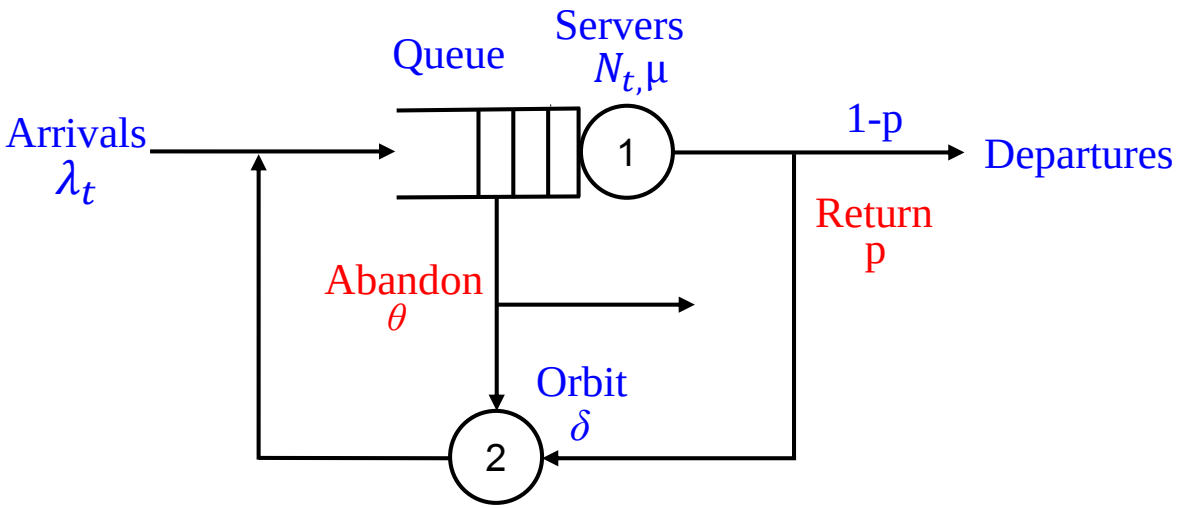
Impatient Customers - Isolate or Aggregate

ILTelecom 9/3/2008



Model Selection: As Simple as Possible but Not Simpler

Service with Retrials and Abandonment; w/ Massey, Reiman, Stolyar



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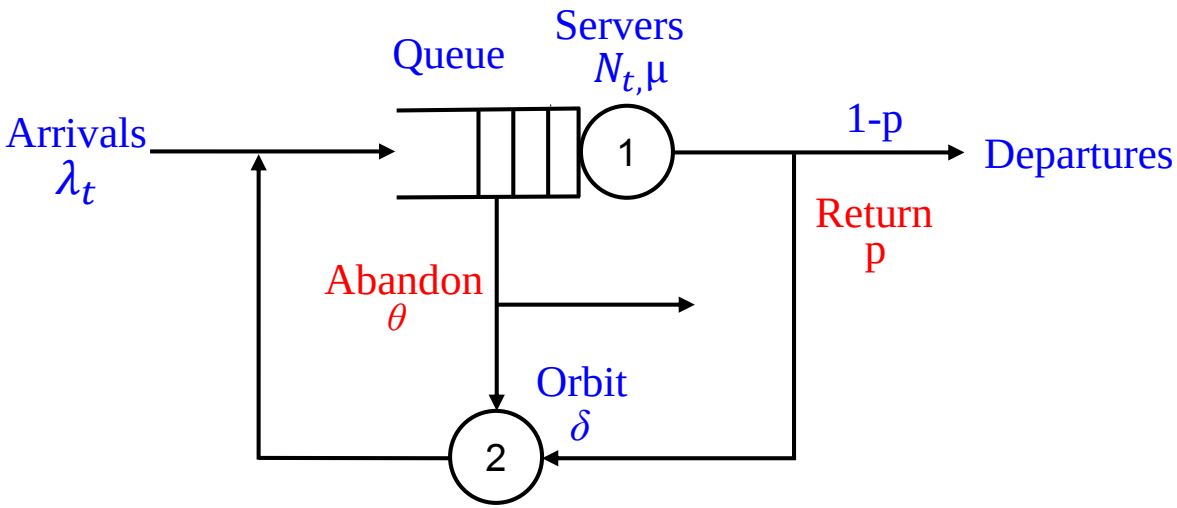
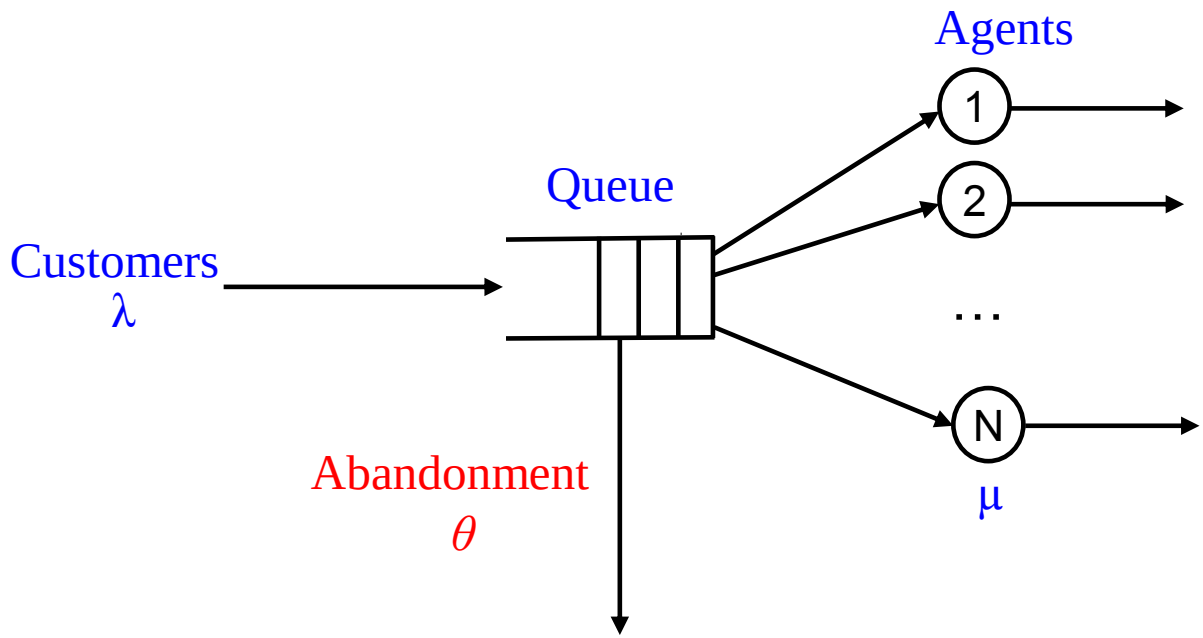


- Call centers: Visit durations naturally measured in minutes
  - Arrival rates are “constant" during visit
  - Returns occur hours after visit

$\Rightarrow$ “Select" Base Model (of 1/2 hour):

Stationary, Abandonment

A Basic Staffing Model: Erlang-A



w/ O. Garnett

"Birth & Death" Queue = M/M/N + M (Palm 1940's):

- λ – **Arrival** rate (Poisson)
- μ – **Service** rate (Exponential; $E[S] = \frac{1}{\mu}$)
- θ – **Patience** rate (Exponential, $E[\text{Patience}] = \frac{1}{\theta}$)
- N – Number of **Servers** (Agents).

Erlang-A: Is it Relevant?

Experience:

- Arrival process **not pure Poisson** (time-varying, σ^2 too large)
- Service times **not Exponential** (typically close to LogNormal)
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- Customers and Servers **not homogeneous** (classes, skills): **w/ R. Atar, G. Shaikhet; R. Atar, I. Gurvich,** . . .
- Customers return for service (after busy, abandonment; dependently: **P. Khudiakov, R. Ghebali, M. Gorfine, P. Feigin**)
- $\cdots$, and more.

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Question: **Is Erlang-A Relevant?** **Robust enough?** **YES !**

- **Practice**: Staffing engine of Work-Force Management software
- **Theory**: Theoretical engine of Operational Regimes
  QD, **ED**, **QED**

Asymptotic Erlang-X (Markovian Q's)

- Pre-History, 1914: **Erlang** (Erlang-B = M/M/n/n, Erlang-C = M/M/n)
- Pre-History, 1974: Jagerman (Erlang-B)
- History Milestone, 1981: **Halfin-Whitt** (Erlang-C, GI/M/n)
- Erlang-A (M/M/N+M), 2002: w/ **Garnett** & Reiman
- Erlang-A with General (Im)Patience (M/M/N+G), 2005: w/ Zeltyn
- Erlang-C (ED+QED), 2009: w/ Zeltyn
- **Erlang-B with Retrial, 2010(3): Avram, Janssen, van Leeuwaarden**
- **Refined Asymptotics (Erlang A/B/C, . . .), 2008-2013: Janssen, van Leeuwaarden, Zhang, Zwart**
- Production Q's, 2011: Reed & Zhang
- Universal Erlang-A, ongoing: w/ Gurvich & Huang
- Queueing Networks:
  - (Semi-)Closed: Nurse Staffing (Jennings & de Vericourt), CCs with IVR (w/ Khudiakov), Erlang-R (w/ Yom-Tov)
  - CCs with Abandonment and Retrials: w. Massey, Reiman, Rider, Stolyar
  - Markovian Service Networks: w/ Massey & Reiman
- Leaving out:
  - **Non-Exponential Service Times**: M/D/n (Erlang-D), G/Ph/n, · · · , G/GI/n+GI, Measure-Valued Diffusions
  - **Dimensioning** (Staffing): M/M/n, · · · , time-varying Q's, V- and Reversed-V, · · ·
  - **Control**: V-network, Reversed-V, · · · , SBRNets

Asymptotic Landscape: 9 Operational Regimes, and then some

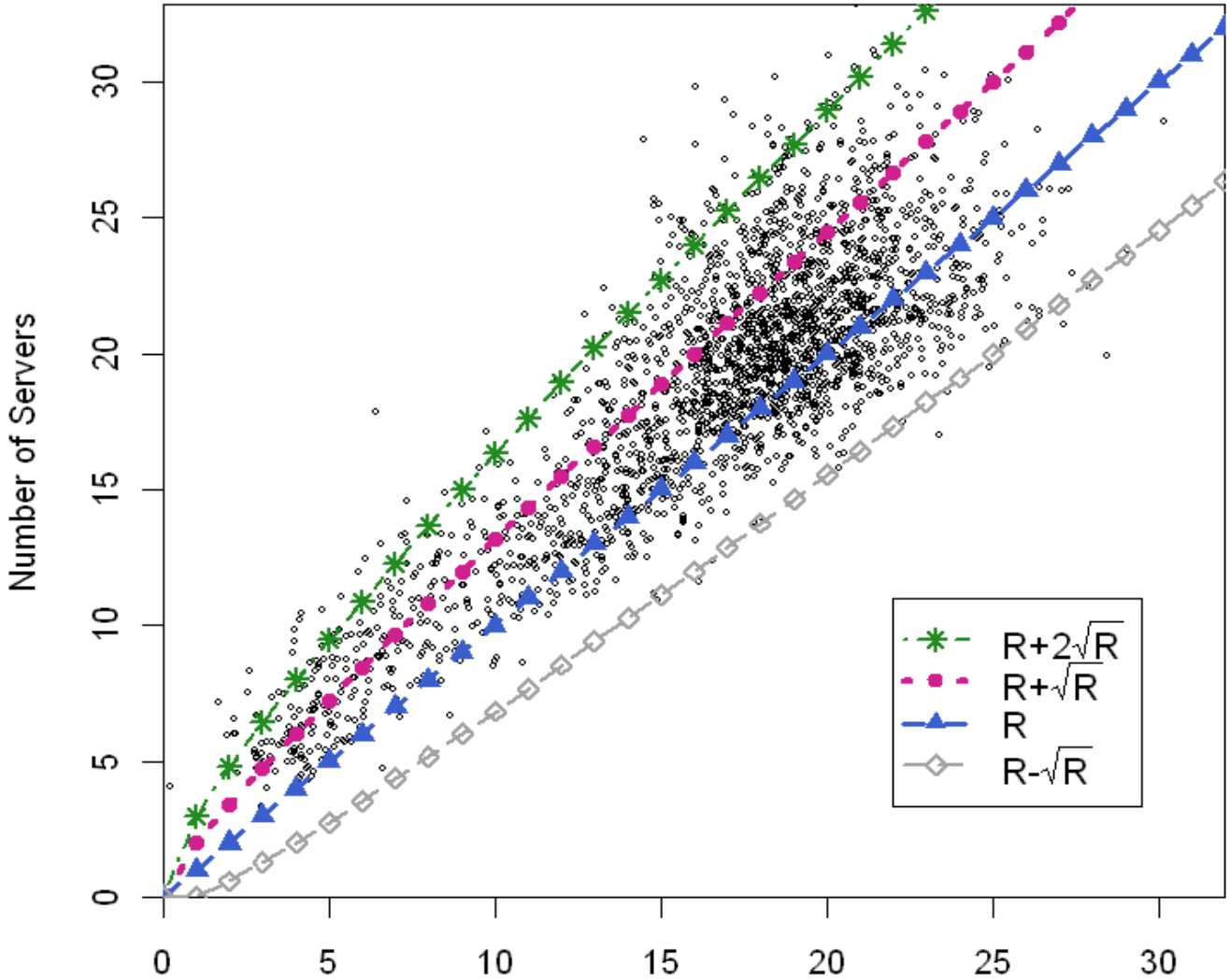
Erlang-A, w/ I. Gurvich & J. Huang

Erlang-A μ & θ fixed	Conventional scaling			Many-Server scaling			NDS scaling		
	Sub	Critical	Over	QD	QED	ED	Sub	Critical	Over
Offered load per server	$\frac{1}{1+\delta}$	$1-\frac{\beta}{\sqrt{n}}$	$\frac{1}{1-\gamma}$	$\frac{1}{1+\delta}$	$1-\frac{\beta}{\sqrt{n}}$	$\frac{1}{1-\gamma}$	$\frac{1}{1+\delta}$	$1-\frac{\beta}{n}$	$\frac{1}{1-\gamma}$
Arrival rate λ	$\frac{\mu}{1+\delta}$	$\mu-\frac{\beta}{\sqrt{n}}\mu$	$\frac{\mu}{1-\gamma}$	$\frac{n\mu}{1+\delta}$	$n\mu-\beta\mu\sqrt{n}$	$\frac{n\mu}{1-\gamma}$	$\frac{n\mu}{1+\delta}$	$n\mu-\beta\mu$	$\frac{n\mu}{1-\gamma}$
# servers	1			n			n		
Time-scale	n			1			n		
Impatience rate	θ/n			θ			θ/n		
Staffing level	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu}(1+\frac{\beta}{\sqrt{n}})$	$\frac{\lambda}{\mu}(1-\gamma)$	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu}+\beta\sqrt{\frac{\lambda}{\mu}}$	$\frac{\lambda}{\mu}(1-\gamma)$	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu}+\beta$	$\frac{\lambda}{\mu}(1-\gamma)$
Utilization	$\frac{1}{1+\delta}$	$1-\sqrt{\frac{\theta}{\mu}}\frac{h(\hat{\beta})}{\sqrt{n}}$	1	$\frac{1}{1+\delta}$	$1-\sqrt{\frac{\theta}{\mu}}\frac{\hat{h}(\hat{\beta})}{\sqrt{n}}$	1	$\frac{1}{1+\delta}$	$1-\sqrt{\frac{\theta}{\mu}}\frac{h(\hat{\beta})}{n}$	1
$\mathbb{E}(Q)$	$\frac{1}{\delta(1+\delta)}$	$\sqrt{n}g(\hat{\beta})$	$\frac{n\mu\gamma}{\theta(1-\gamma)}$	$\frac{1}{\delta}\varrho_n$	$\sqrt{n}g(\hat{\beta})\alpha$	$\frac{n\mu\gamma}{\theta(1-\gamma)}$	$o(1)$	$ng(\hat{\beta})$	$\frac{n^2\mu\gamma}{\theta(1-\gamma)}$
$\mathbb{P}(Ab)$	$\frac{1}{n}\frac{1}{\delta}\frac{\theta}{\mu}$	$\frac{\theta}{\sqrt{n}\mu}g(\hat{\beta})$	γ	$\frac{1}{n}\frac{(1+\delta)}{\delta}\frac{\theta}{\mu}\varrho_n$	$\frac{\theta}{\sqrt{n}\mu}g(\hat{\beta})\alpha$	γ	$o(\frac{1}{n^2})$	$\frac{\theta}{n\mu}g(\hat{\beta})$	γ
$\mathbb{P}(W_q>0)$	$\frac{1}{1+\delta}$	≈ 1		ϱ_n	$\alpha\in(0,1)$	≈ 1	≈ 0	≈ 1	
$\mathbb{P}(W_q>T)$	$\frac{1}{1+\delta}e^{-\frac{\delta}{1+\delta}\mu T}$	$1+O(\frac{1}{\sqrt{n}})$	$1+O(\frac{1}{n})$	≈ 0		$f(T)$	≈ 0	$\frac{\bar{\Phi}(\hat{\beta}+\sqrt{\theta\mu}T)}{\bar{\Phi}(\hat{\beta})}$	$1+O(\frac{1}{n})$
Congestion $\frac{\mathbb{E}W_q}{\mathbb{E}S}$	$\frac{1}{\delta}$	$\sqrt{n}g(\hat{\beta})$	$n\mu\gamma/\theta$	$\frac{1}{n}\frac{(1+\delta)}{\delta}\varrho_n$	$\frac{\alpha}{\sqrt{n}}g(\hat{\beta})$	$\frac{\mu\gamma}{\theta}$	$o(\frac{1}{n})$	$g(\hat{\beta})$	$n\mu\gamma/\theta$

- Conventional: Ward & Glynn (03, $G/G/1+G$)
- Many-Server:
  - QED: Halfin-Whitt (81), w/ Garnett & Reiman (02)
  - ED: Whitt (04)
  - NDS: Atar (12)
- **"Missing"**: ED+QED; Hazard-rate scaling (M/M/N+G); Time-Varying, Non-Parametric; Moderate- and Large-Deviation; Networks (multi-regimes)

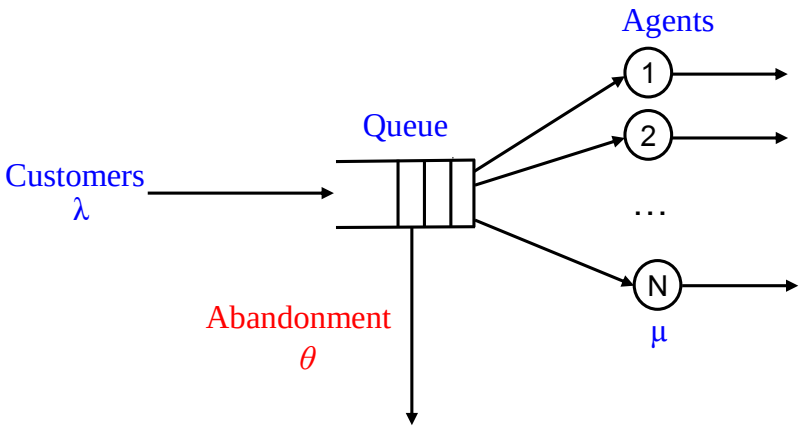
Beyond Fluid: #Agents vs. Offered-Load ($N \approx R + \beta\sqrt{R}$)

IL Telecom; June-September, 2004 (2205 30min intervals, over 13 weeks, week-days)



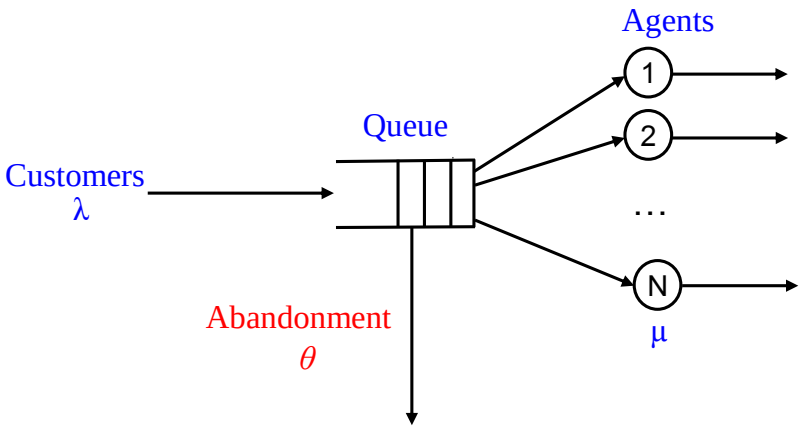
e.g. Offered-load R $\stackrel{avg}{=}$ **5 calls per min** $\times$ **3.2 min per call = 16 Erlangs**

Universal Approximations: Erlang-A (M/M/N + M)



w/ **I. Gurvich & J. Huang**

Universal Approximations: Erlang-A (M/M/N + M)

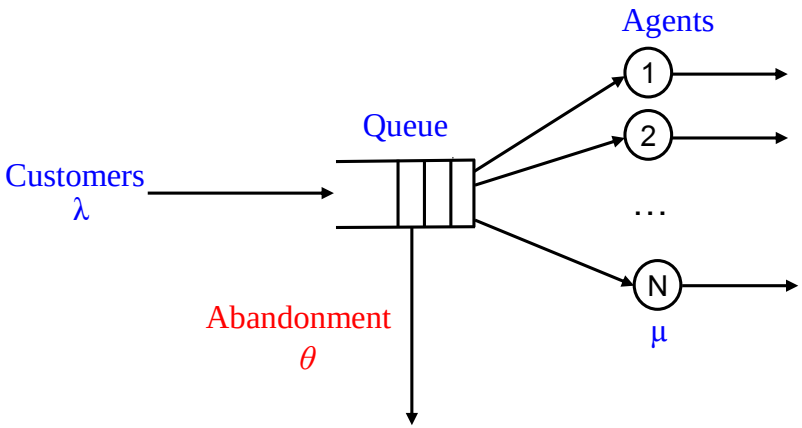


w/ **I. Gurvich & J. Huang**

- **QNet**: Birth & Death Queue, with B - D rates

$$F(q) = \lambda - \mu \cdot (q \wedge n) - \theta \cdot (q - n)^+, \quad q = 0, 1, \ldots$$

Universal Approximations: Erlang-A (M/M/N + M)



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- **FNet**: Dynamical (Deterministic) System – ODE

$$dx_t = F(x_t)dt, \quad t \geq 0$$

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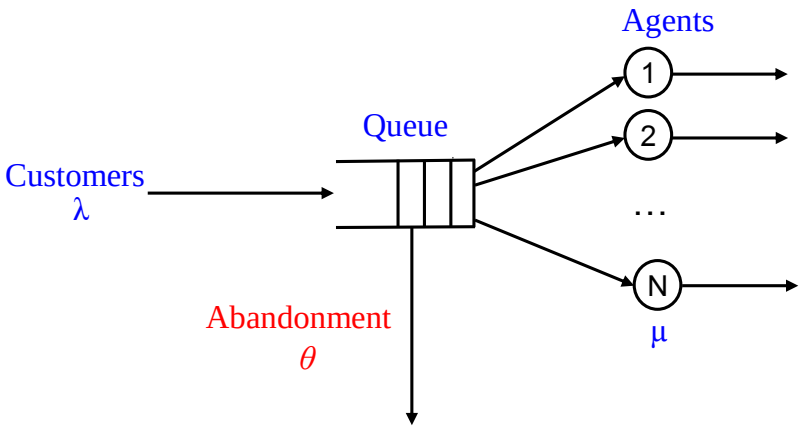

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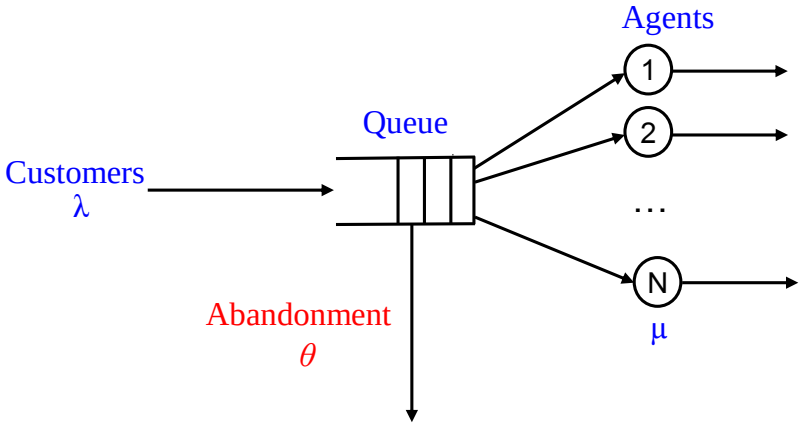
$$dY_t = F(Y_t)dt + \sqrt{2\lambda}\, dB_t, \quad t \geq 0$$

Universal Approximations: Erlang-A (M/M/N + M)



w/ **I. Gurvich & J. Huang**

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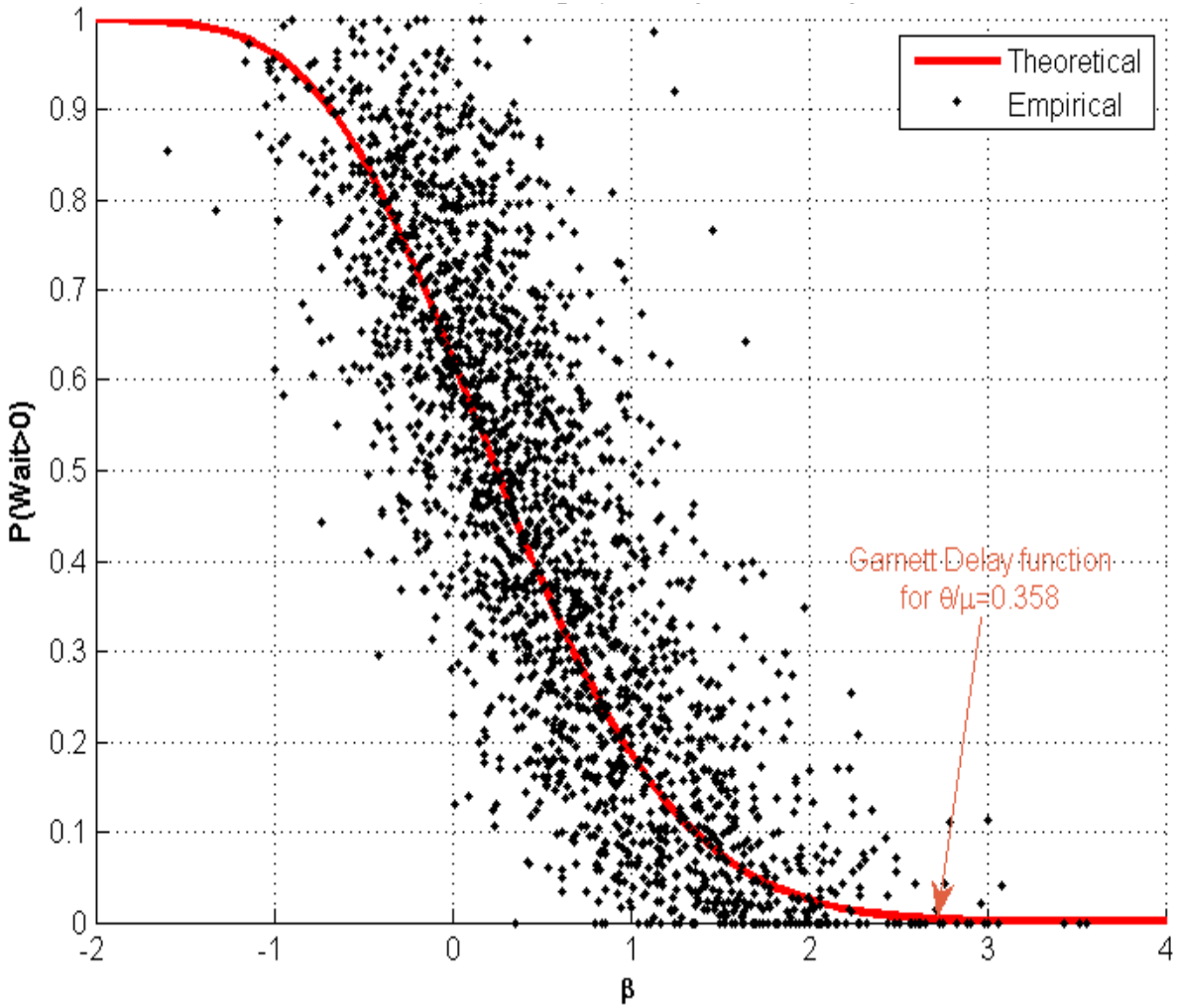
eg. $\mu = \theta$: $\dot{x} = \lambda - \mu \cdot x$, $Y =$ OU process

Parsimonious (Tractable, Robust), Accurate, Valuable

Erlang-A Value: DNet $P(W_q > 0)$ vs. Data

IL Telecom; June-September, 2004 (2205 30min intervals, weekdays)



- Approximations, w/ Patience $\approx 3\times$ Service-Duration ($\mu/\theta \approx 3$)

Accuracy: DNet vs. QNet

- Δ^λ is the "balancing" state, obtained by solving

$$\lambda = \mu(n \wedge \Delta^\lambda) + \theta(\Delta^\lambda - n)^+.$$

Solution: $\Delta^\lambda = \frac{\lambda}{\mu} - \left(\frac{\lambda}{\mu} - n\right)^+ \left(1 - \frac{\mu}{\theta}\right).$

Specifically: **QD** = $\frac{\lambda}{\mu}$; **ED** = $n + \frac{1}{\theta}(\lambda - n\mu)$; **QED** = $n + \mathcal{O}(\sqrt{\lambda})$)

- Centered processes (excursions):

$$\tilde{Q}^\lambda(\cdot) = Q(\cdot) - \Delta^\lambda, \quad \tilde{Y}^\lambda(\cdot) = Y(\cdot) - \Delta^\lambda.$$

Theorem: For f bounded by an m-degree polynomial ($m \geq 0$):

$$\mathbb{E}f(\tilde{Q}^\lambda(\infty)) - \mathbb{E}f(\tilde{Y}^\lambda(\infty)) = \mathcal{O}(\sqrt{\lambda}^{m-1}).$$

- **Accurate**: more than heavy-traffic *limits*

Simplicity: Why 2λ?

- Semi-martingale representation of the B&D process: Fluid + Martingale
- Predictable quadratic variation:

$$\int_0^t [\lambda + \mu(Q_s \wedge n) + \theta(Q_s - n)^+] ds$$

- In steady-state, arrival rate $\equiv$ departure rate:

$$\lambda = \mathbb{E}[\mu(Q_s \wedge n) + \theta(Q_s - n)^+]$$

- Expectation of the predictable quadratic variation:

$$\mathbb{E} \int_0^t [\lambda + \mu(Q_s \wedge n) + \theta(Q_s - n)^+] ds = 2\lambda t$$

- **Simple** $\Rightarrow$ **Tractable, Robust**: $\text{dMartingale}_t \approx \sqrt{2\lambda} \cdot \text{dBrownian}_t$

Reconciling Time-Varying and Steady-State Models

- **Rigid** (fixed) staffing level during a time-varying shift:
  Doomed to alternate between overloading and underloading
- **Flexible** staffing:
  Can design **time-varying staffing** that achieves, **at all times, Steady-State performance**
  via Square-Root Staffing (Modified Offered-Load)

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- **History**:
  - Jennings, M., Reiman, Whitt (1996): Emergence of the phenomenon, with infinite-server heuristics
  - Feldman, M., Massey, Whitt (2008): Stabilize delay probability with QED staffing, with little theory
  - Liu and Whitt (2012): Stabilize abandonment probability, with ED theory
  - w/ Huang, Gurvich (ongoing): QED theory

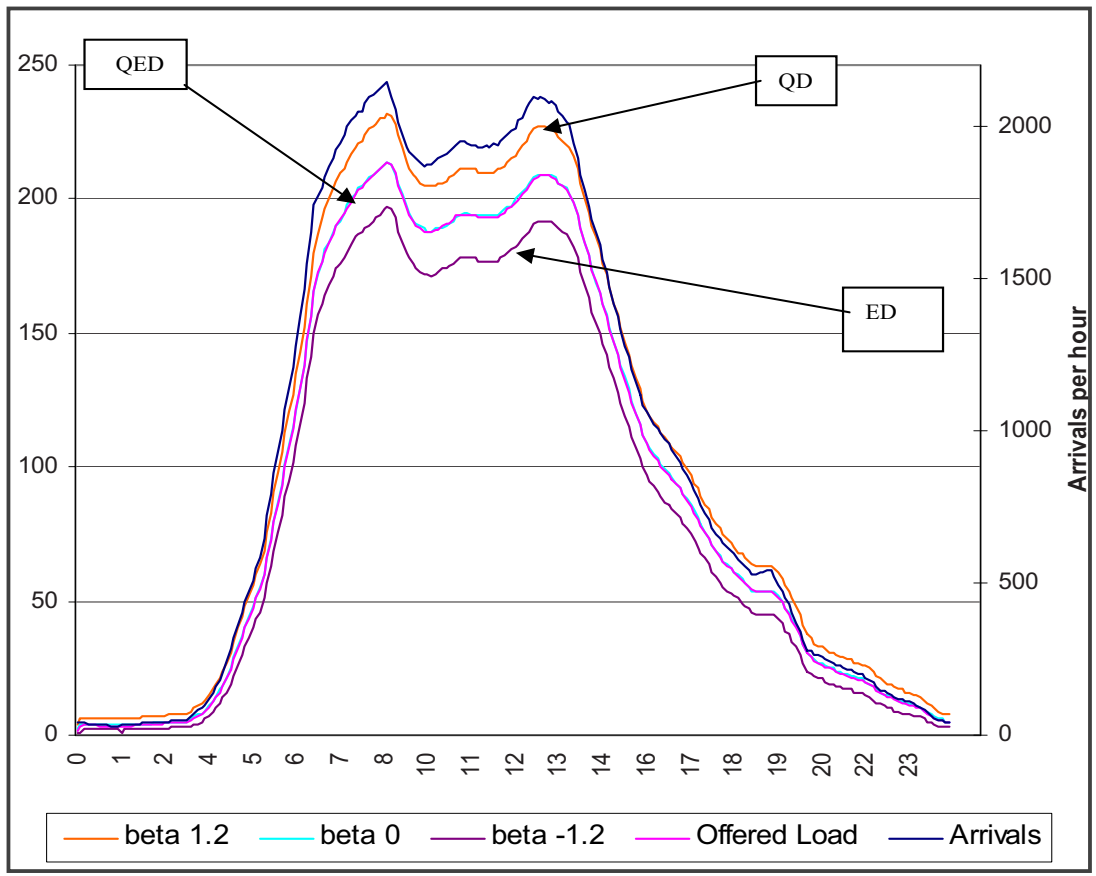
Time-Varying Arrival Rates

Square-Root Staffing:

$$N(t) = R(t) + \beta\sqrt{R(t)}, \quad -\infty < \beta < \infty.$$

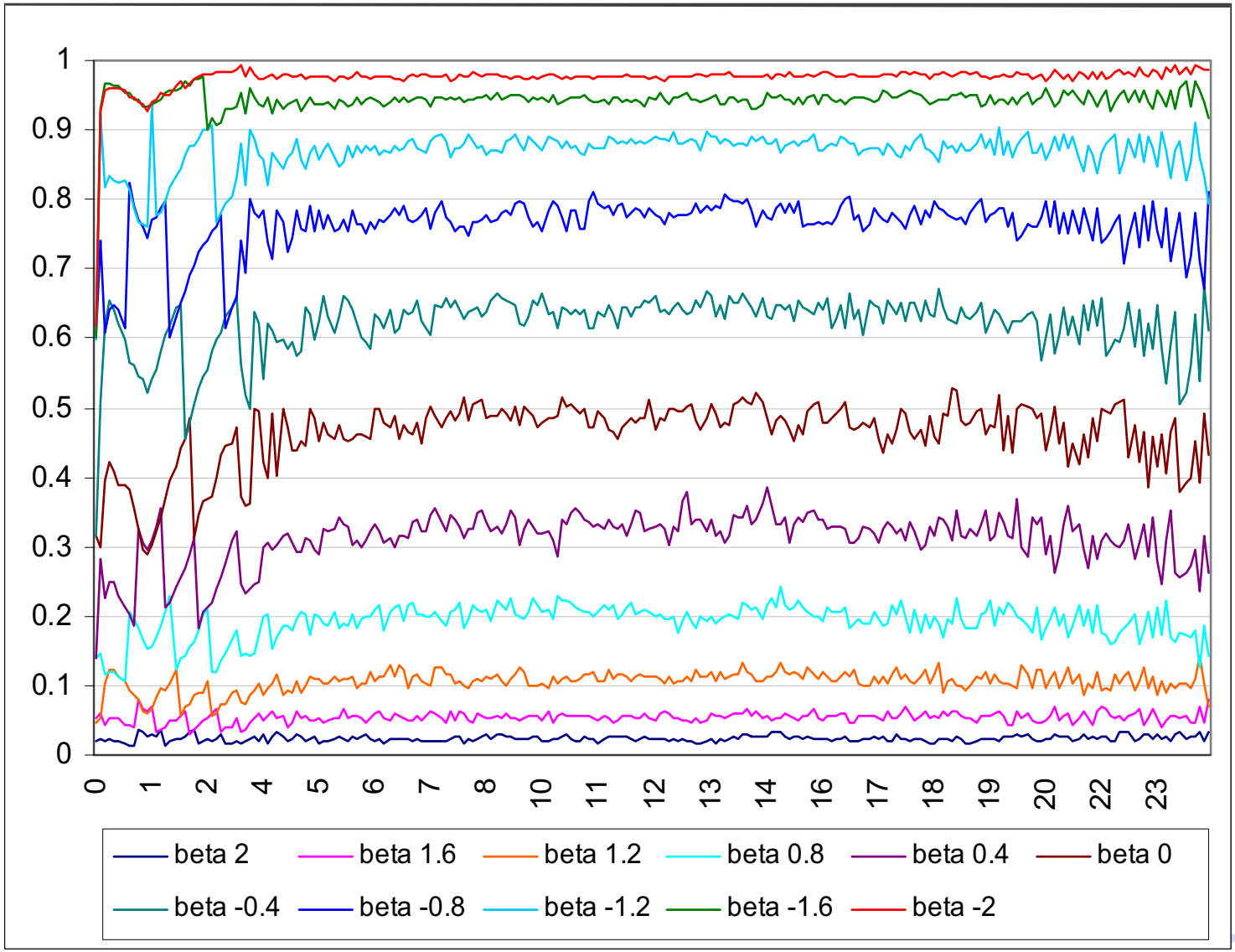
$R(t)$ is the **Offered-Load** at time t $\quad (\, R(t) \neq \lambda(t) \times \mathrm{E[S]} \,)$

Arrivals, Offered-Load and Staffing



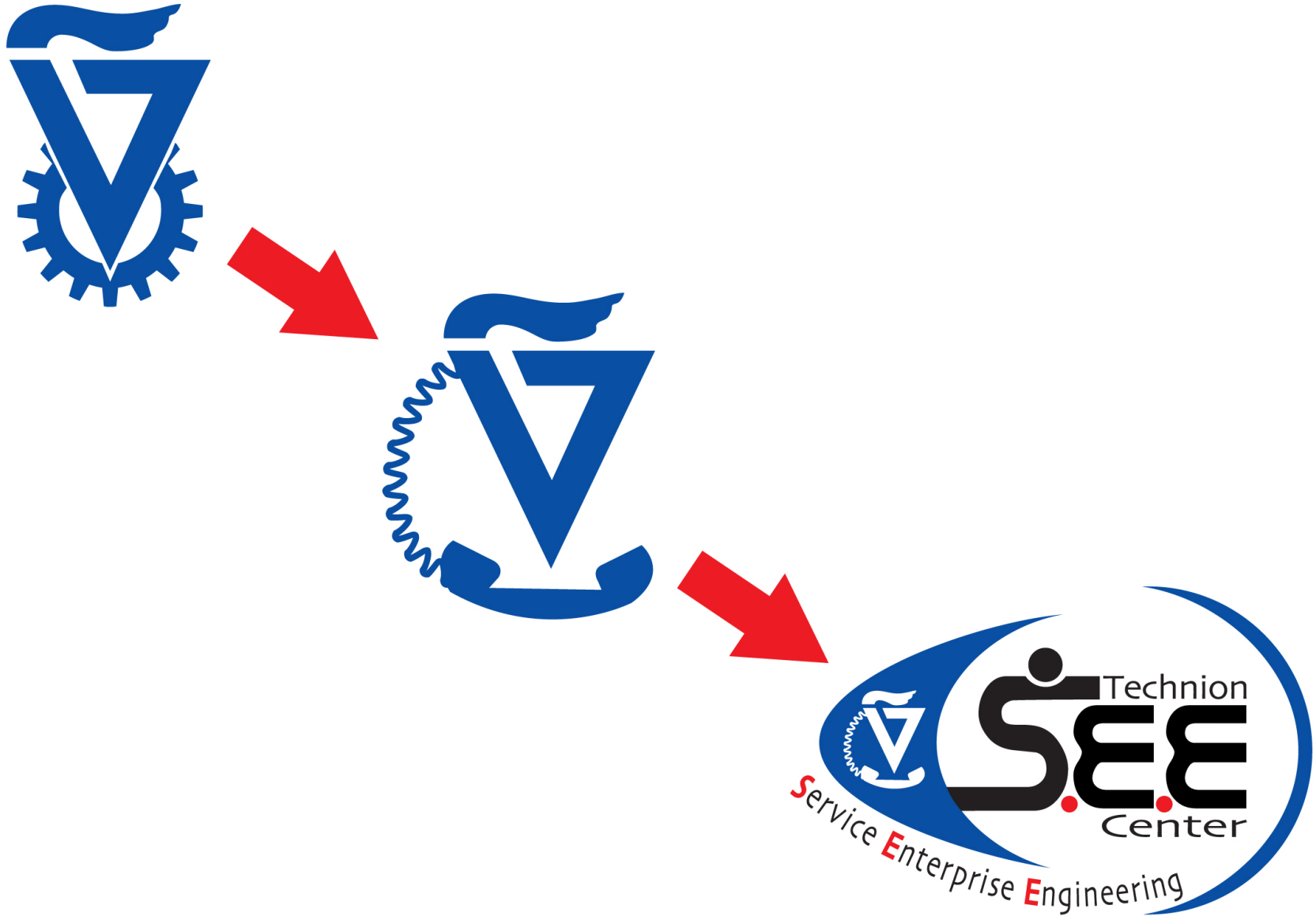
Time-Stable Performance of Time-Varying Systems

Delay Probability = as in the **Stationary Erlang-A / R**



Pause for a Commercial:

Pause for a Commercial: The Technion SEELab



Technion SEE = Service Enterprise Engineering

SEELab: Data-repositories for research and teaching

- Detailed **operational histories** (customers, servers), e.g.
  1. * Bank Anonymous : **1 year, 350K calls by 15 agents** - in 2000, which paved the way to:
  2. *U.S. Bank : **2.5 years, 220M calls, 40M by 1000 agents**
  3. Israeli Cellular: **2.5 years, 110M calls, 25M calls by 750 agents**
  4. Israeli Bank: **from January 2010, daily-deposit at a SEESafe**
  5. *Home (Rambam) Hospital : **4 years, 1000 beds**, ward-level flow
  6. Emergency Department (ED) patient flow:
     - 5 EDs in Israel: 1-2 years, **late David Sinreich**, ED arrivals & LOS
     - ED in Seoul: 2 months, **K. Song-Hee & W. Cha**, pilot
     - ED in Singapore: 2 years, **S. He & M. Sim**, pilot
  7. Service Engineering **internet site**: click-stream data (2 years)

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Environment for graphical EDA in real-time:

- ***SEEStat** : primitives (arrivals, services, patience)
- **SEEGraph**: structure, animation (protocols $\Rightarrow$ simulation)

*** Open & Free for academic use**

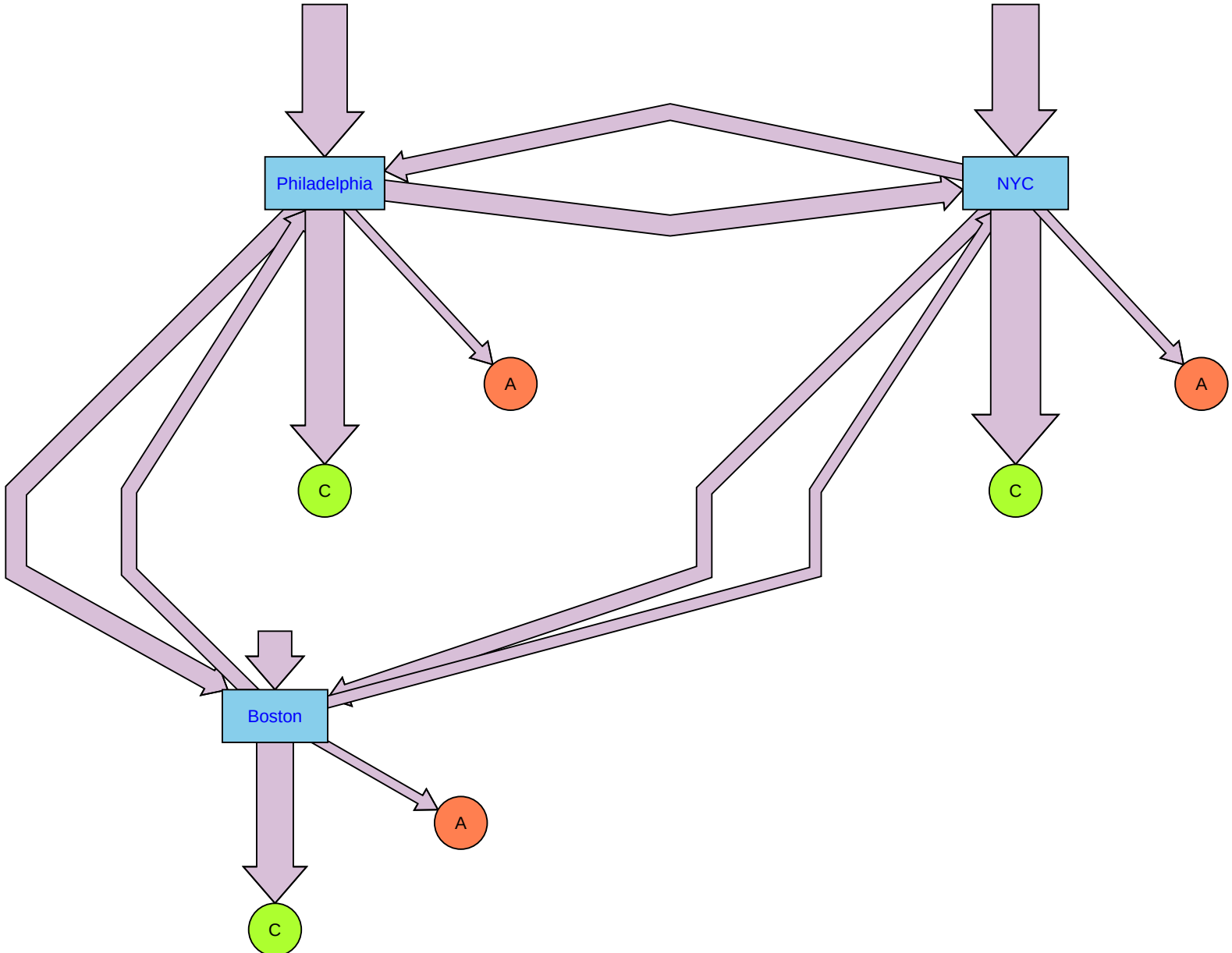
Empirical Adventures at the SEELab

SEELab History suggests possible guidelines for ServNet Mining:

1. **Primitives**: arrivals, services, (im)patience
2. **Structure**: static process-maps
3. **Protocols**: Load Balancing, Dynamic Priority, Information

EDA $\Rightarrow$ open questions, new directions, uncharted territories

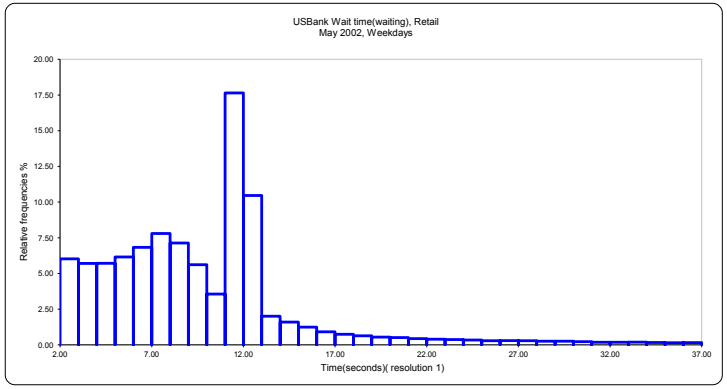
Protocol Mining: Snapshots of Connectivity



Protocols: Waiting Time in a Call Center

Routing via Thresholds (sec.)

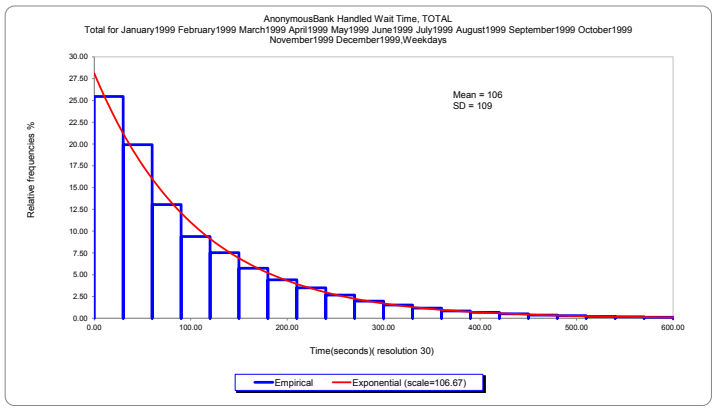
Large U.S. Bank



Protocols: Waiting Time in a Call Center

Exponential in Heavy-Traffic (min.)

Small Israeli Bank



Routing via Thresholds (sec.)

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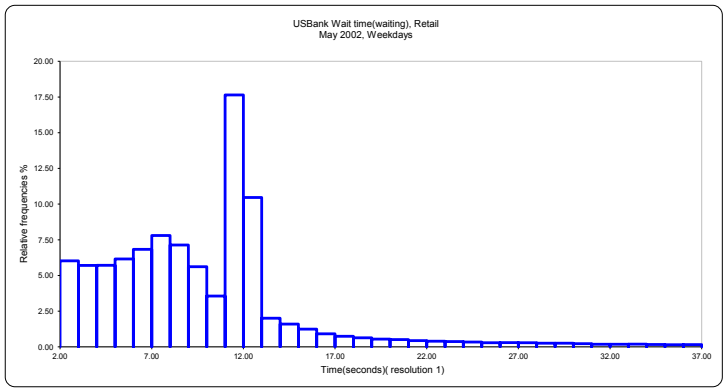
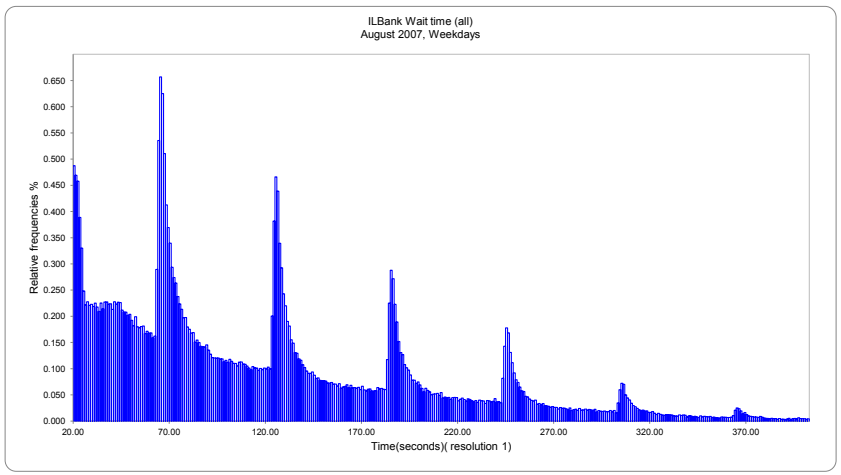


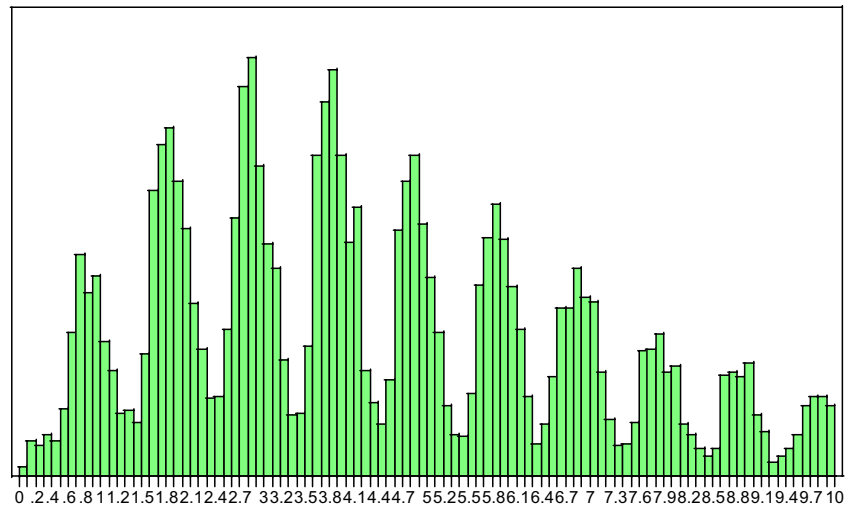
Scheduling Priorities (sec.) [compare Hospital LOS (hours)]

Medium Israeli Bank



Protocols: LOS in Hospitals

In Hours: 2 Time Scales, Mixture



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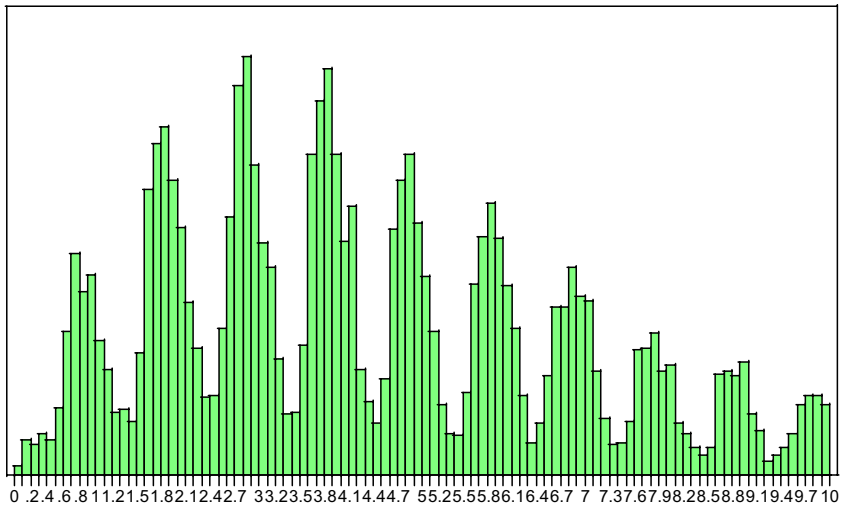
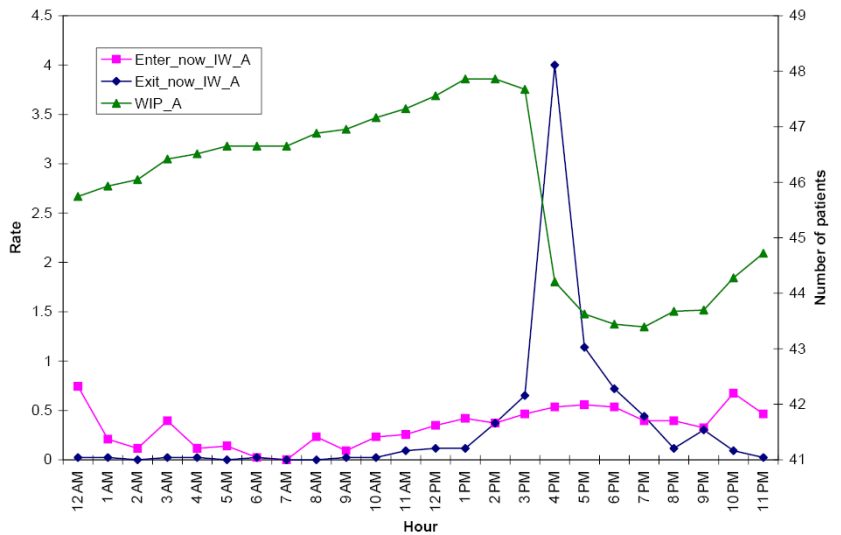


Explanation: Patients released around **3pm** (2-3 in Singapore, 2-4 in UNC Hospital)

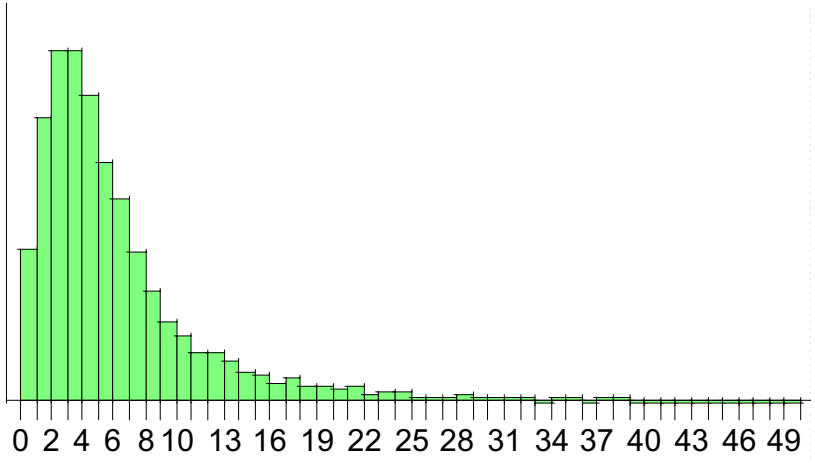


Why Bother ?

- Hourly Scale: Staffing,. . .
- Daily: Flow / Bed Control,. . .

Protocols: LOS in Hospitals

Israeli Hospital, in Days: LN



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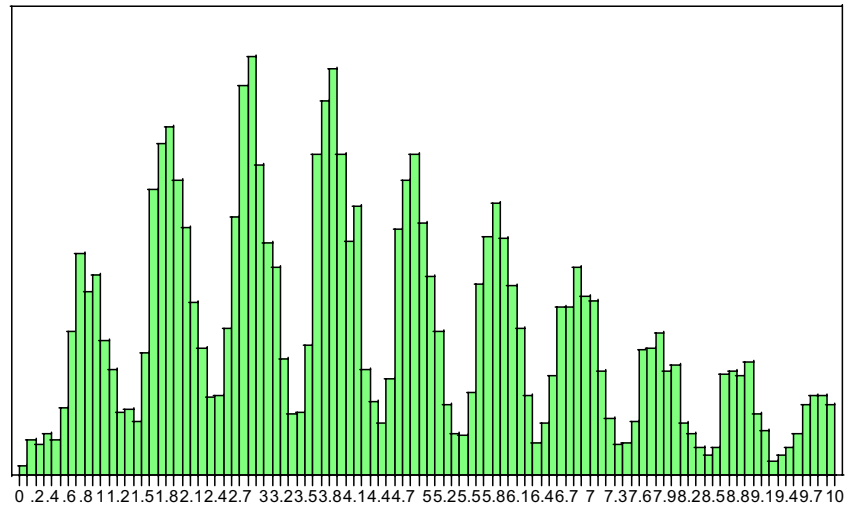
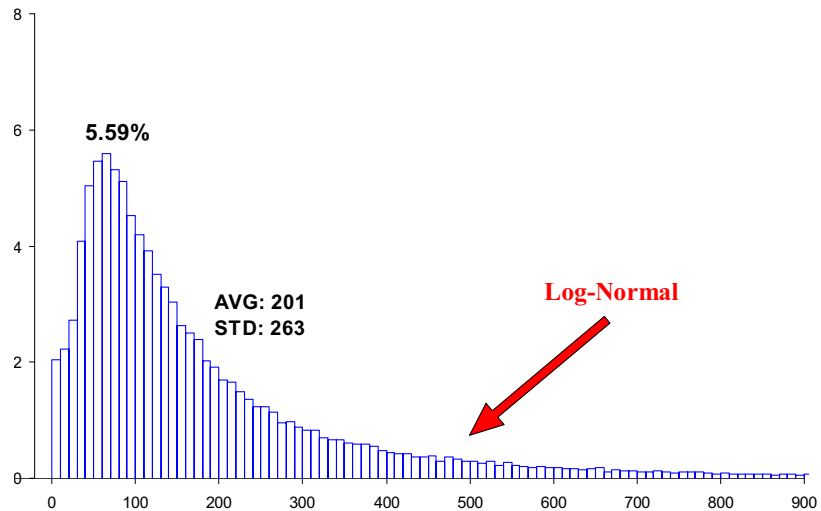


Primitives: Services (Durations)

Histogram of Service-Duration in an Israeli Call Center, 1999

Why LogNormal?

November-December

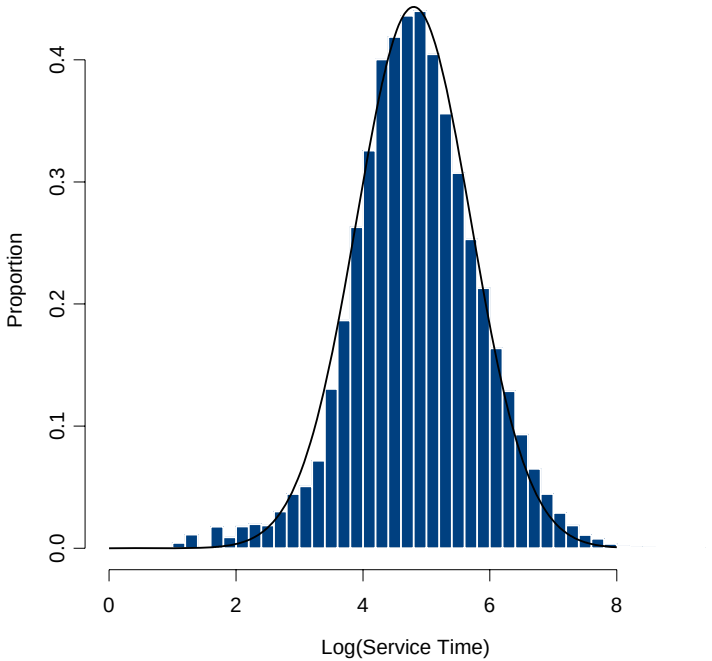


- November-December: **LogNormal** durations (common) ?

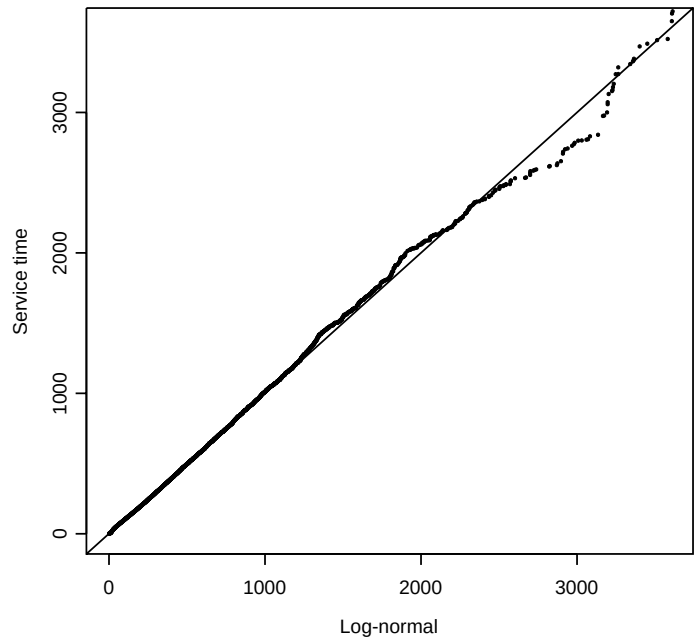
Durations: Phone Calls

Israeli Call Center, Nov–Dec, 1999

Log(Service Times)



LogNormal QQPlot



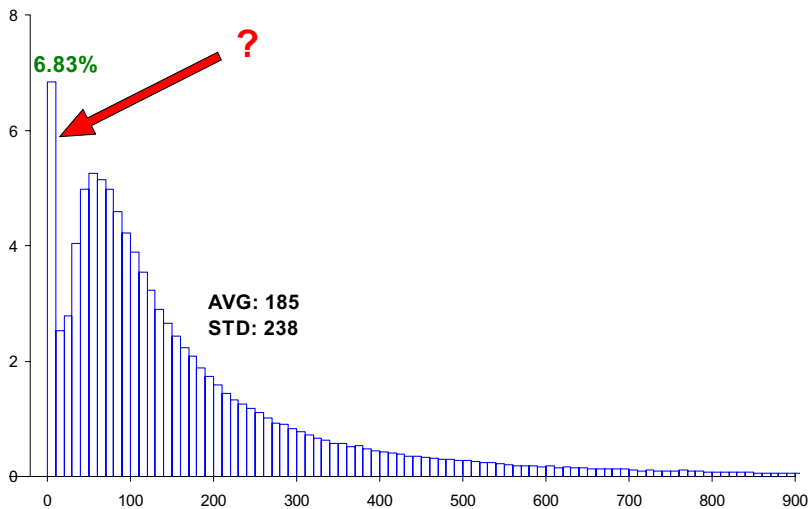
- **Practically Important**: (mean, std)(log) characterization
- **Theoretically Intriguing**: Why LogNormal ? Naturally multiplicative but, in fact, also **Infinitely-Divisible** (Generalized Gamma-Convolutions)

Primitives: Services (Durations)

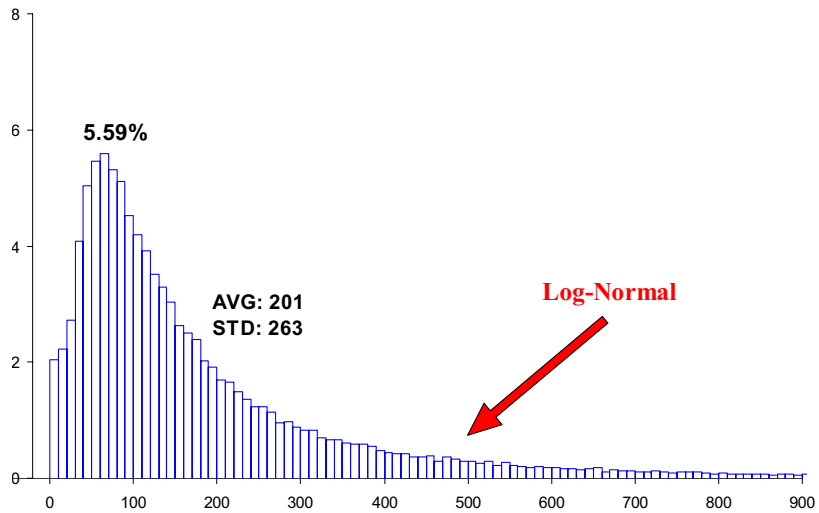
Histogram of Service-Duration in an Israeli Call Center, 1999

Why short services? Why LogNormal?

January-October



November-December

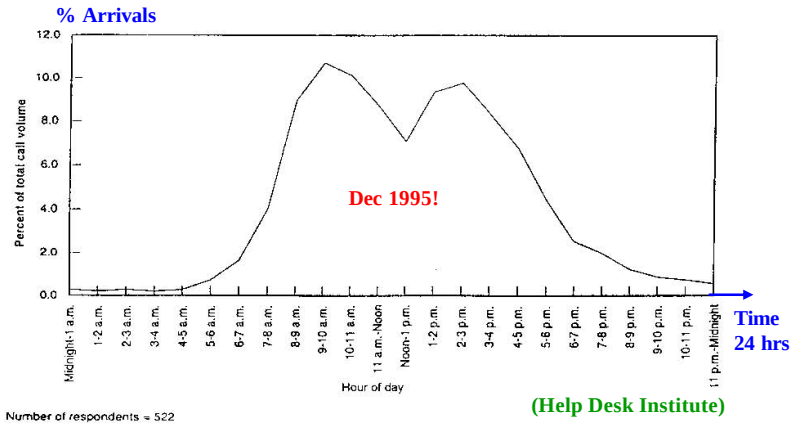


- January-October: **6.8% Short-Services** ($\leq$ 10 seconds) ?
- November-December: **LogNormal** durations (common) ?

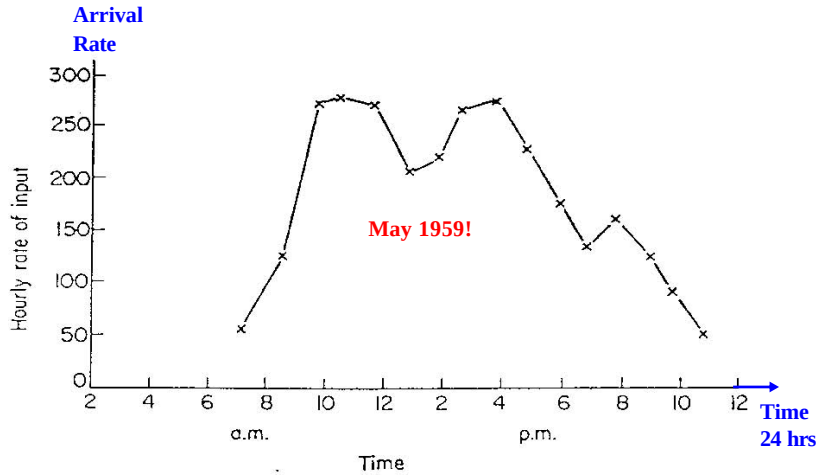
Primitives: Arrival (Rates) to Service

Why 2 Daily Peaks?

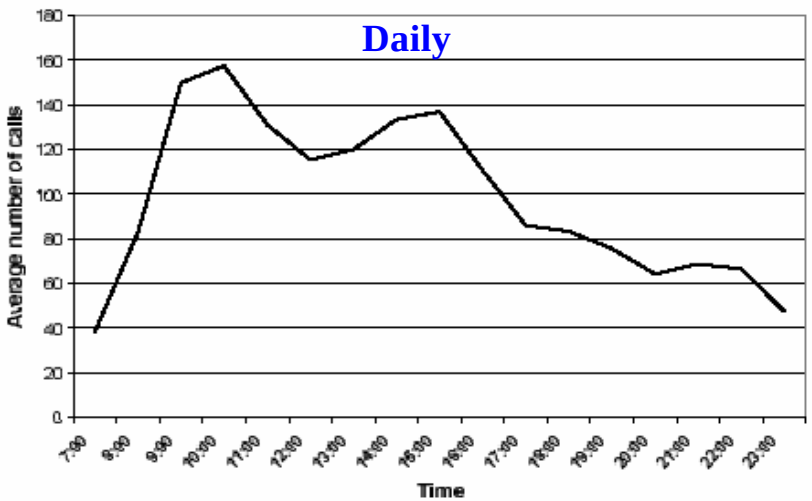
CC: Dec. **1995**, (USA, 700 Helpdesks)



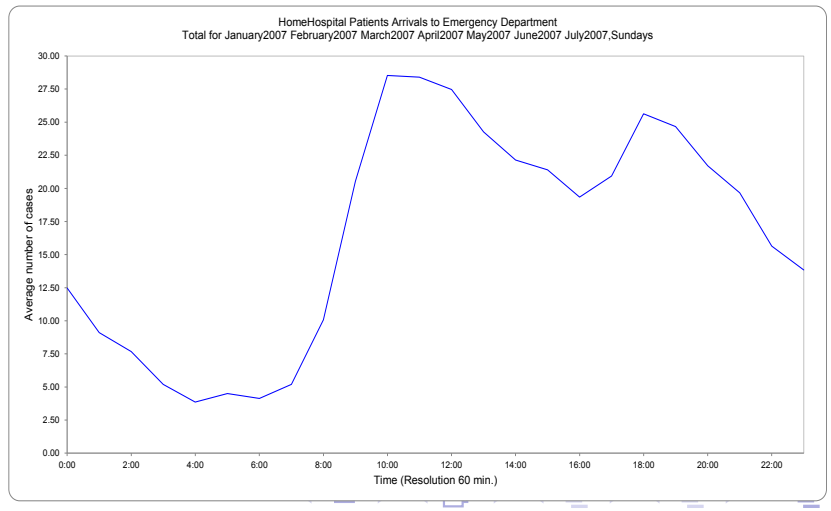
CC: May **1959** (England)



CC: Nov. 1999 (Israel)

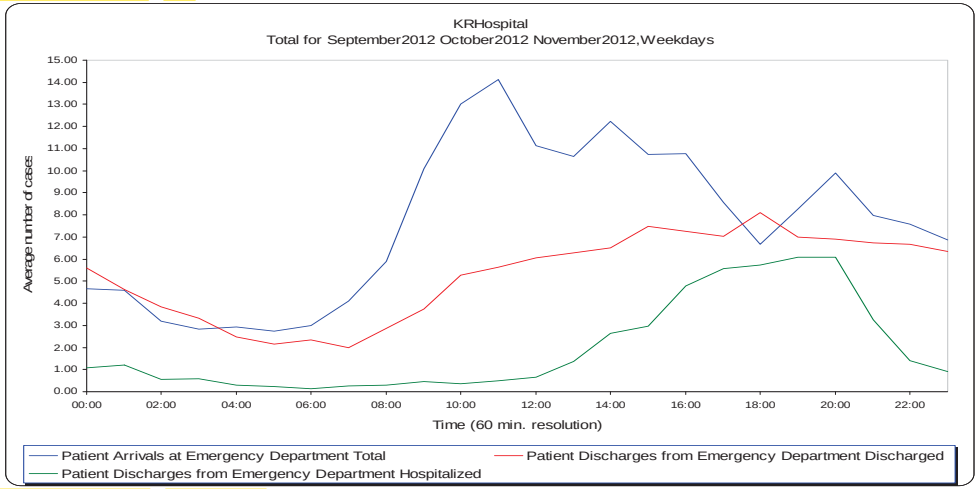


ED: Jan.–July 2007 (Israel)

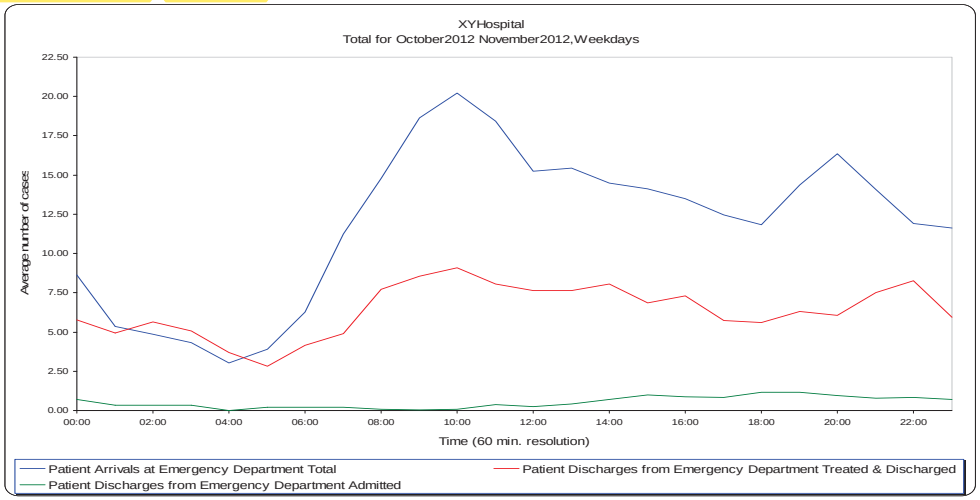


Arrival (Discharge) Rates in Korea and Singapore

KRHospital, all ED patients



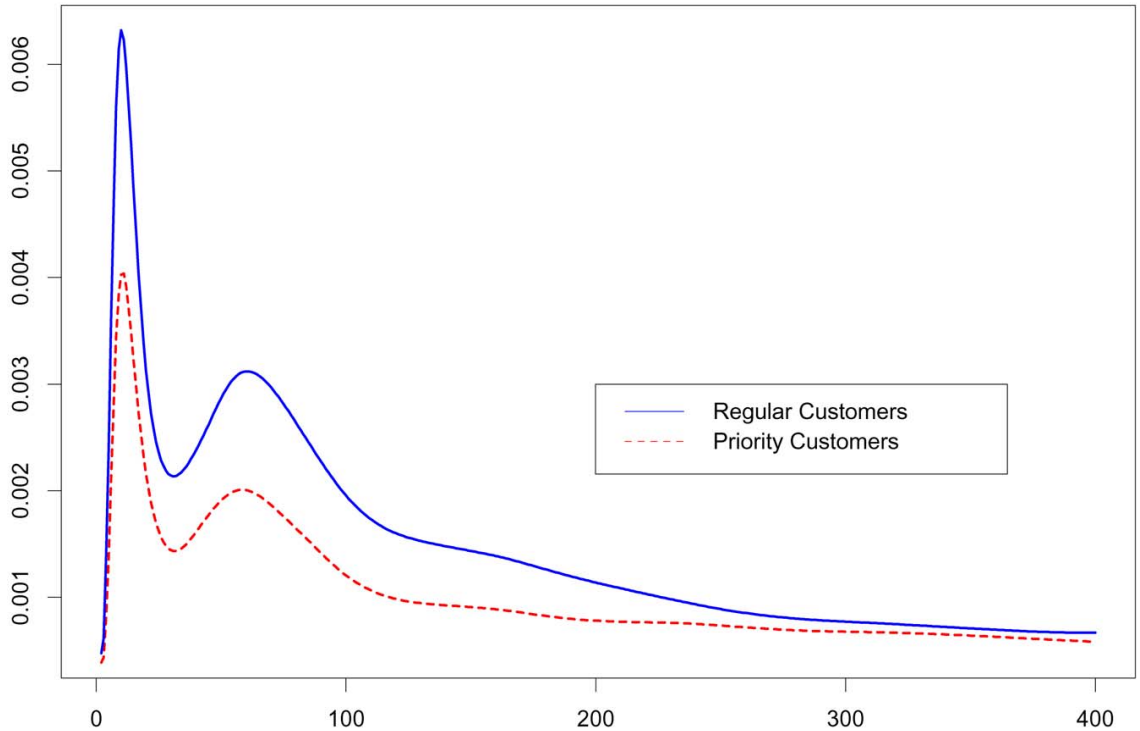
XYHospital, walking patients



Protocols: (Im)Patience while Waiting (Psychology)

Palm: (1943–53): **Irritation** $\propto$ **Hazard Rate**

Regular over **VIP** Customers: VIP more patient here (Israeli Bank)



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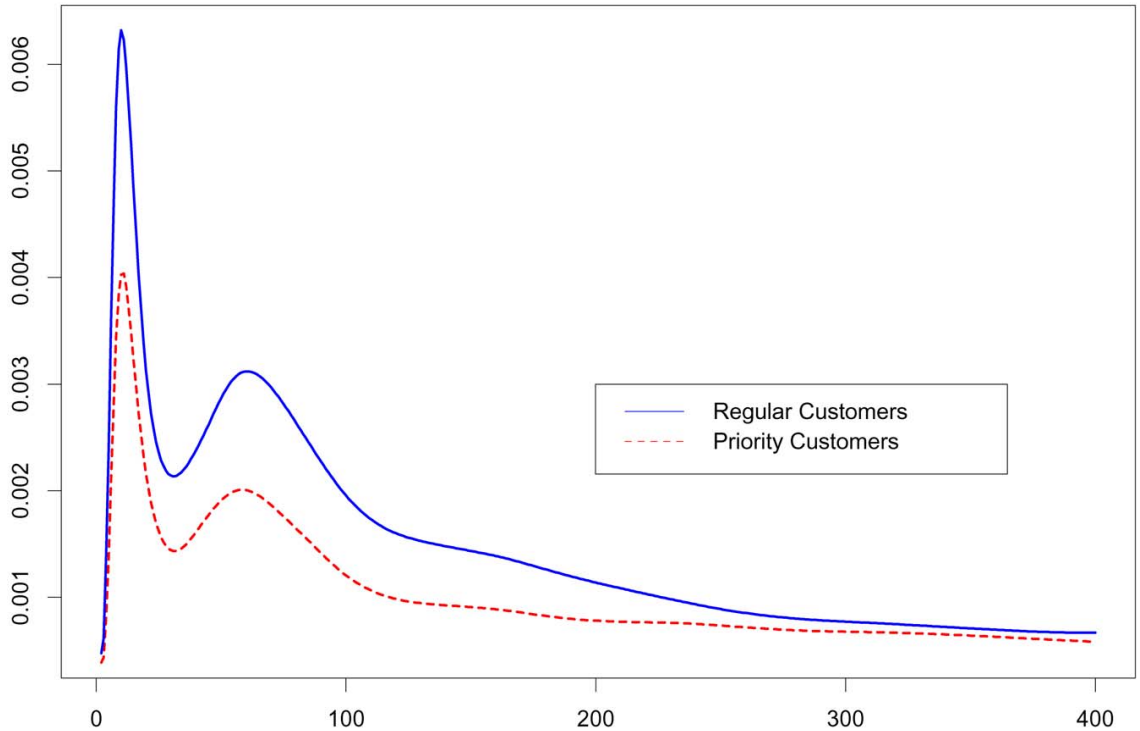
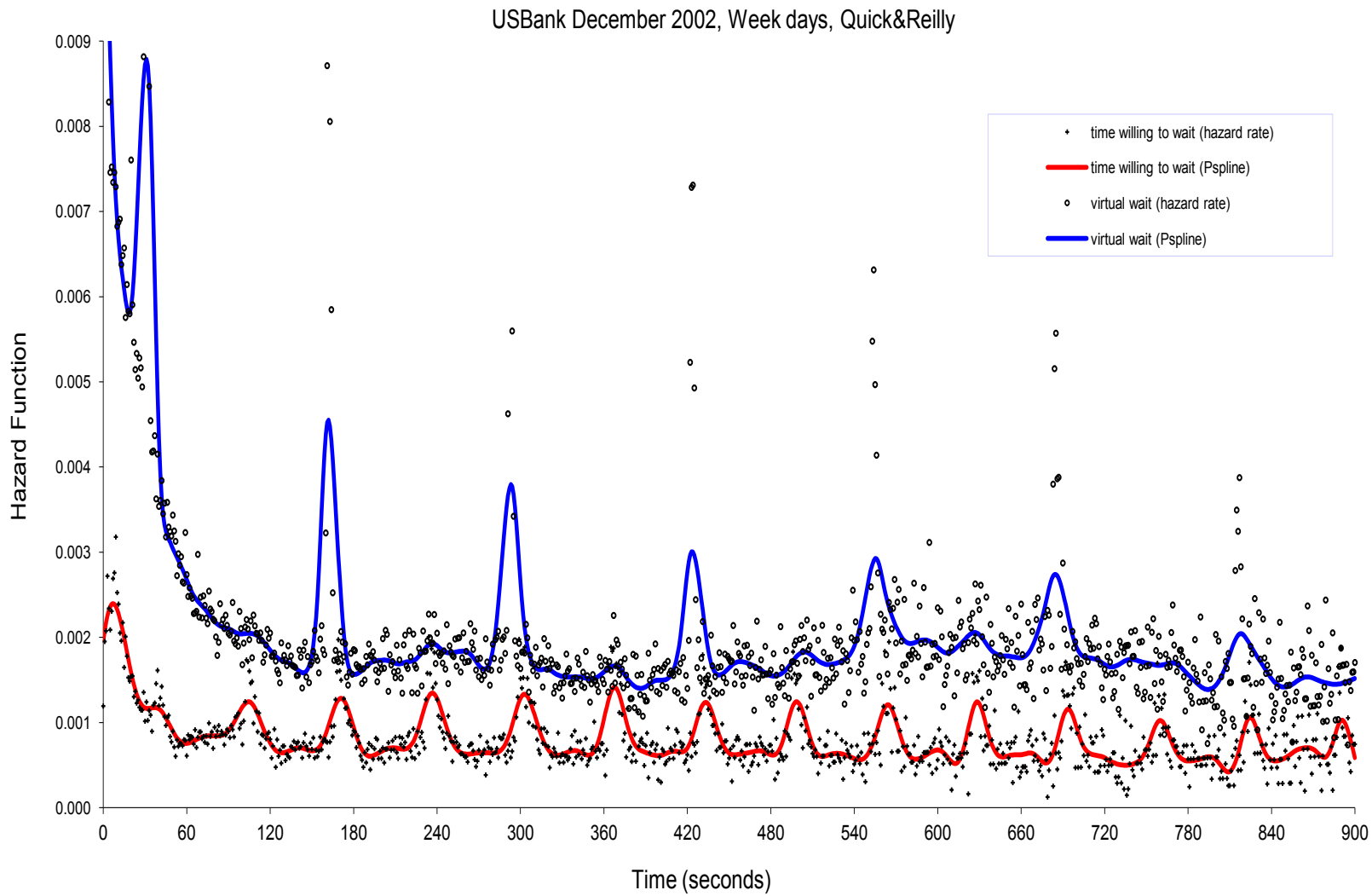


- Why **Peaks** of **abandonment**? **Announcement** epochs
  - Control abandonment w/ info: encourage, discourage
  - Technical Challenges, **w/ J. Huang, J. Zhang, H. Zhang**
- Statistical challenges: **Un-Censoring**, Smoothing

Protocols + Psychology

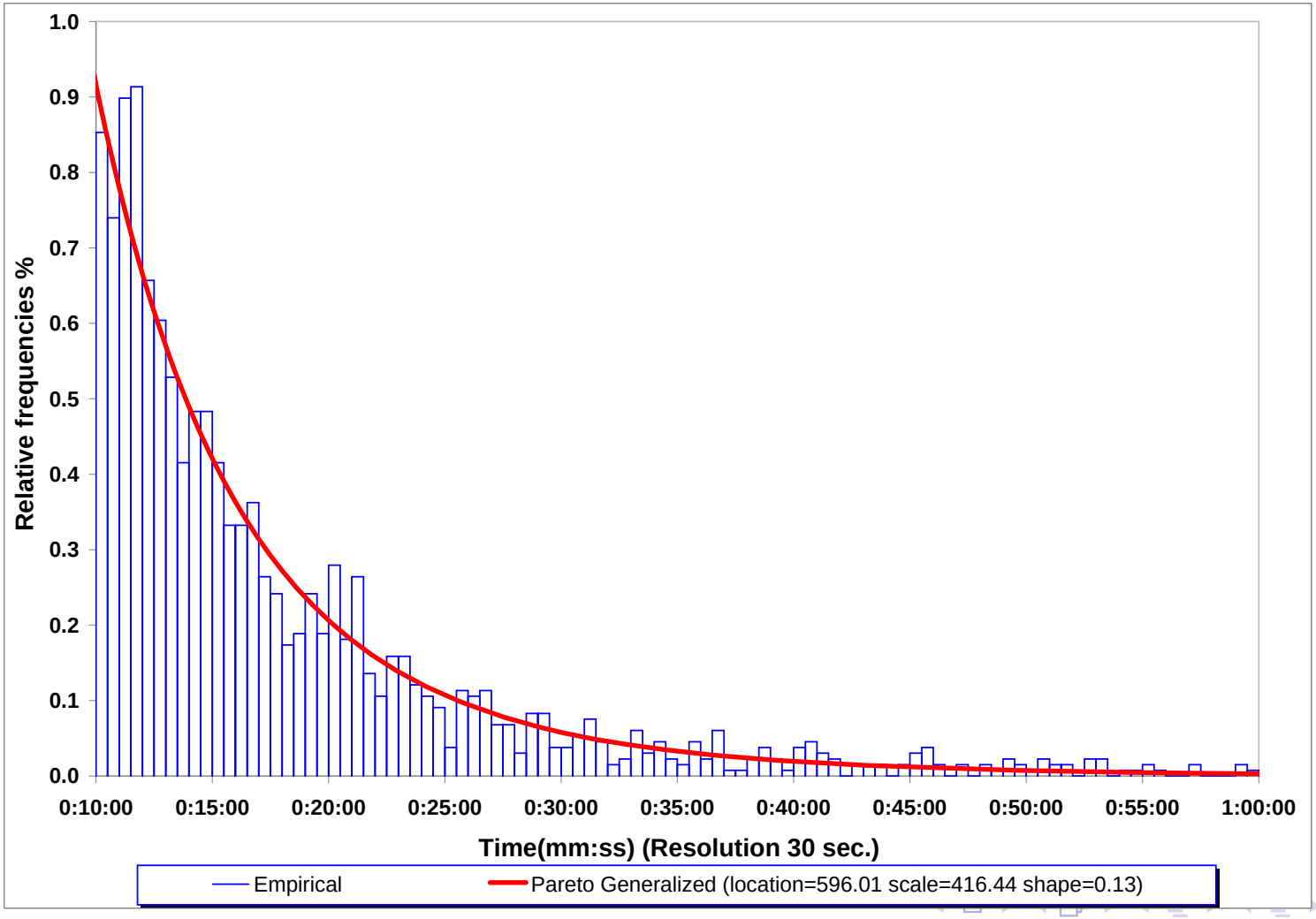
Patient Customers, Announcements, Priority Upgrades



Primitives: (Im)Patience

Israeli Bank: Uncensored 13,000 Customers, 24/11/2008

Patience $\geq 10min$: Why Pareto Tail?



On Data-Based Research

w/ V. Trofimov, E. Nadjharov, I. Gavako = Technion SEELab

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- **SimNets** of Service Systems = **Virtual Realities**, where “complex" models meet “simple" models, towards enhancement and credibility

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- Beyond the prevalent: "single researcher (with a PhD student) obtaining small data for a single research project": unprofessional, no learning across generations, no sharing among researchers, irreproducible research
- Data-based Research: Tradition in Physics, Chemistry, Biology; Psychology (now also in Transportation (Science) and (Behavioral) Economics)
- **Why not in Service/Queueing** Science / Engineering / Management ?

On Data-Based Research

w/ V. Trofimov, E. Nadjharov, I. Gavako = Technion SEELab

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- **Glad to see this happening in DSC/e**

Data-Based Creation of ServNets: some Technicalities

- **ServNets** = QNets, SimNets, FNets, DNets
- **Graph Layout**: Adapted from but significantly extends Graphviz (AT&T, 90's); eg. *edge-width*, which must be restricted to *poly-lines*, since there are "no parallel Bezier (Cubic) curves ($B_n(p) = E_p F[B(n,p)], 0 \leq p \leq 1$)
- **Algorithm**: Dot Layout (but with cycles), based on Sugiyama, Tagawa, Toda ('81): "Visual Understanding of **Hierarchical** System Structures"

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- Draws data directly from **SEELab** data-bases:
  - Relational DBs (Large! eg. USBank Full Binary = 37GB, Summary Tables = 7GB)
  - Structure: Sequence of events/states, which (due to size) partitioned (yet integrated) into days (eg. call centers) or months (eg. hospitals)
  - Differs from industry DBs (in call centers, hospitals, websites)

Applying (Queueing) Asymptotics

There are by now numerous insightful asymptotic queueing models at our disposal, and many arise from, and create, deep beautiful theory:

Has it helped one approximate or simulate a service system more efficiently, estimate its parameter more accurately, teach it to our students more effectively, perhaps even manage the system better?

I am of the opinion that the answers to such questions have been too often negative, and that positive answers must have theory and applications nurture each other, which is good.

How to make this good happen? My approach has been to marry theory with data , which has been supported by (what I only recently came to realize is) process (Q-Net) mining: building-blocks, structure, protocols; laws of "nature".

Prevalent (Asymptotic) Approximations

System (Data)

Model

SimNet **Q-Net**

Prevalent (Asymptotic) Approximations

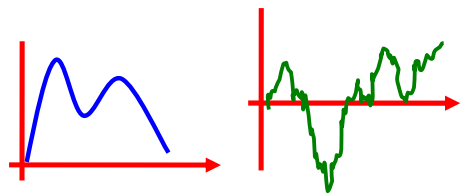
System (Data)

Phenomenon

Accuracy

Model ⟷ **Approximation**

SimNet **Q-Net** **F-Net** **D-Net**



Data–Based Prevalent (Asymptotic) Approximations Models

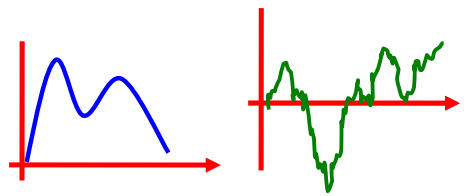
System (Data)

Phenomenon

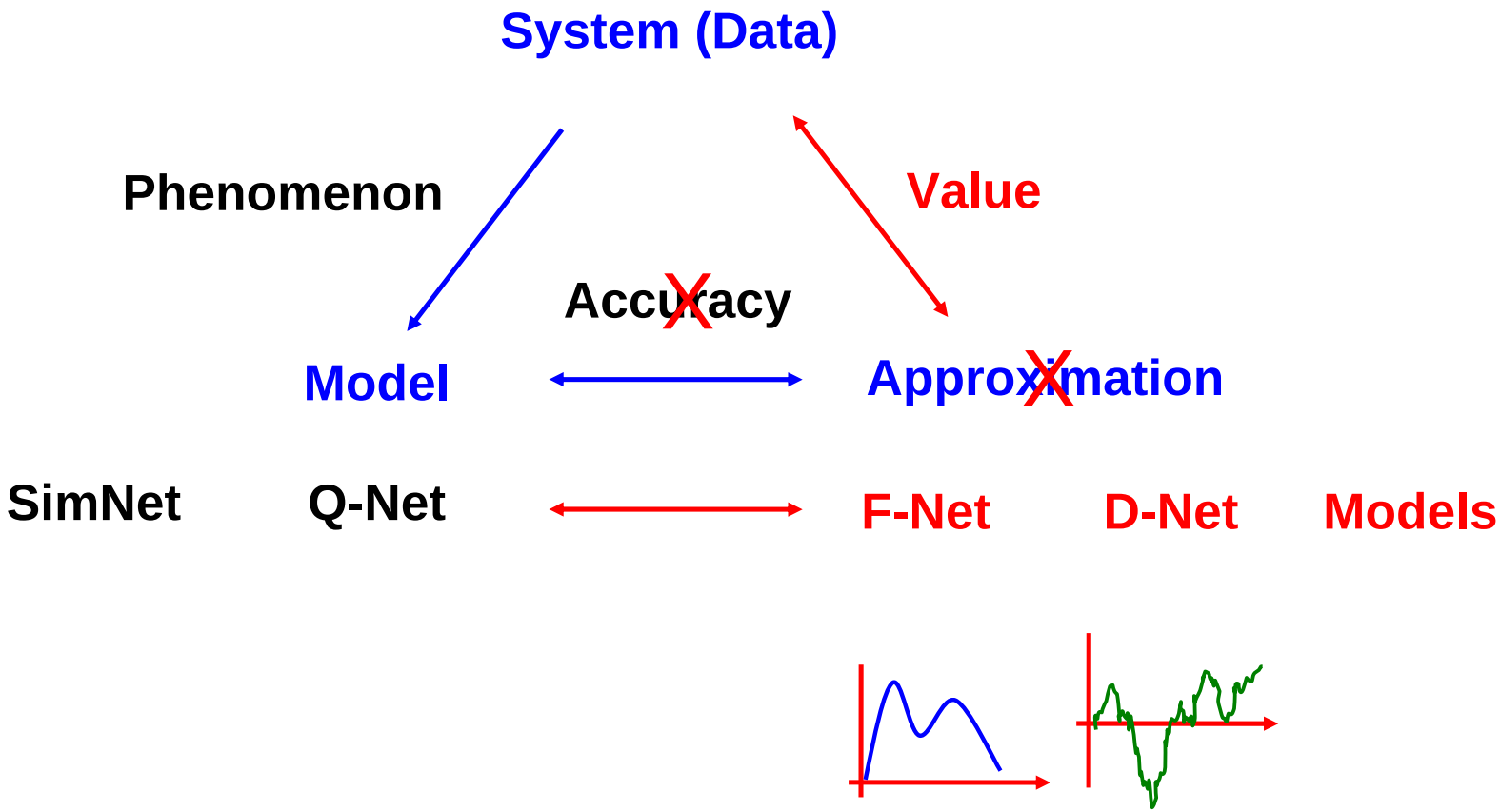
Accuracy

Model ⟷ Approximation

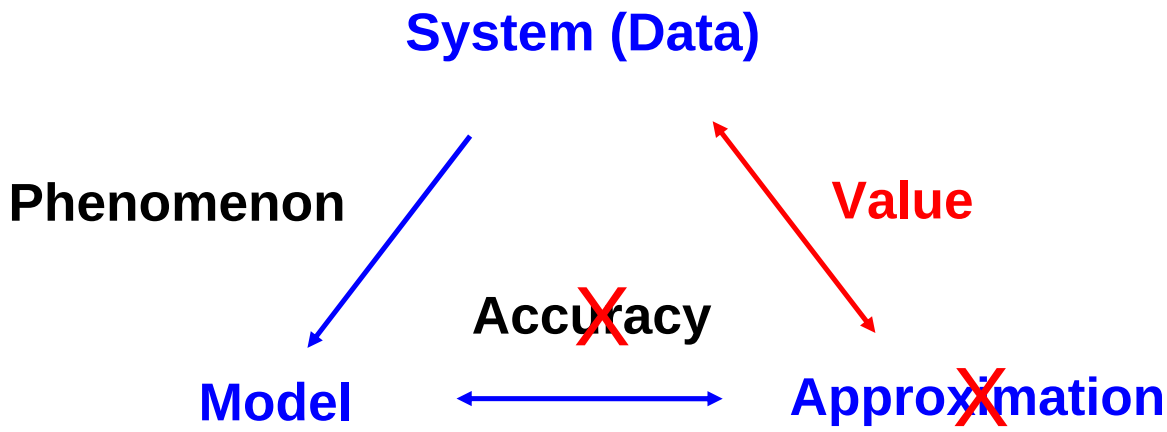
SimNet Q-Net ⟷ F-Net D-Net Models



Data–Based Prevalent (Asymptotic) Approximations Models



Data–Based Prevalent (Asymptotic) Approximations Models



System = Coin Tossing, **Model** = Binomial ; de Moivre 1738

Approx. / Models: SLLN **(FNets)**, CLT **(DNets)** ; Laplace 1810

Value: Exceeds Value of originating stylized model

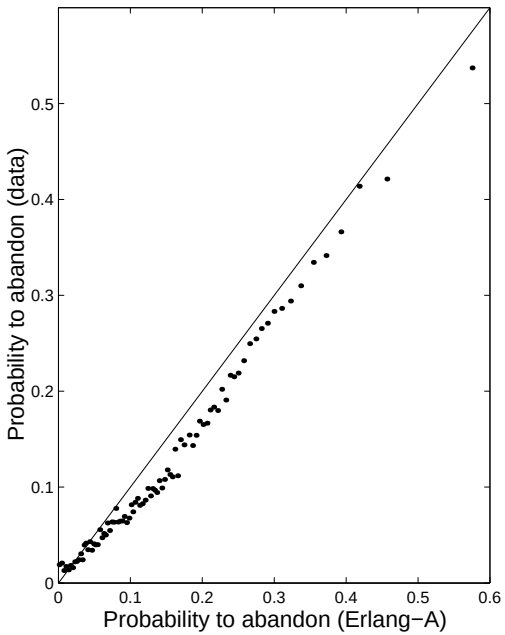
Normal, Brownian Motion ; Bachalier 1900

Poisson ; Poisson 1838

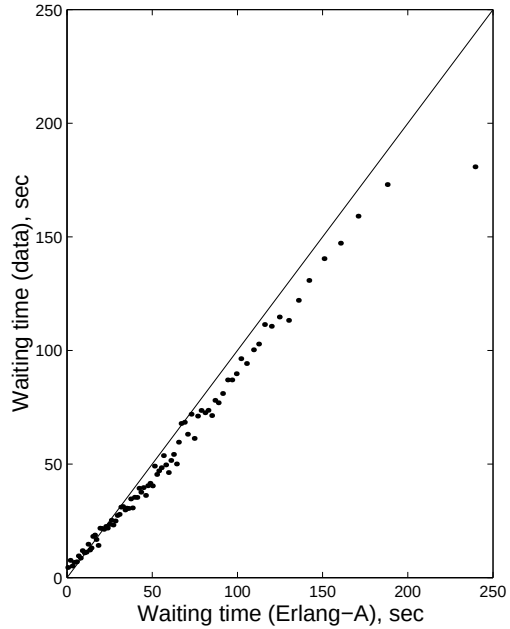
Erlang-A: Fitting a Simple Model to a Complex Reality

Hourly Performance vs. Erlang-A Predictions (1 year)

% Abandon



E[*Wait*]



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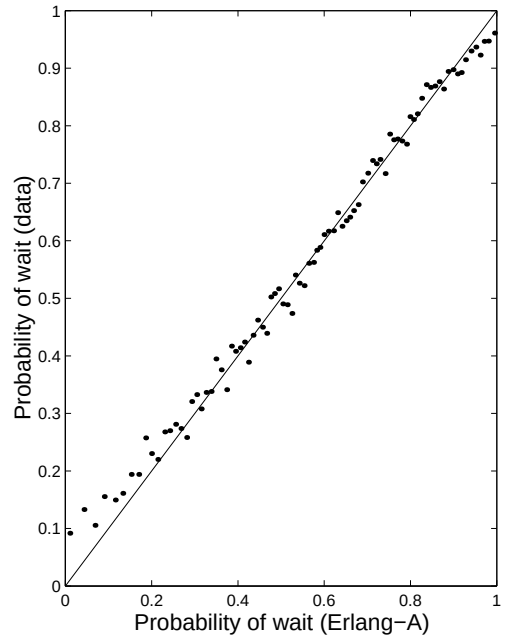
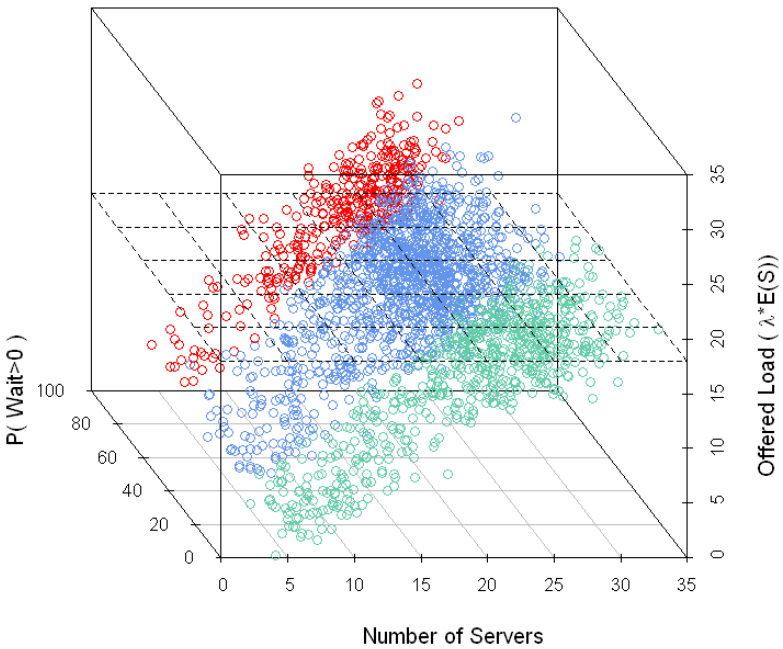


- Empirically-Based & Theoretically-Supported Estimation of (Im)Patience: $\hat{\theta} = P\{Ab\}/\mathrm{E}[W_q]$)
- Small Israeli Bank (more examples in progress)
- Hourly performance vs. Erlang-A predictions, 1 year: aggregated groups of 40 similar hours

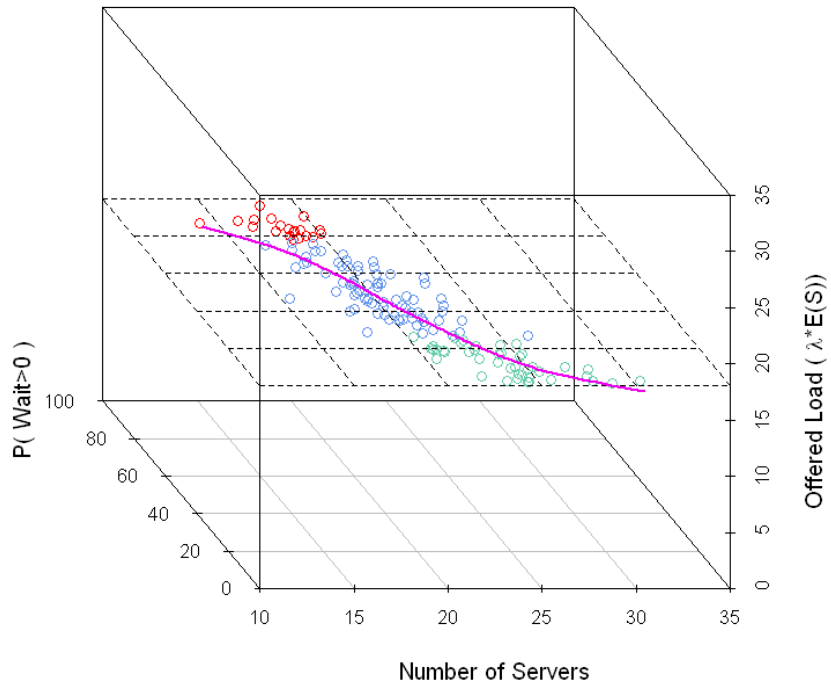
Operational Regimes: Q(uality) vs. E(fficiency)

IL Telecom; June-September, 2004

P{$W_q > 0$} vs. (R, N)



R-Slice: P{$W_q > 0$} vs. N



3 Operational Regimes:

- **QD**: $\leq 25\%$
- **QED**: $25\% - 75\%$
- **ED**: $\geq 75\%$

Calculating the Offered-Load $R(t)$, Theoretically

- Offered-Load Process: $\boldsymbol{L(\cdot)}$ = **Least** number of **servers** that guarantees **no delay**.
- **Offered-Load** Function $\boldsymbol{R(t)} = E[L(t)]$, $t \geq 0$.
  Think $M_t/G/N_t^? + G$ vs. $\boldsymbol{M_t/G/\infty}$: **Ample-Servers**.

Four (all useful) representations, capturing **"workload before t"**:

$$R(t) = E[L(t)] = \int_{-\infty}^{t} \lambda(u) \cdot P(S > t-u)du = E\Big[A(t) - A(t-S)\Big] =$$

$$= E\left[\int_{t-S}^{t} \lambda(u)du\right] = E[\lambda(t-S_e)] \cdot E[S] \approx \ldots .$$

- $\{\boldsymbol{A(t)},\ t \geq 0\}$ Arrival-Process, rate $\boldsymbol{\lambda(\cdot)}$;
- $\boldsymbol{S}$ ($\boldsymbol{S_e}$) generic Service-Time (Residual Service-Time).
- Relating L, λ, S ("W"): **Time-Varying Little's Formula**.
  Stationary models: $\lambda(t) \equiv \lambda$ then $R(t) \equiv \boldsymbol{\lambda} \times \boldsymbol{E[S]}$.

QED-c: $\boldsymbol{N_t = R_t + \beta R_t^c}$, $1/2 \leq c < 1$; ($c = 1$ separate analysis).

Data Cleaning: MCE with RFID Support

Data-base				Company report		comment
Asset id	order	**Entry** date	**Exit** date	Entry date	Exit date	
4	1	1:14:07 PM		1:14:00 PM		
6	1	12:02:02 PM	12:33:10 PM	12:02:00 PM	12:33:00 PM	
8	1	11:37:15 AM	12:40:17 PM	11:37:00 AM		**exit is missing**
10	1	12:23:32 PM	12:38:23 PM	12:23:00 PM		
12	1	12:12:47 PM	12:35:33 PM		12:35:00 PM	**entry is missing**
15	1	1:07:15 PM		1:07:00 PM		
16	1	11:18:19 AM	11:31:04 AM	11:18:00 AM	11:31:00 AM	
17	1	1:03:31 PM		1:03:00 PM		
18	1	1:07:54 PM		1:07:00 PM		
19	1	12:01:58 PM		12:01:00 PM		
20	1	11:37:21 AM	12:57:02 PM	11:37:00 AM	12:57:00 PM	
21	1	12:01:16 PM	12:37:16 PM	12:01:00 PM		
22	1	12:04:31 PM	12:20:40 PM			first customer is missing
22	2	12:27:37 PM		12:27:00 PM		
25	1	12:27:35 PM	1:07:28 PM	12:27:00 PM	1:07:00 PM	
27	1	12:06:53 PM		12:06:00 PM		
28	1	11:21:34 AM	11:41:06 AM	11:41:00 AM	11:53:00 AM	**exit time instead of entry time**
29	1	12:21:06 PM	12:54:29 PM	12:21:00 PM	12:54:00 PM	
31	1	11:40:54 AM	12:30:16 PM	11:40:00 AM	12:30:00 PM	
31	2	12:37:57 PM	12:54:51 PM	12:37:00 PM	12:54:00 PM	
32	1	11:27:11 AM	12:15:17 PM	11:27:00 AM	12:15:00 PM	
33	1	12:05:50 PM	12:13:12 PM	12:05:00 PM	12:15:00 PM	wrong exit time
35	1	11:31:48 AM	11:40:50 AM	11:31:00 AM	11:40:00 AM	
36	1	12:06:23 PM	12:29:30 PM	12:06:00 PM	12:29:00 PM	
37	1	11:31:50 AM	11:48:18 AM	11:31:00 AM	11:48:00 AM	
37	2	12:59:21 PM		12:59:00 PM		

- Imagine **“Cleaning" 60,000+ customers per day** (call centers) !
- “Psychology" of Data Trust and Transfer (e.g. 2 years till transfer)

Event-Logs in a Call Center (Bank Anonymous)

A Data Sample (Excel worksheet)

vru+line	call_id	customer_id	priority	type	date	vru_entry	vru_exit	vru_time	q_start	q_exit	q_time	outcome	ser_start	ser_exit	ser_time	server
AA0101	44749	27644400	2	PS	990901	11:45:33	11:45:39	6	11:45:39	11:46:58	79	AGENT	11:46:57	11:51:00	243	DORIT
AA0101	44750	12887816	1	PS	990905	14:49:00	14:49:06	6	14:49:06	14:53:00	234	AGENT	14:52:59	14:54:29	90	ROTH
AA0101	44967	58660291	2	PS	990905	14:58:42	14:58:48	6	14:58:48	15:02:31	223	AGENT	15:02:31	15:04:10	99	ROTH
AA0101	44968	0	0	NW	990905	15:10:17	15:10:26	9	15:10:26	15:13:19	173	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44969	63193346	2	PS	990905	15:22:07	15:22:13	6	15:22:13	15:23:21	68	AGENT	15:23:20	15:25:25	125	STEREN
AA0101	44970	0	0	NW	990905	15:31:33	15:31:47	14	00:00:00	00:00:00	0	AGENT	15:31:45	15:34:16	151	STEREN
AA0101	44971	41630443	2	PS	990905	15:37:29	15:37:34	5	15:37:34	15:38:20	46	AGENT	15:38:18	15:40:56	158	TOVA
AA0101	44972	64185333	2	PS	990905	15:44:32	15:44:37	5	15:44:37	15:47:57	200	AGENT	15:47:56	15:49:02	66	TOVA
AA0101	44973	3.06E+08	1	PS	990905	15:53:05	15:53:11	6	15:53:11	15:56:39	208	AGENT	15:56:38	15:56:47	9	MORIAH
AA0101	44974	74780917	2	NE	990905	15:59:34	15:59:40	6	15:59:40	16:02:33	173	AGENT	16:02:33	16:26:04	1411	ELI
AA0101	44975	55920755	2	PS	990905	16:07:46	16:07:51	5	16:07:51	16:08:01	10	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44976	0	0	NW	990905	16:11:38	16:11:48	10	16:11:48	16:11:50	2	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44977	33689787	2	PS	990905	16:14:27	16:14:33	6	16:14:33	16:14:54	21	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44978	23817067	2	PS	990905	16:19:11	16:19:17	6	16:19:17	16:19:39	22	AGENT	16:19:38	16:21:57	139	TOVA
AA0101	44764	0	0	PS	990901	15:03:26	15:03:36	10	00:00:00	00:00:00	0	AGENT	15:03:35	15:06:36	181	ZOHARI
AA0101	44765	25219700	2	PS	990901	15:14:46	15:14:51	5	15:14:51	15:15:10	19	AGENT	15:15:09	15:17:00	111	SHARON
AA0101	44766	0	0	PS	990901	15:25:48	15:26:00	12	00:00:00	00:00:00	0	AGENT	15:25:59	15:28:15	136	ANAT
AA0101	44767	58859752	2	PS	990901	15:34:57	15:35:03	6	15:35:03	15:35:14	11	AGENT	15:35:13	15:35:15	2	MORIAH
AA0101	44768	0	0	PS	990901	15:46:30	15:46:39	9	00:00:00	00:00:00	0	AGENT	15:46:38	15:51:51	313	ANAT
AA0101	44769	78191137	2	PS	990901	15:56:03	15:56:09	6	15:56:09	15:56:28	19	AGENT	15:56:28	15:59:02	154	MORIAH
AA0101	44770	0	0	PS	990901	16:14:31	16:14:46	15	00:00:00	00:00:00	0	AGENT	16:14:44	16:16:02	78	BENSION
AA0101	44771	0	0	PS	990901	16:38:59	16:39:12	13	00:00:00	00:00:00	0	AGENT	16:39:11	16:43:35	264	VICKY
AA0101	44772	0	0	PS	990901	16:51:40	16:51:50	10	00:00:00	00:00:00	0	AGENT	16:51:49	16:53:52	123	ANAT
AA0101	44773	0	0	PS	990901	17:02:19	17:02:28	9	00:00:00	00:00:00	0	AGENT	17:02:28	17:07:42	314	VICKY
AA0101	44774	32387482	1	PS	990901	17:18:18	17:18:24	6	17:18:24	17:19:01	37	AGENT	17:19:00	17:19:35	35	VICKY
AA0101	44775	0	0	PS	990901	17:38:53	17:39:05	12	00:00:00	00:00:00	0	AGENT	17:39:04	17:40:43	99	TOVA

- Unsynchronized transition times, consistently