

**Data-Driven Service Networks:
Models of congested service systems
(e.g. hospitals, call-centers, courts, ...)**

**“Theompirical” Journeys in Service Systems
OR = SE/IE + OM + DS viewpoints**

Avishai Mandelbaum
IEM & SEELab, Technion
[link](#)

My First Visit to Milan

- Guest of Mediolanum Bank (then approx. 5000 agents)
- Flight, Hotel, San Siro
- Two Lectures: **Management** = fine; **Team Leaders** = **adventure** for 2 reasons:
 1. [Language](#)

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 2. Teaching “staffing” via “Offered-Load calculations” (averages):
 - Suppose 25 calls per min x 4 minute per call = 100 Erlangs = **Offered-Load (min-work per min)**
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 - Much less: **Efficiency-Driven** (ED-regime)
 - Much more: **Quality-Driven** (QD, e.g. 200 agents implies $100/200 = 50\%$ utilization)
 - Aim at the **QED regime** = Quality- and Efficiency-Driven, via **Square-Root Staffing**:
$$\text{Number of agents} = 100 + \beta\sqrt{100}, \beta \text{ in } [-1, 1]$$
$$100 + 10\beta \text{ approx. in } [90, 110], \text{ then refine}$$

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Can be explained also via a universal conservation law:

Little's Law

L = number in system, λ = throughput-rate, W = time in system

$L = \lambda \times W$ (finite-horizon, long-run/steady-state)

Research Partners (35 years)

Students:

Aldor, Baron Yonit, Carmeli-Yuviler Nitzan, Carmeli Boaz, Chen Hong, Cohen Izik, Feldman Zohar, Garnett, Ghebali, Gurvich, Khudyakov, Koren, Maman, Marmor, Reich, Rosenshmidt, Shaikhet, Senderovich, Tseytlin, Yom-Tov, Zaied, Zeltyn, Zychlinski, Zohar Eti, Zviran, ...

Theory:

Armony, Atar, Azriel, Chen Hong*, Cohen Izik*, Garnett*, Gurvich*, Feigin, Gal, Huang Junfei, Jelenkovic, Kaspi, Massey, Momcilovic, Reiman, Shimkin, Stolyar, Trichakis, Trofimov, Wasserkrug, Whitt, Yom-Tov*, Zeltyn*, Zhang Jiheng, Zhang Hanqin, ...

Exploratory Data Analysis, Data Sources, Statistics, Projects:

Brown, Gans, Shen Haipeng*, Sakov, Zhao Linda; Zeltyn*; Ritov, Goldberg*; Gurvich*, Huang Junfei*, Liberman*; Liu Nan, Ye Han; Armony, Marmor*, Tseytlin*, Yom-Tov*; Gorfine, Ghebali*; Tezcan; Kim Song-Hee, Won Chul Cha; Feigin, Azriel*; Rafaeli; Momcilovic, Trichakis; Bunnell, Kadish, Leib; ...

Industry:

Mizrahi Bank, Fleet Bank, Rambam Hospital, IBM Research, Hapoalim Bank, Pelephone Cellular, Samsung Hospital, Singapore Hospitals, Dana Farber Cancer Institute, LivePerson, Cheetah Labs,...

Technion SEE Laboratory (SEELab):

Feigin; Trofimov, Nadjharov, Gavako; Kutsy; Senderovic*, Carmeli*; Liberman*, Koren*, Plonsky*; Research Assistants, Visitors, Postdocs, ...



Note: In many Western countries, there is a short list of popular "first names," but countless "last names." In China, it is just the reverse. The list of last names is short, and the number of first names is in the billions (from chinapage.com/biography/lastname.html).



Service Science & Engineering

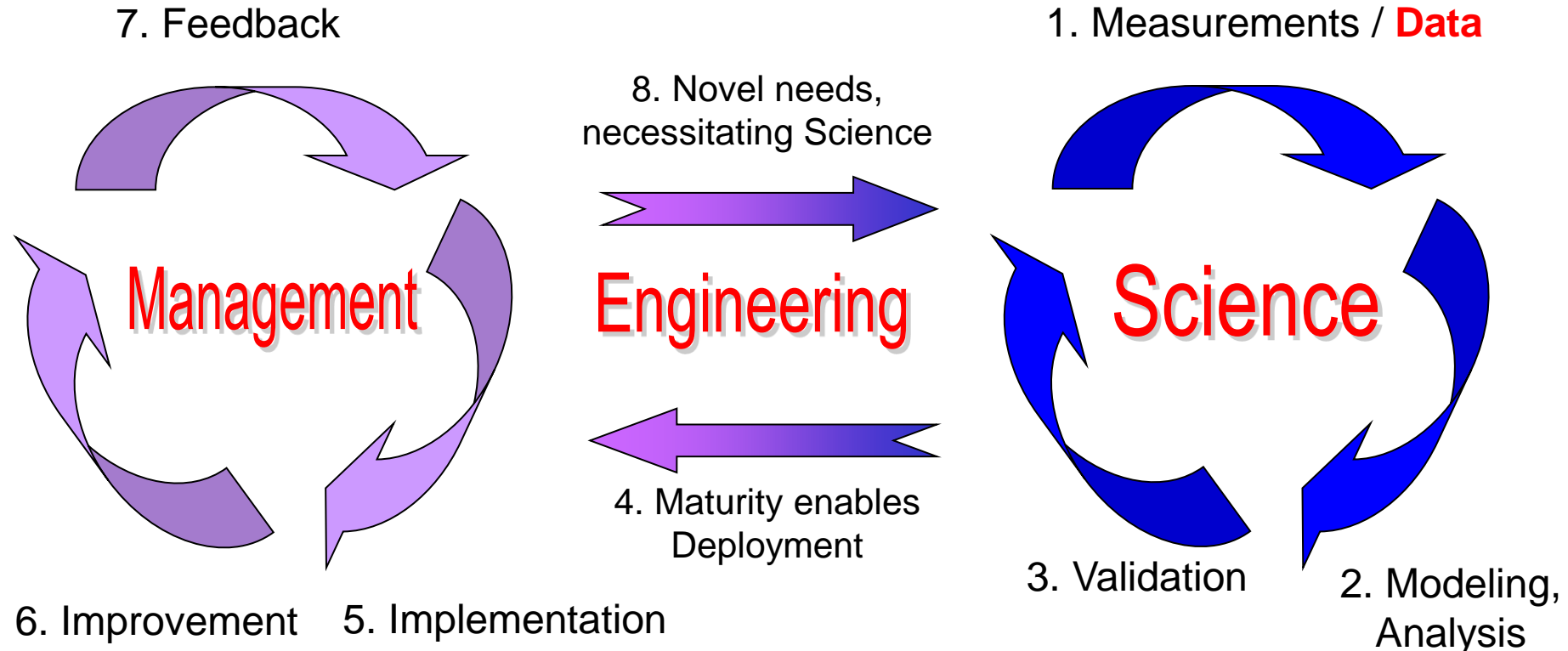
- Service constitutes 60-80% of the west economy
- Creating most new jobs
- Service processes are technology-intensive and data
- Require customization
 - For example, **Hospitals** (Emergency Room): 3M USA nurses
 - Or **Call Center** (IVR): 3M agents

Scope of the Service Industry

Guangzhou Railway Station, Southern China



Service Science, Engineering, Management



ER / ED Environment: Service Network

Acute (Internal, Trauma)



Walking



Multi-Trauma

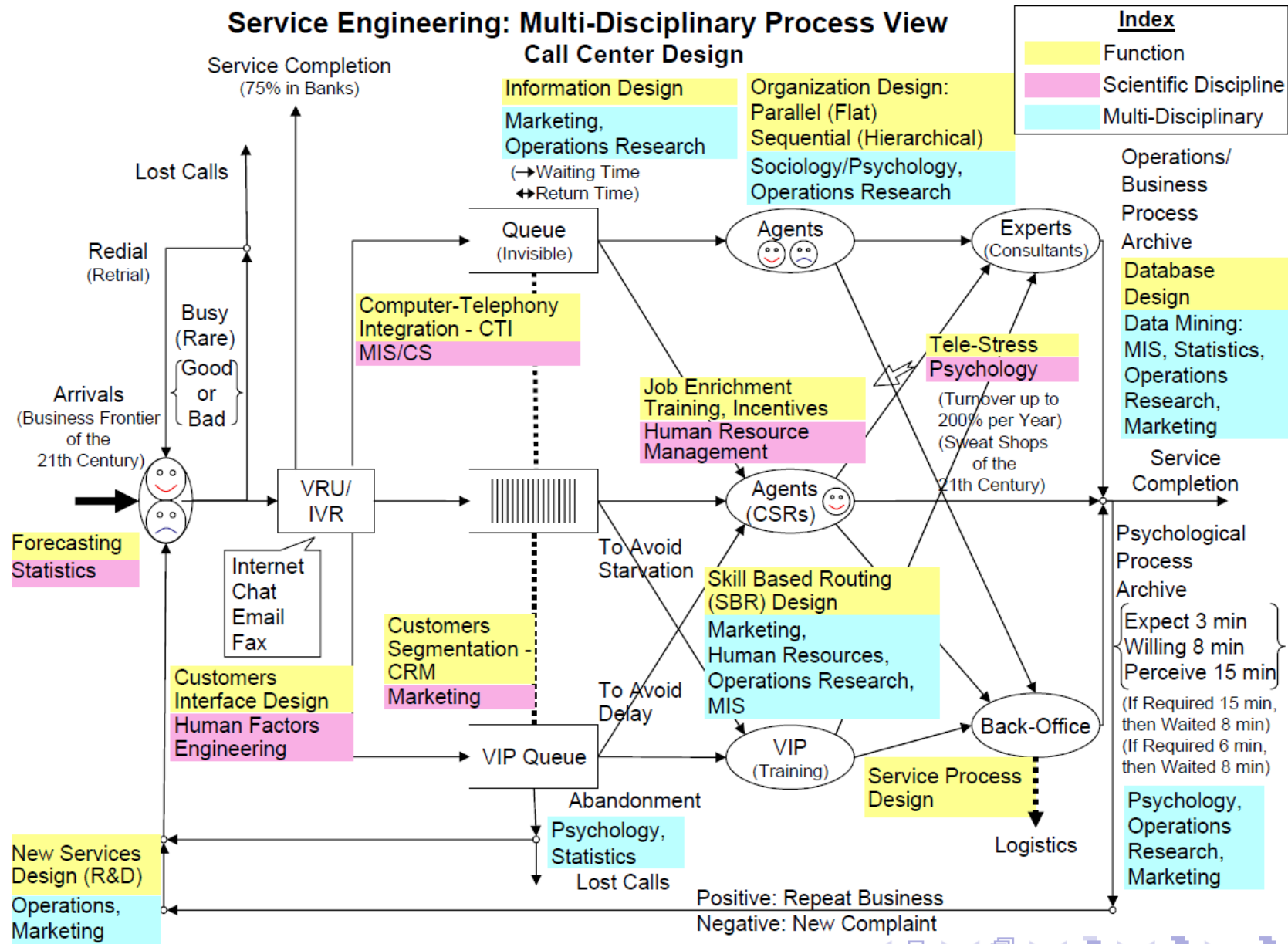


Call-Center Environment: Service Network

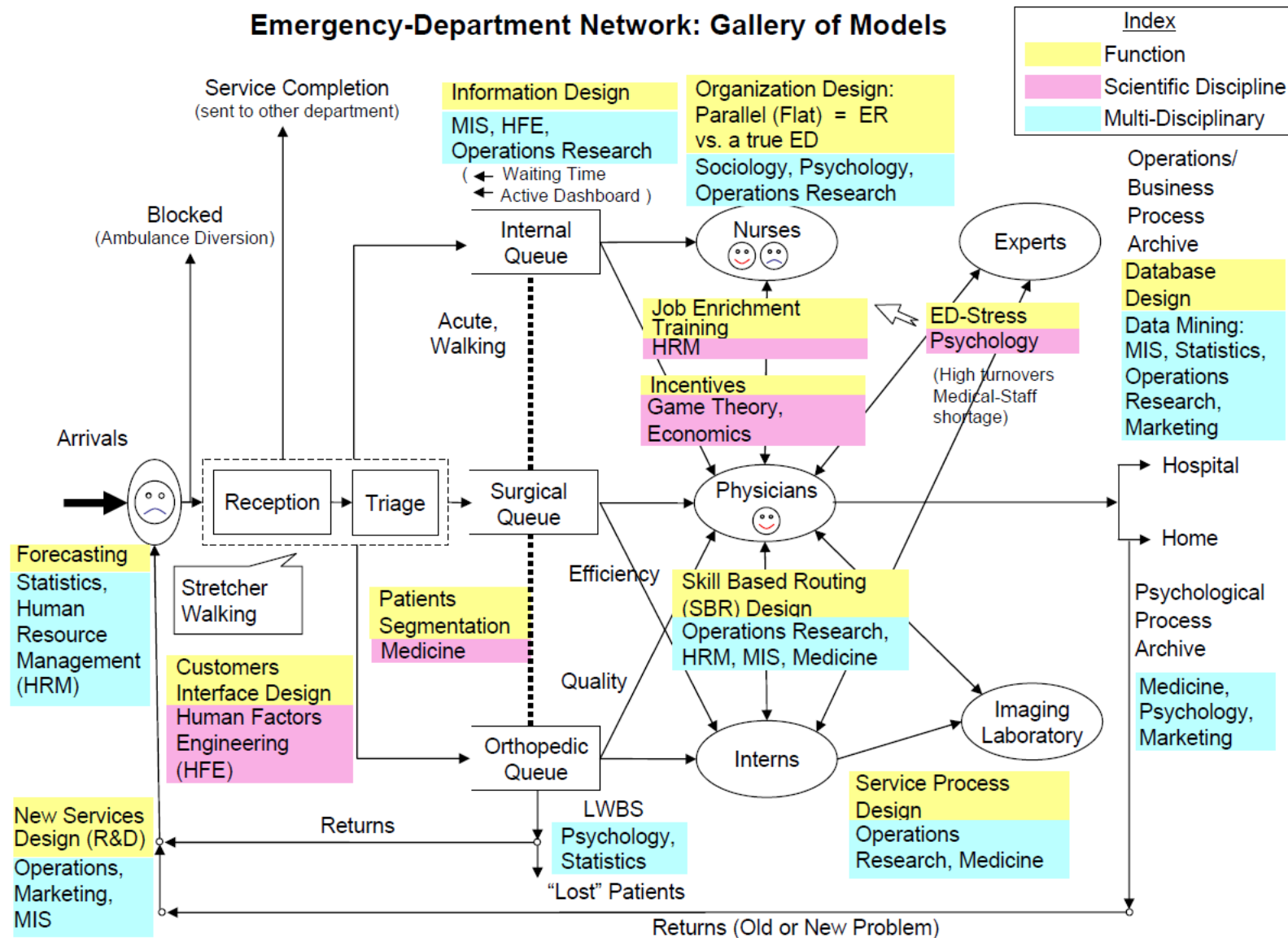
= “Fruit-flies of Hospitals”: fast, low-stake, no IRB, ...
yet highly relevant



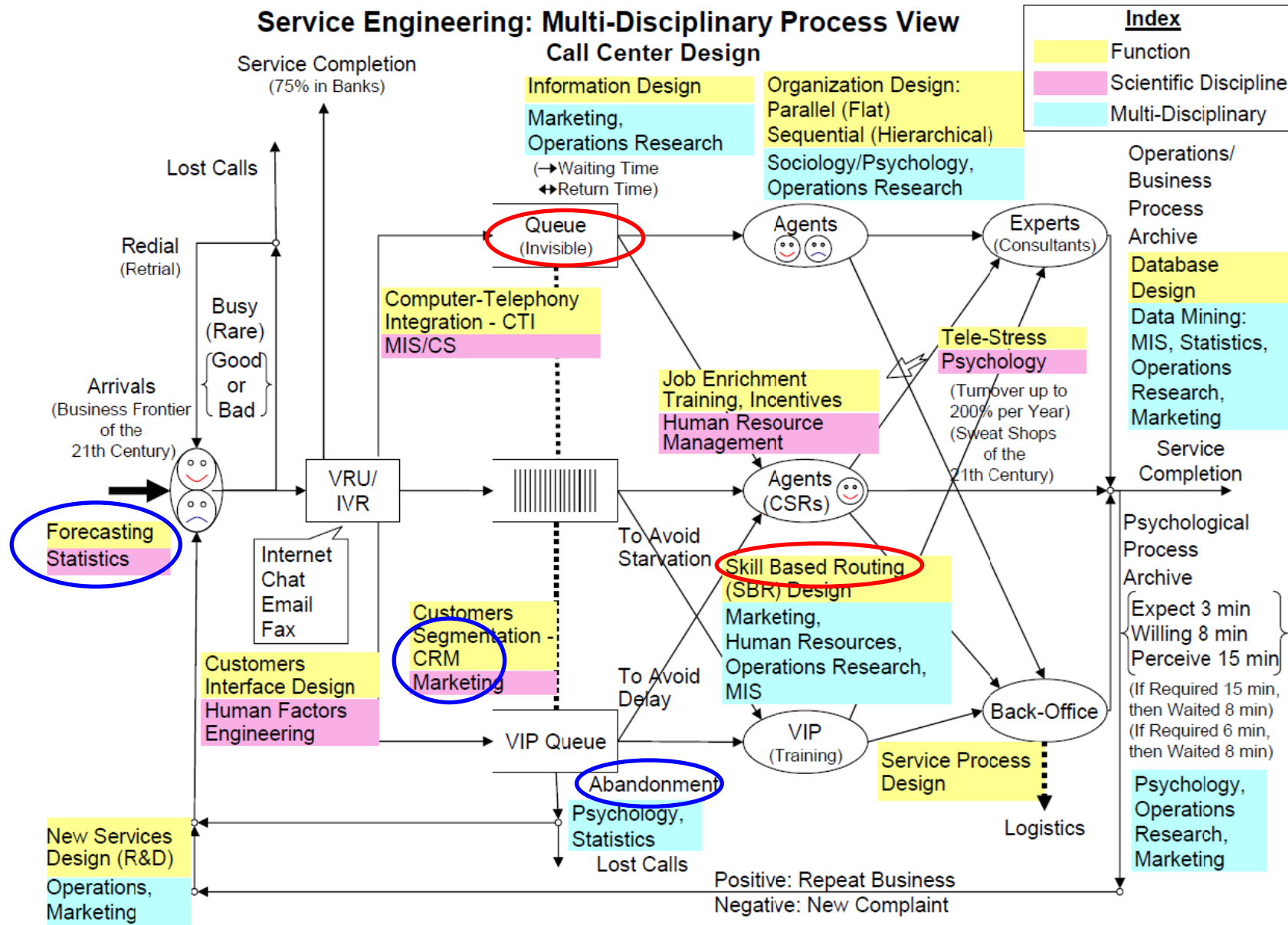
Call-Center Network: Gallery of Models



Emergency-Dept.: Multi-Disciplinary ServEng View



Call Centers = Fruit-Flies of Hospitals



Skeptic View (of Operations-Research, Service Engineering)

prompted

Call Centers = Fruit-Flies of Hospitals

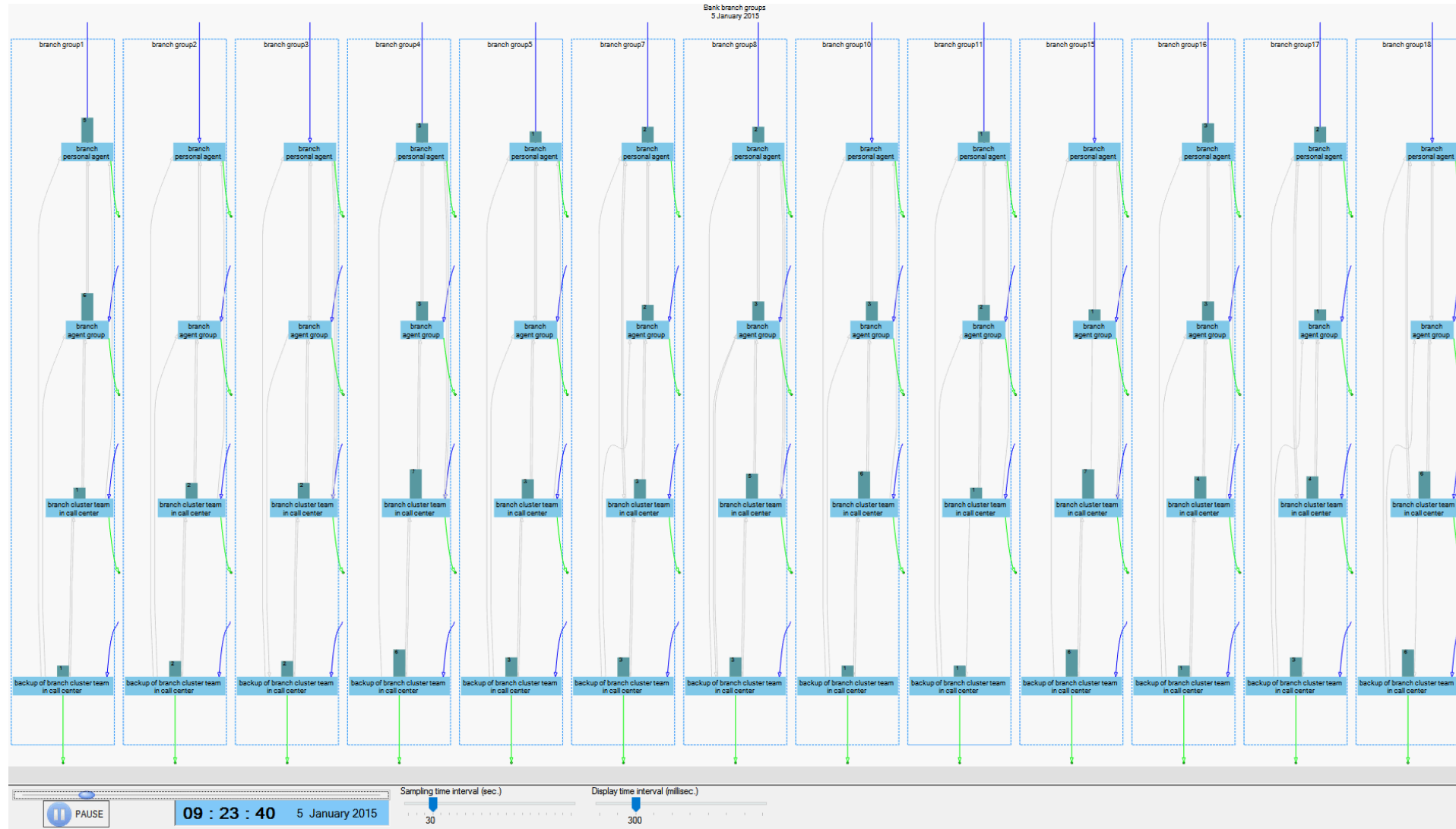
- (very) Short History of Fruit-Flies Research:

<http://www.youtube.com/watch?v=bKrpnfTISaE>

- **A** politician's view-point (also triggered the above):

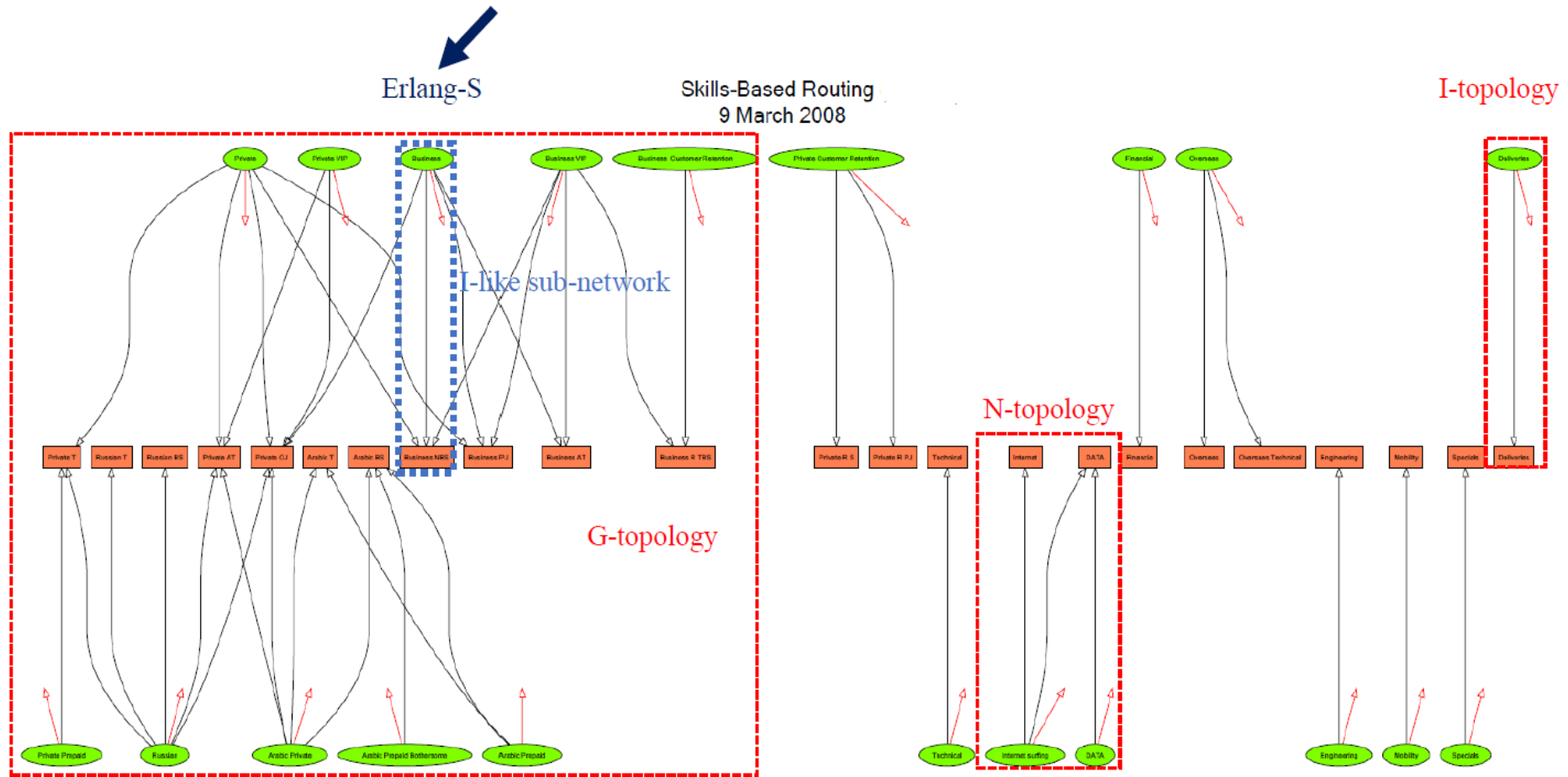
http://www.youtube.com/watch?v=xao_4Y-lOdk

Telephone Queues: 2000 Bank-Agents, in Call-Centers + Branches (ILDU Bank)



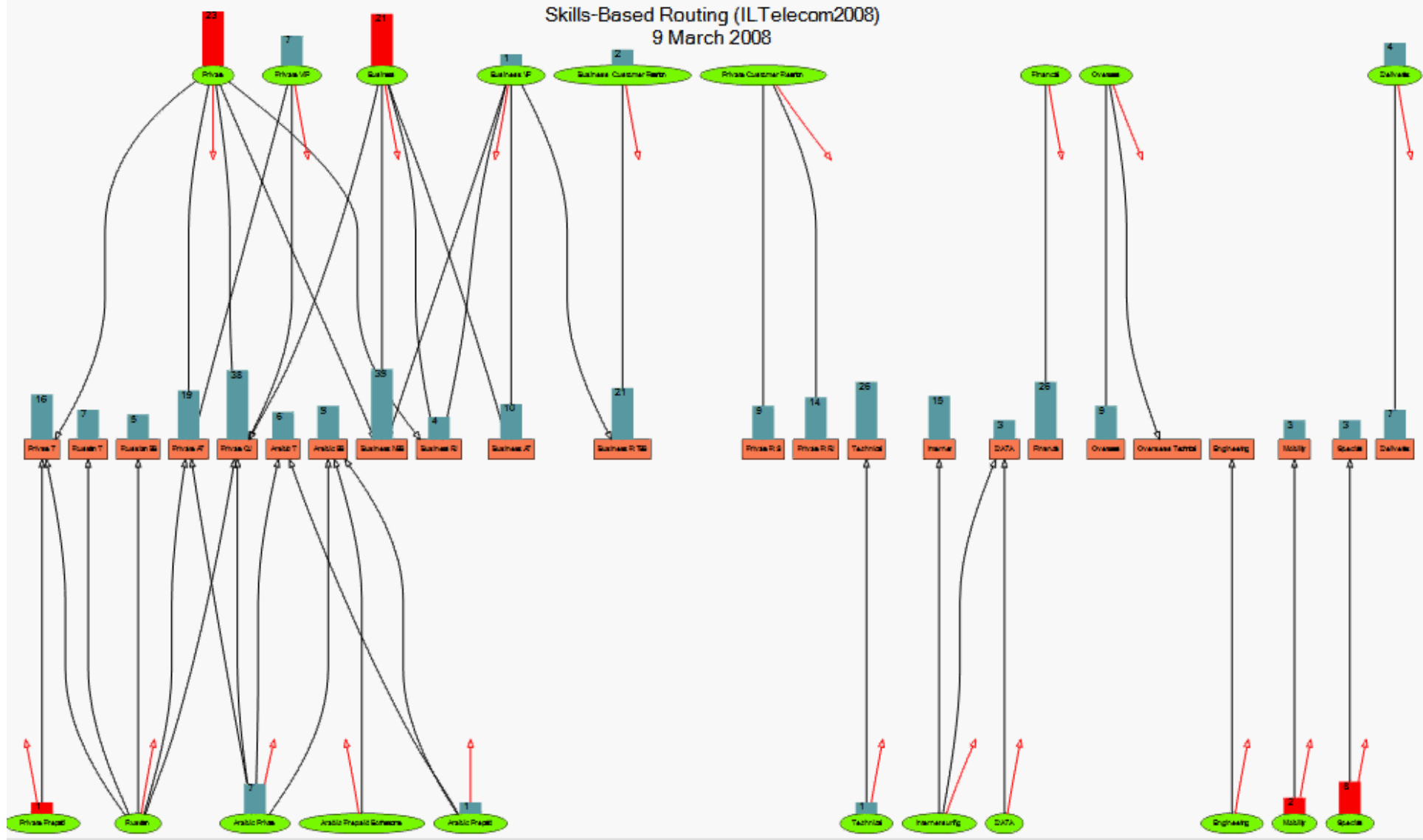
Call Center = Matching Customers & Agents (Needs & Skills)

Hospital = Patients Wards



Topology of a call center:
 Server-queues are in the **rectangles** and customer-queues are in the **ovals**

Skills-Based Routing (ILTecom2008)
9 March 2008

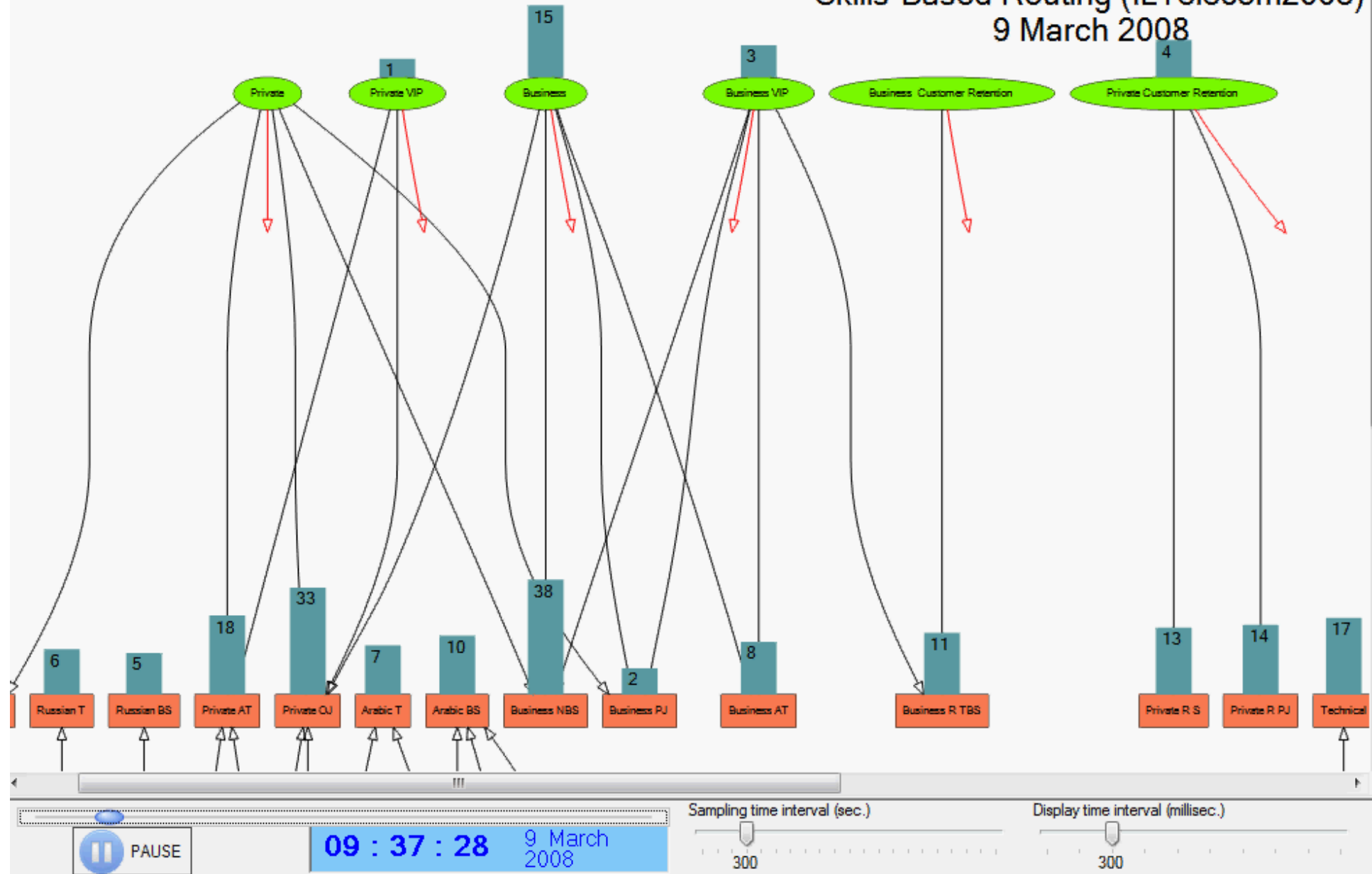


PAUSE 10 : 13 : 00 9 March 2008

Sampling time interval (sec.) 300

Display time interval (millisec.) 300

Skills-Based Routing (ILTelecom2008) 9 March 2008



Skills-Based Routing in a Call Center

- **Customer Classes**
 - **Marketing** segregates customers according to their needs and/or importance – this determines customer priorities
- **Agent Skills**
 - **Human-Resources** Management assigns agent skills according to capabilities, experience (training) – this determines agent constituencies
- **Matching Class & Skill (Demand and Supply)**
 - **Operations-Researchers** develop matching algorithms so that customers don't wait long for an agent (service-level) and agents don't wait long for a customer (efficiency)
- **Information Infrastructure (IS/CS)**
- **Data Management (Statisticians)**

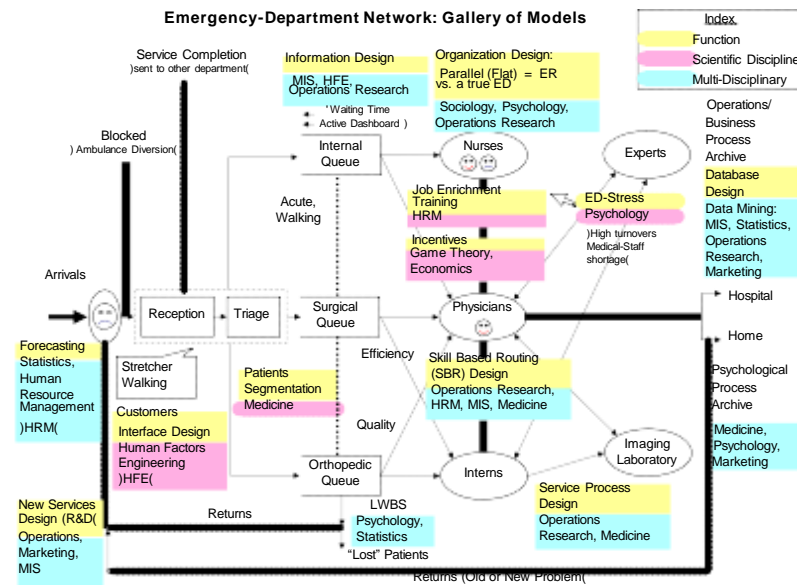
OR SBR Theoretical Support: Asymptotic Analysis in Heavy-Traffic (Stochastic Control)

Evidence-Based Routing in an ER/ED

- **Patient Priorities (Customer Classes)**
 - **Doctors** classify patients according to their urgency and/or needs – this determines customer priorities
- **Ward “Skills” (Agent Skills)**
 - **Management** assigns “skills” to wards according to clinical relevance and capabilities – this determines customer constituencies
- **Matching Class & Skill (Demand and Supply)**
 - **Operations-Researchers** develop matching algorithms so that patients don’t wait long for a ward (service-level) and wards don’t wait long for a patient (efficiency)
- **Information Infrastructure (IS/CS)**
- **Data Management (Statisticians)**

OR SBR Theoretical Support: Asymptotic Analysis in Heavy-Traffic (Stochastic Control)

Theses: Control of Patient Flow (Hospital Network)



- Environment-Dependent **ED Flow Design**, w/ **Marmor, Golany, Israelit**
- **Fair ED-to-IW Routing** (Patients vs. Staff), w/ **Momcilovic, Tseytlin**
- **Staffing Time-Varying Q's with Re-Entrant Customers**, w/ **Yom-Tov**
- **Queueing-Science/Congestion-Laws**, w/ **Armony, Marmor, Tseytlin, Yom-Tov; Israelit**
- **Blocking (ED to IW, IW to Geriatric-Institutions)**, w/ **Zychlinski, Cohen, Momcilovic**
- **Triage (Deadlines) vs. In-Process (Queueing)**, w/ **Huang, Carmeli**

On Asymptotic Research of Queueing Systems

Queueing asymptotics has grown to become a central research theme in Operations Research and Applied Probability, beyond just queueing theory. Its claim to fame has been the deep insights that it provides into the dynamics of Queueing Networks (**QNets**), and rightly so:

- ▶ Kingman's invariance principle in conventional heavy-traffic
- ▶ Whitt's sample-path (functional) framework
- ▶ Reiman's network analysis via oblique reflection
- ▶ Bramson-Williams' framework for state-space collapse
- ▶ Laws' resource pooling
- ▶ Harrison's paradigm for asymptotic control (Wein; van Mieghem's $Gc\mu$)
- ▶ Dai's fluid-based stability
- ▶ Halfin-Whitt's (QED regime) ($\sqrt{\cdot}$ -staffing for many-server queues)
- ▶ $P = NP$: Atar's equivalence of Preemptive and Non-Preemptive SBR; Stolyar, Gurvich
- ▶ Massey-Whitt's research of time-varying queues

Applying Queueing Asymptotics

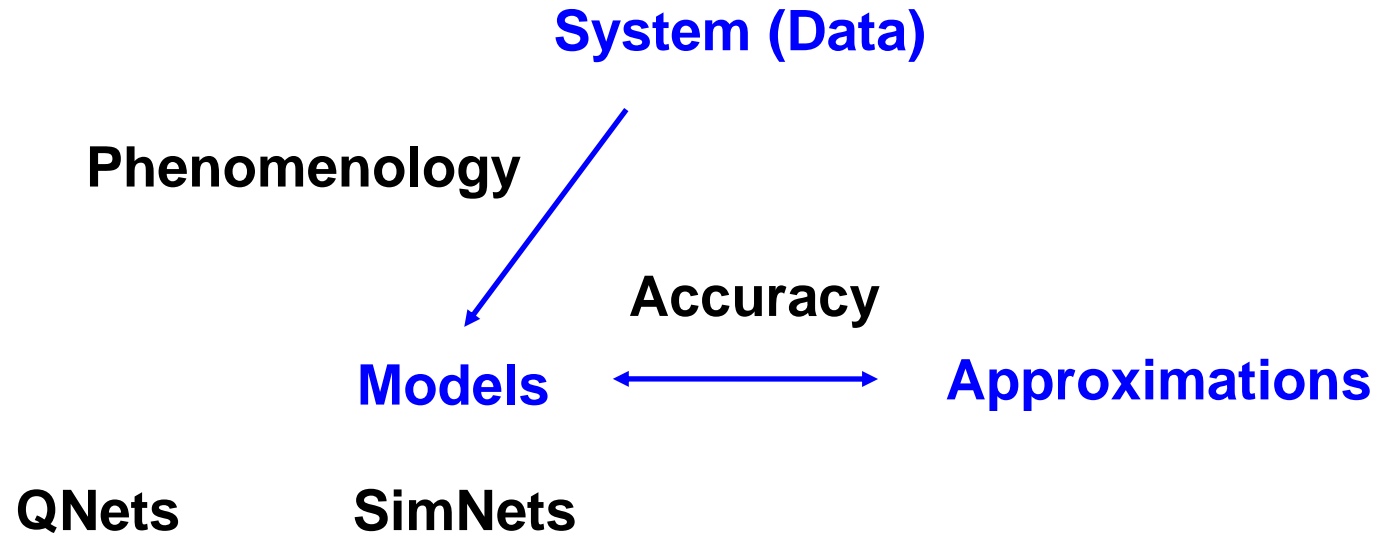
- Has asymptotic research helped one approximate or simulate a service system more efficiently, estimate its parameters more accurately, teach it to our students more effectively, perhaps even manage the system better?
- I am of the opinion that the answer to such questions **could and should have been “yes”** more often than it has been.
- How to change this?
My approach has been to **marry theory with data (“theompirical” research)**, supported by (what only in recent years I came to realize is) **Process Mining (= of Stochastic Networks**: their building-blocks, structure, protocols; flows and laws).

Why be optimistic? Hopefully this course!

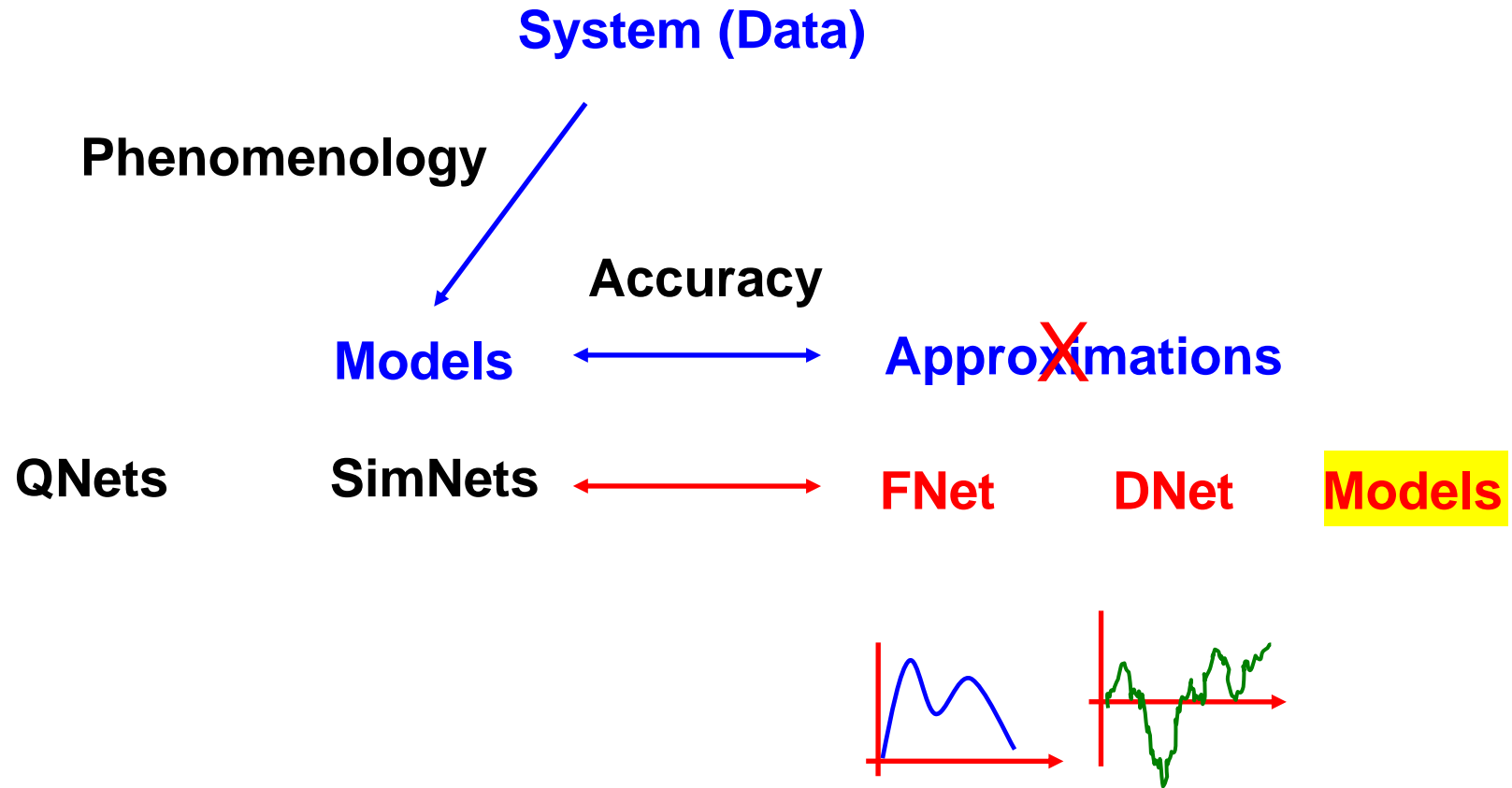
Accuracy vs. Value

Applies to any approximation scheme that approximates theoretical models by other theoretical models.

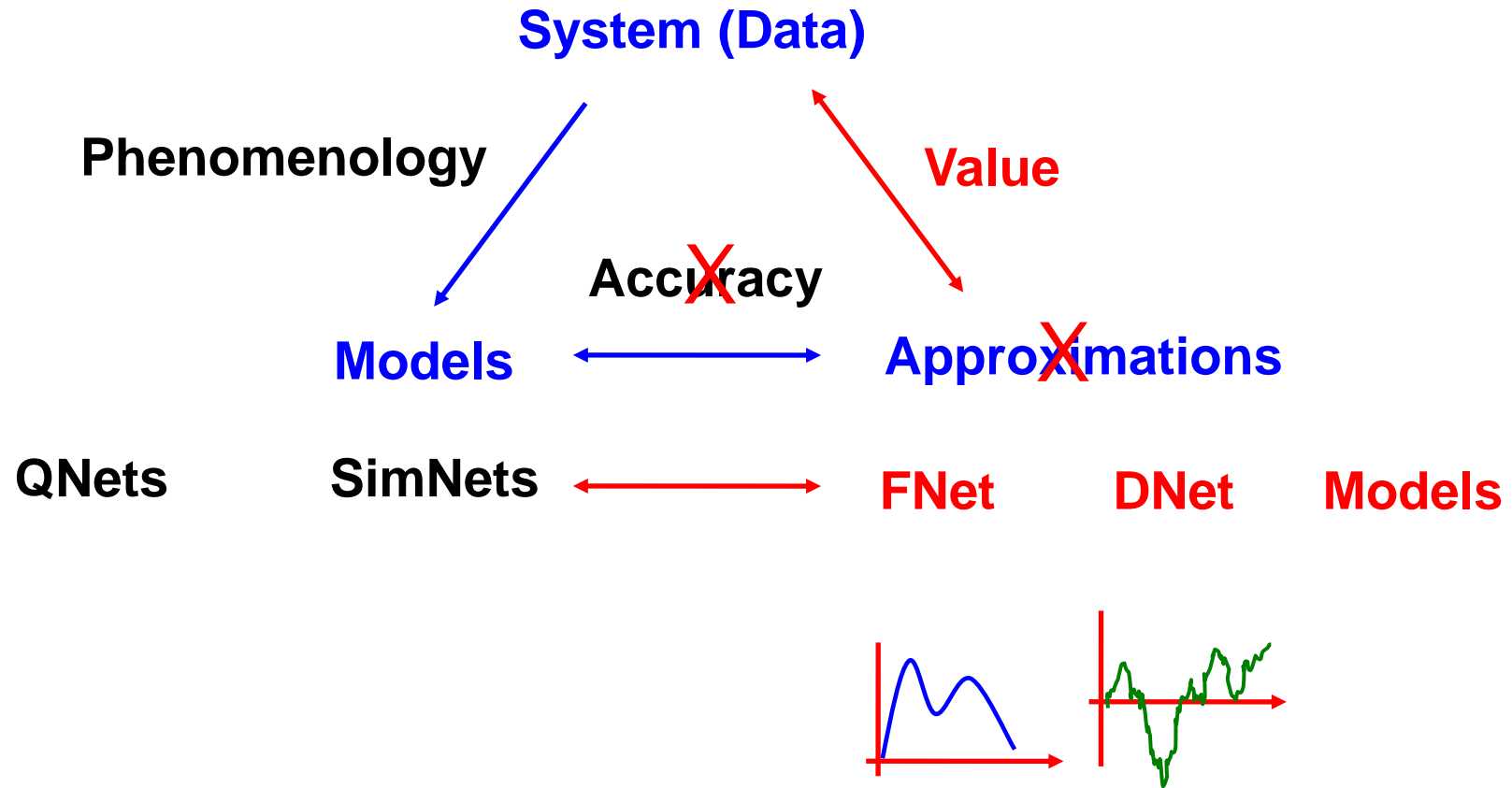
Prevalent (Asymptotic) Approximations



Data-Based ~~Prevalent~~ Asymptotic ~~Approximations~~ Models



Data-Based ~~Prevalent~~ Asymptotic ~~Approximations~~ Models



Lecture 2: Start Here

Technion SEELab

SEE = Service Enterprise Engineering

Home for my “Theompirical” Research



Since 2007:

Data for Research & Teaching

\$1M seed: **Hal & Inge Marcus**

3 Researches (professionals)

- **Students, PostDocs**
- **Academic Visitors**
- **Mirror Servers**

Technion SEELab

SEE = Service Enterprise Engineering

(There is a “SEE Lab” in Bocconi: SEE = Space Economy Evolution, Born 4/6/18)



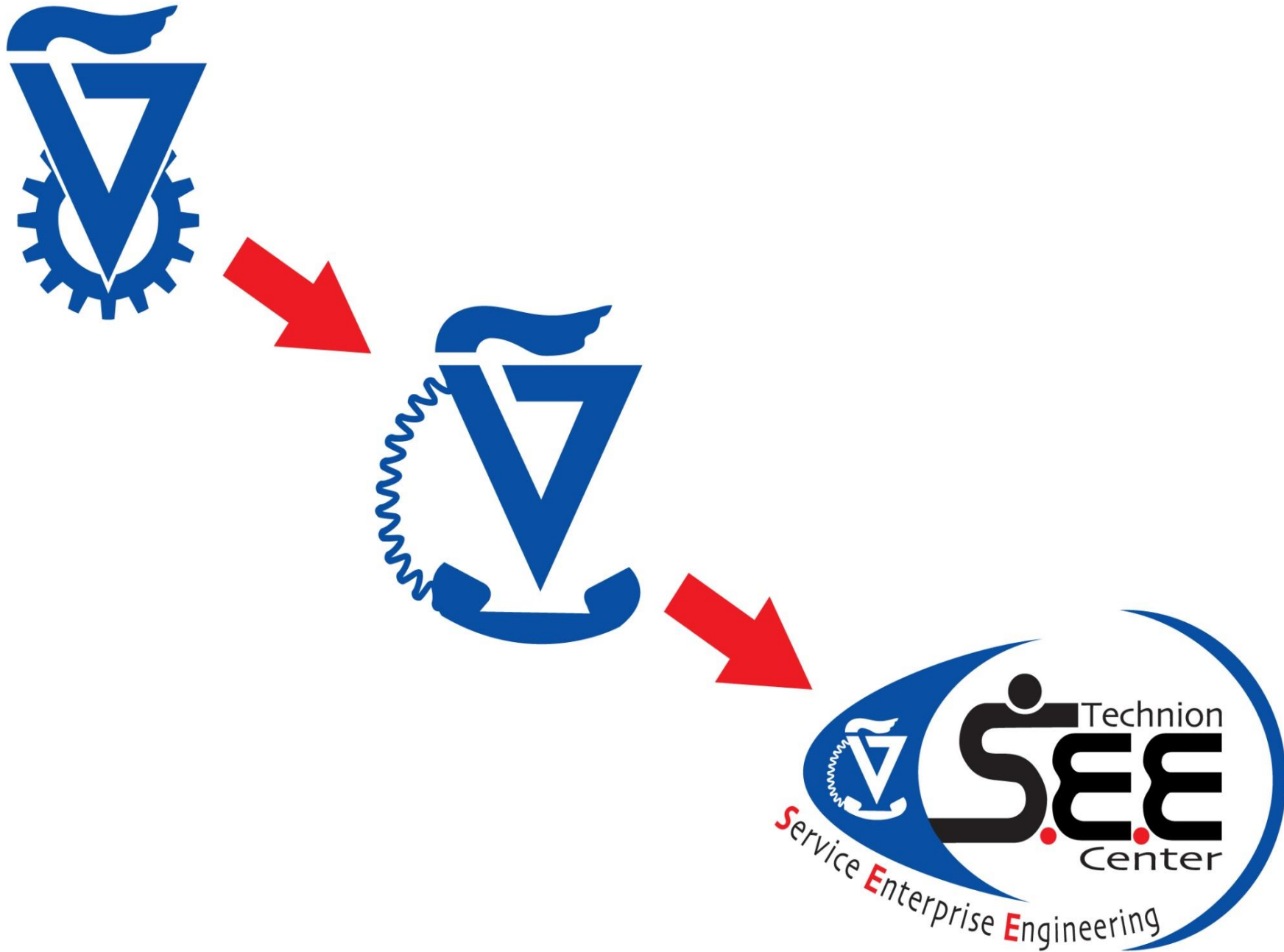
Since 2007:

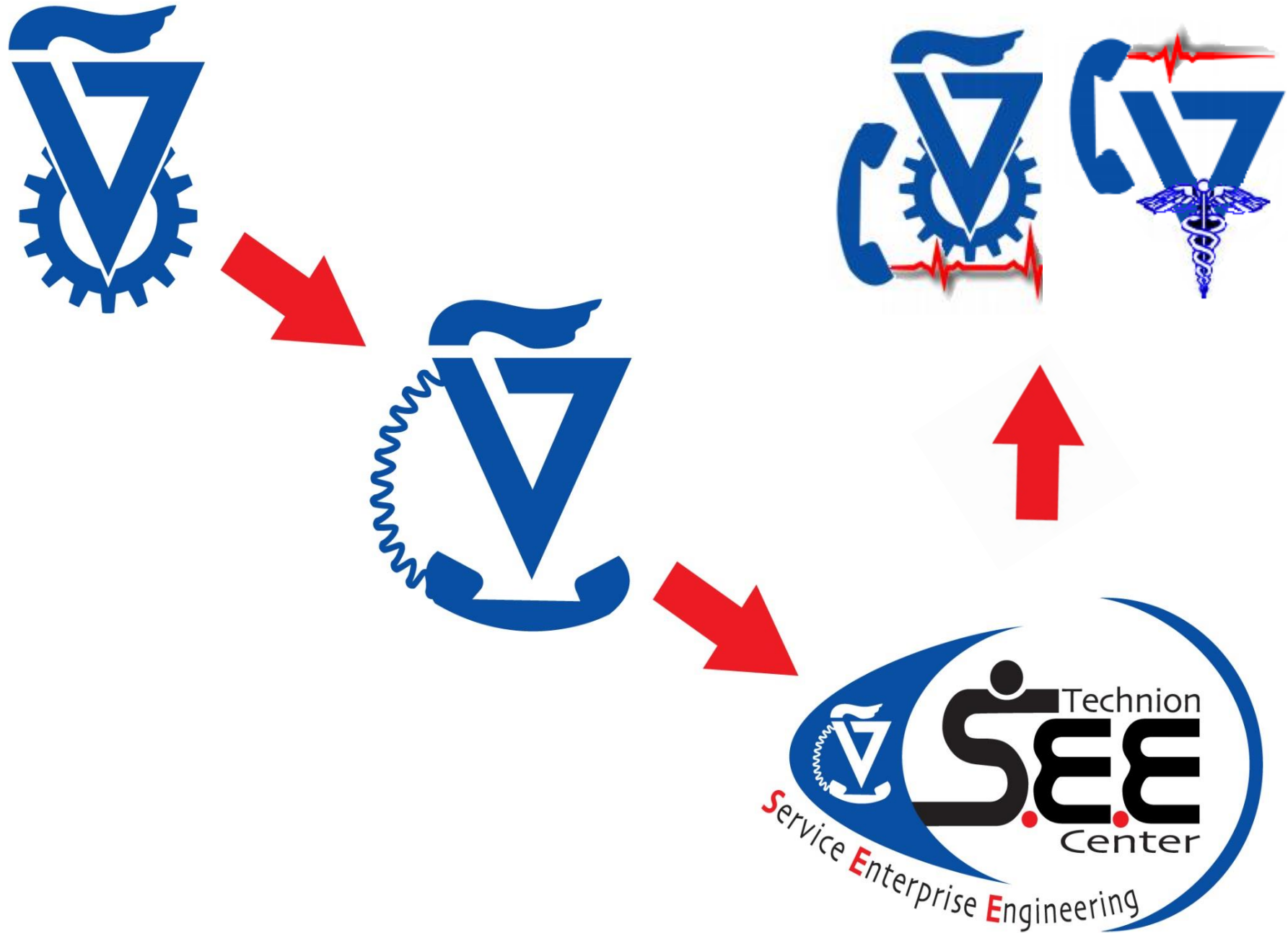
Data for Research & Teaching

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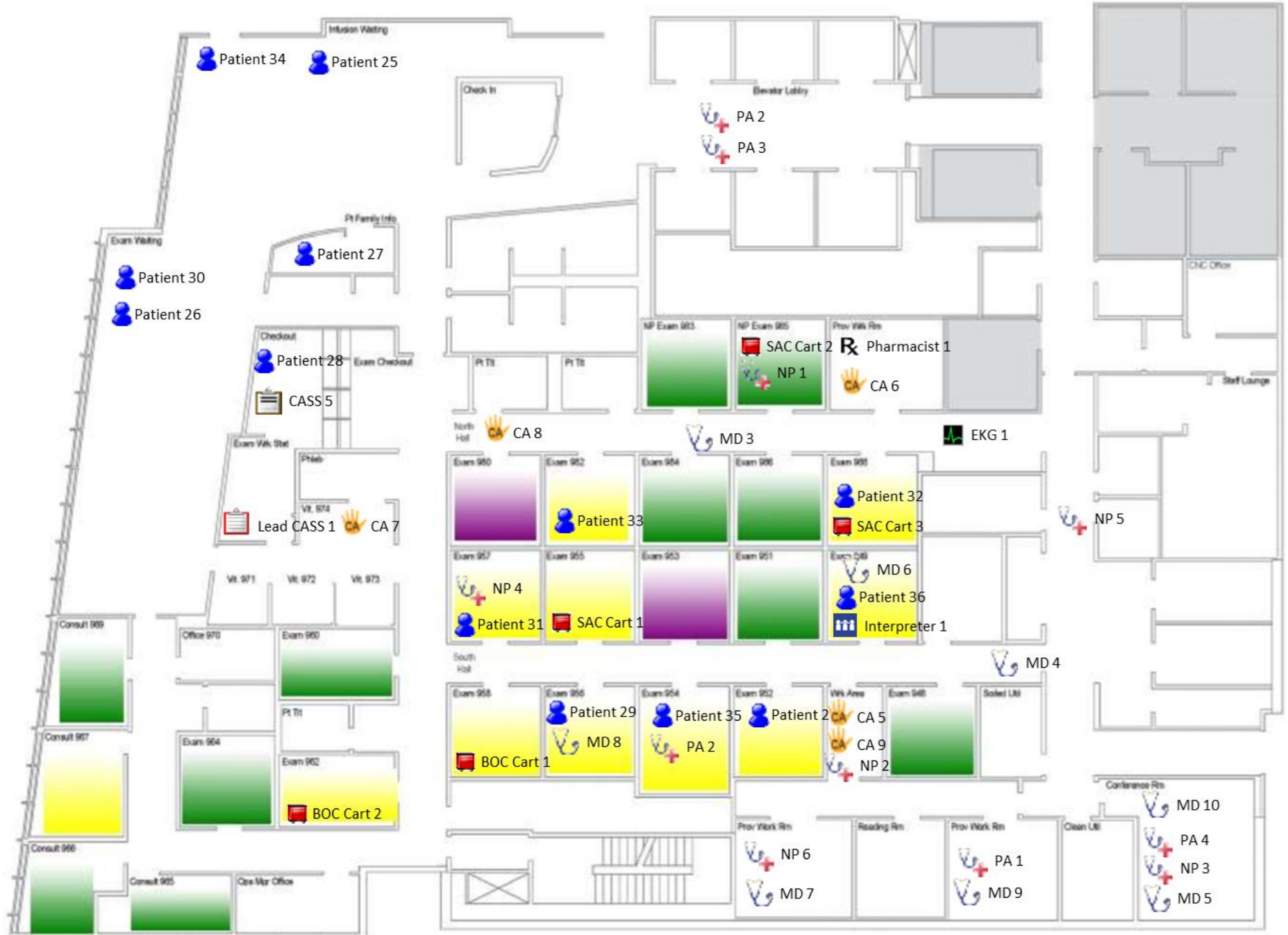


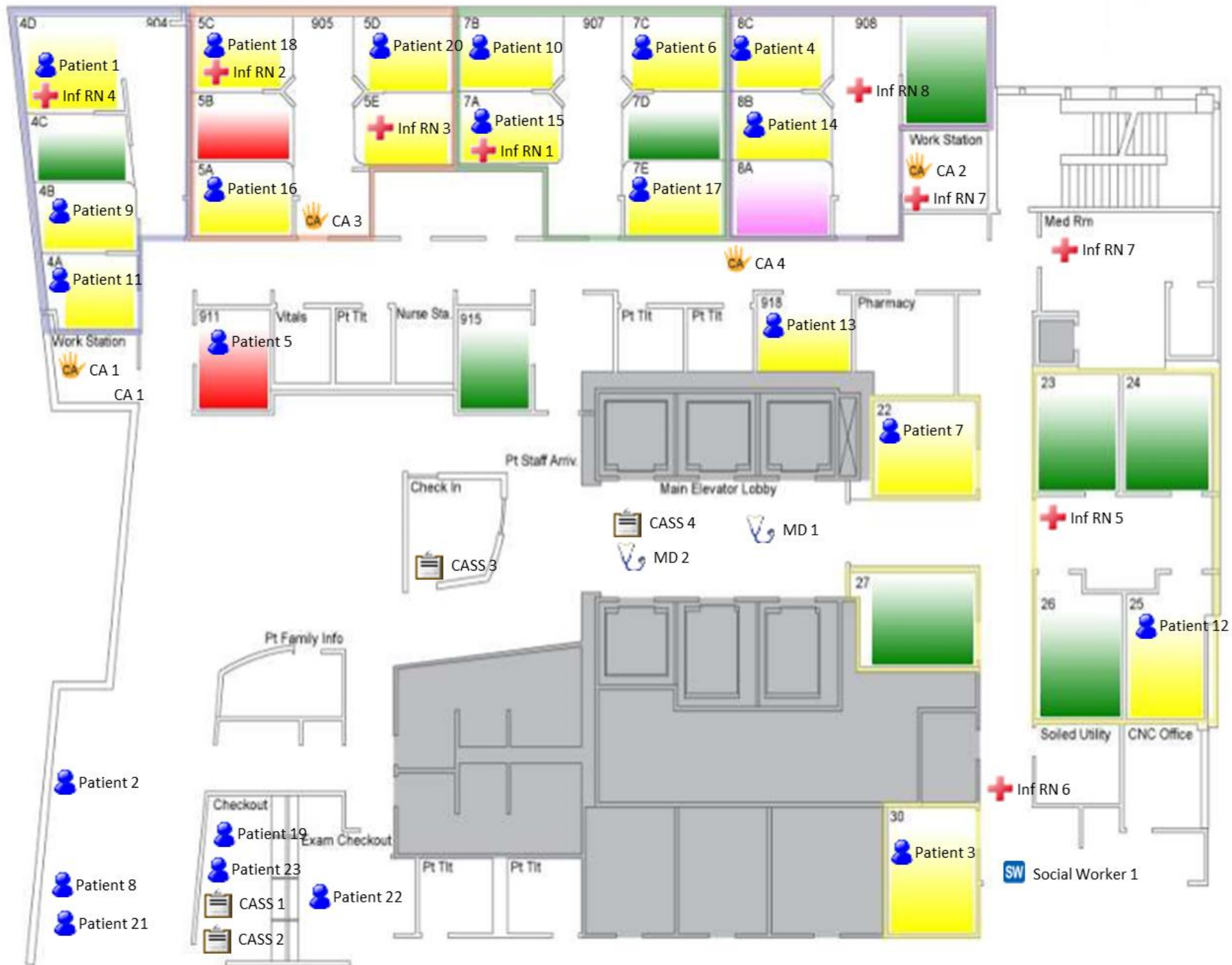


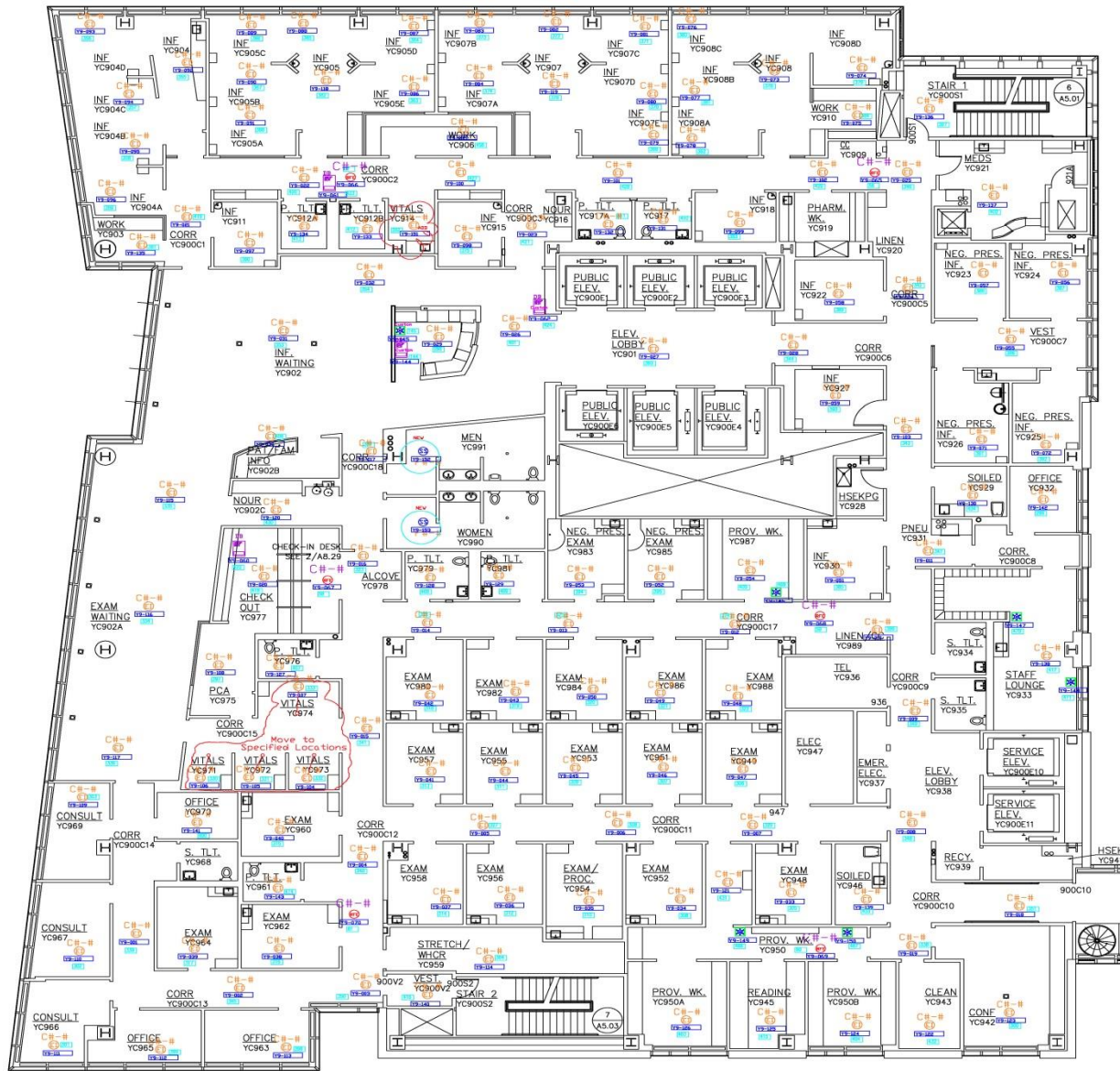


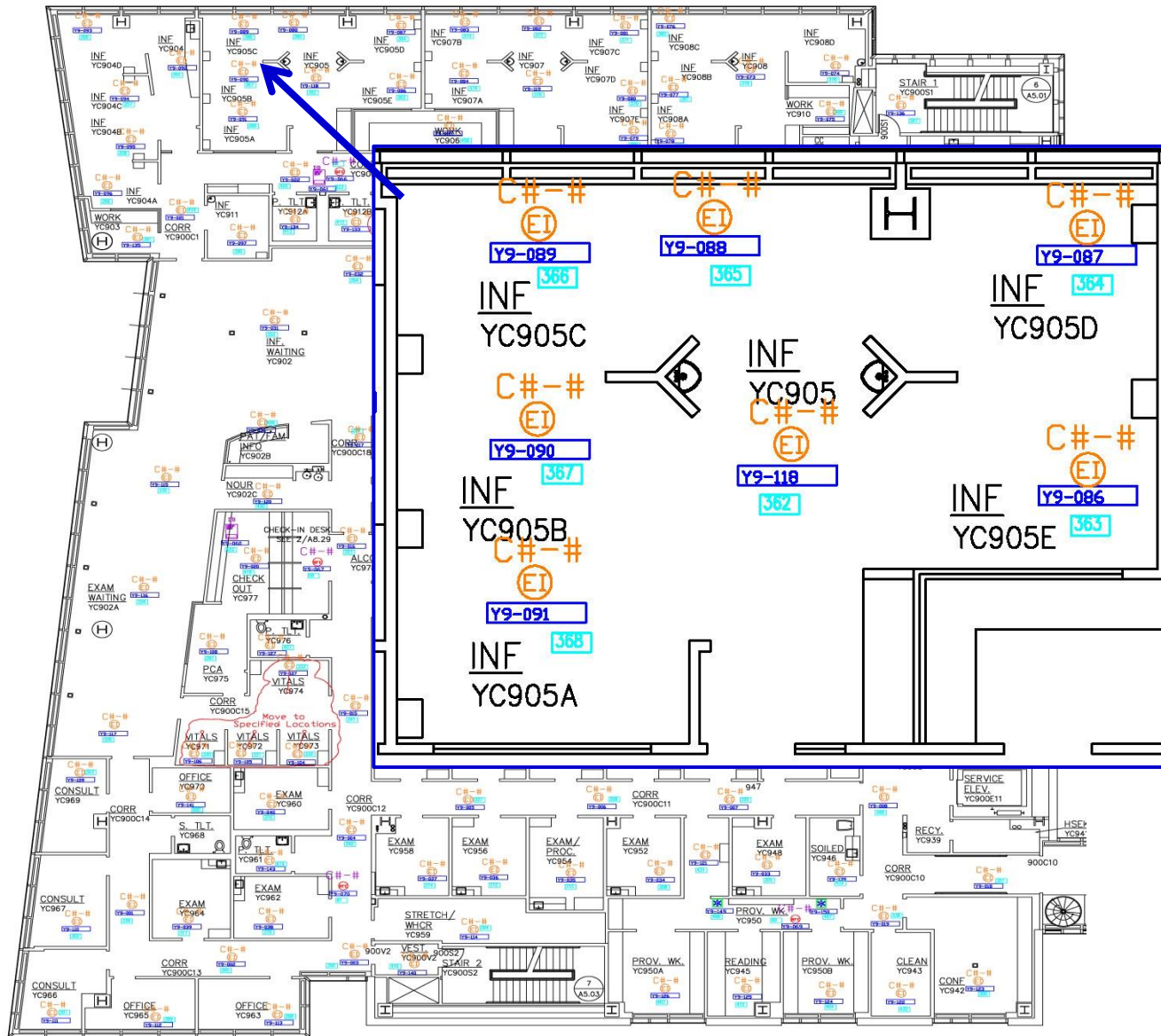
Closing the Data-Gap: from Call-Centers to Hospitals, now Banks

- **Large call center:**
 - 1000s of agents
 - Hundreds of thousands of calls per day
 - Data: operational, psychological, financial – **automatic** collection
- **Large hospital:**
 - 1000+ Beds
 - 1000s of patients & nurses, hundreds of doctors
 - Data: operational, clinical, financial – mostly **inaccessible (to academia)**
- **Large Bank:** “Enjoys” characteristics of both of the above







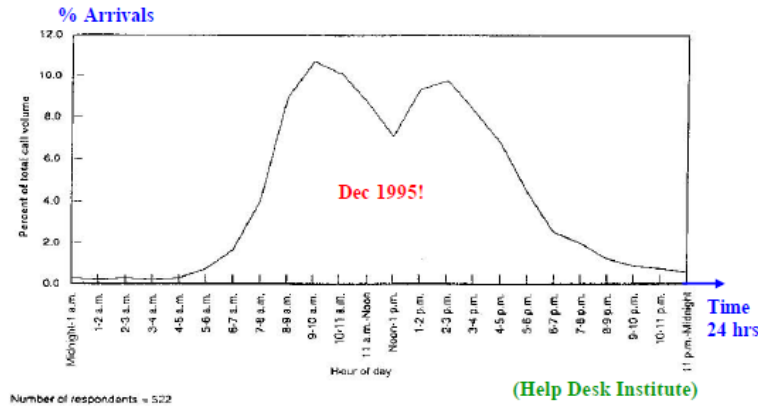


Mining Model-Primitives / Building-Blocks

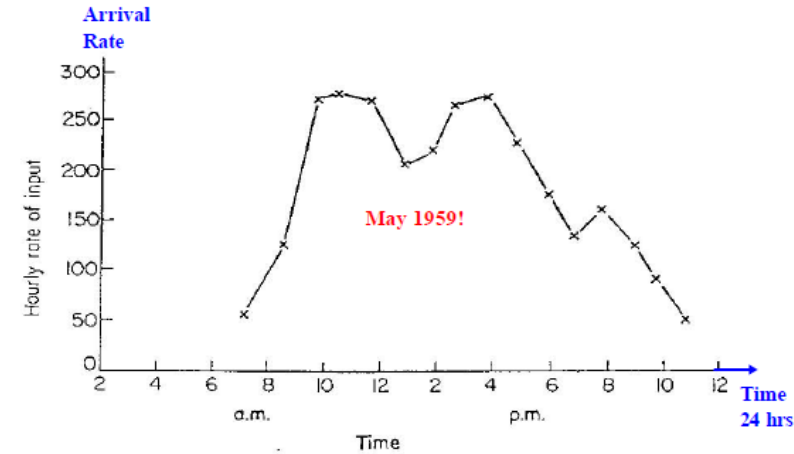
Arrivals to Service

Arrival-Rates to Three Call Centers

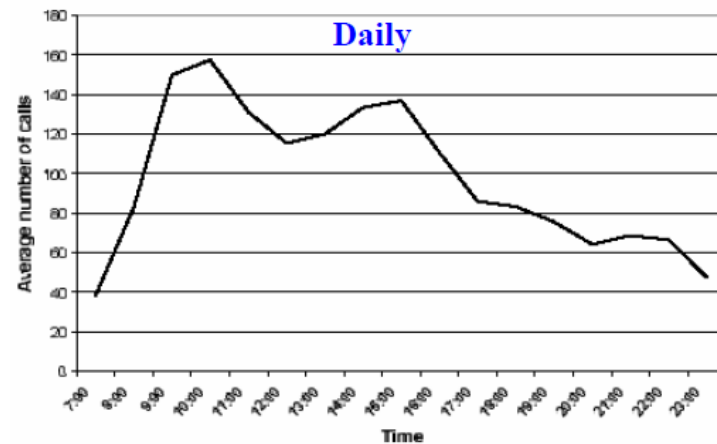
Dec. 1995 (U.S. 700 Helpdesks)



May 1959 (England)



November 1999 (Israel)

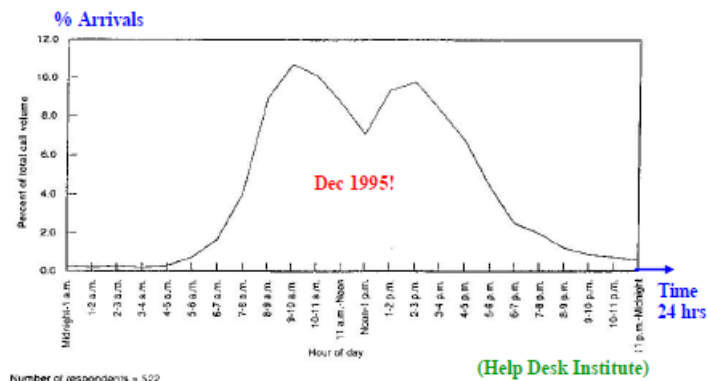


Random Arrivals "must be"
(Axiomatically)
Time-Inhomogeneous Poisson

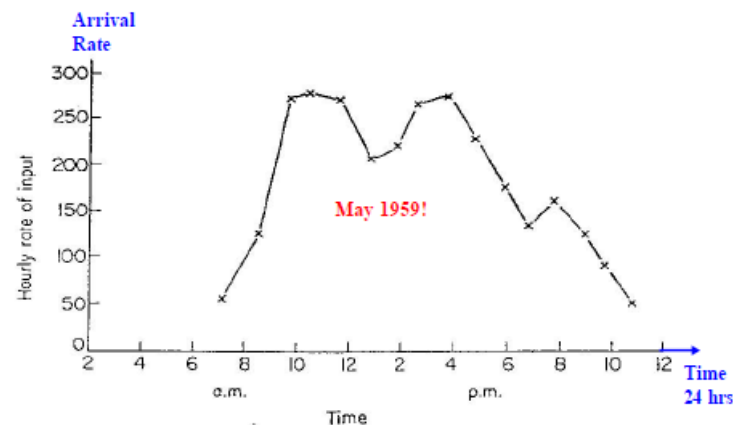
Primitives: Arrival (Rates) to Service

Why 2 Daily Peaks?

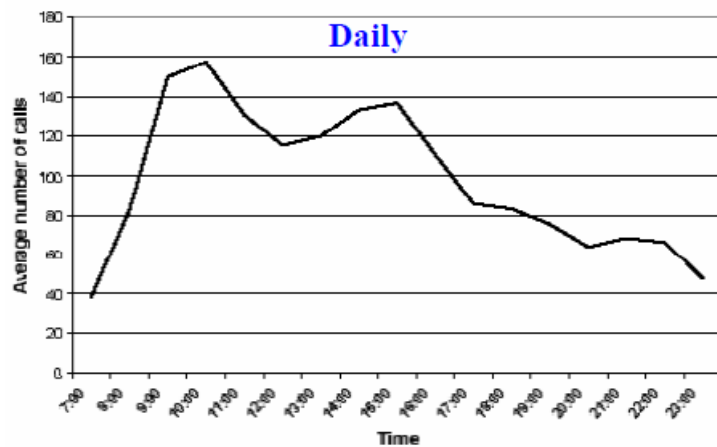
CC: Dec. 1995, (USA, 700 Helpdesks)



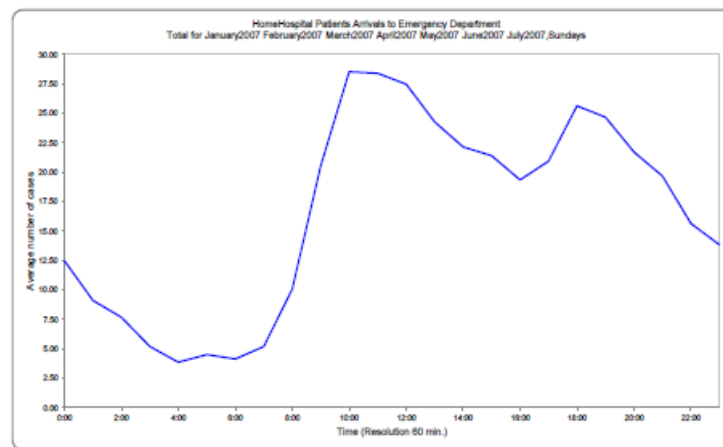
CC: May 1959 (England)



CC: Nov. 1999 (Israel)

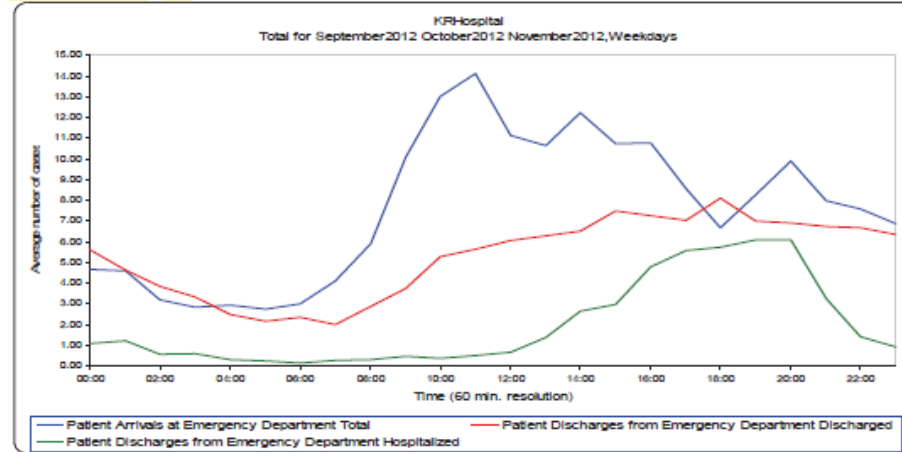


ED: Jan.–July 2007 (Israel)

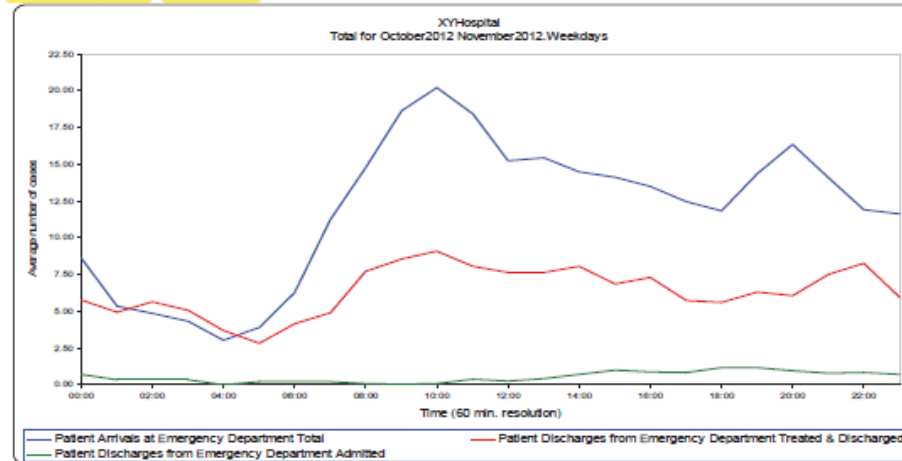


Arrival (Discharge) Rates in Korea and Singapore

KRHospital, all ED patients



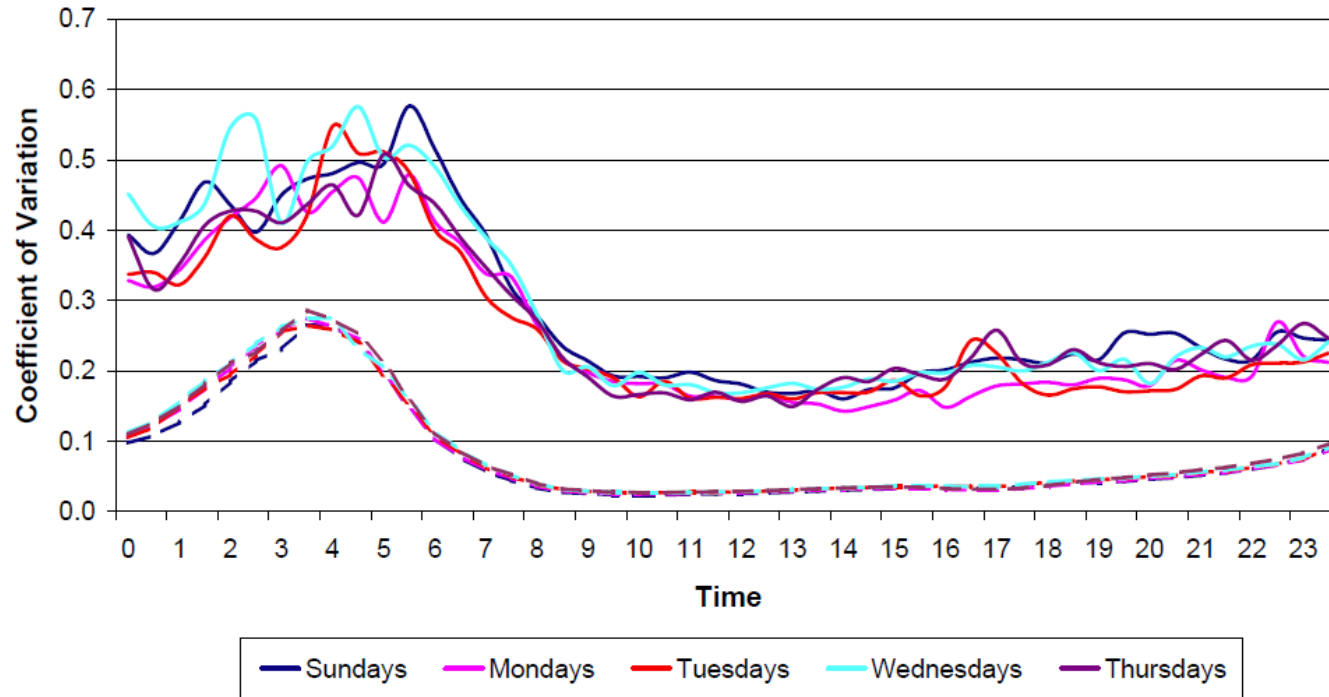
XYHospital, walking patients



Arrivals to Service: only Poisson-Relatives

Arrival-Counts: Coefficient-of-Variation (CV), per 30 min.

Israeli-Bank Call-Center, 263 regular days (4/2007 - 3/2008)

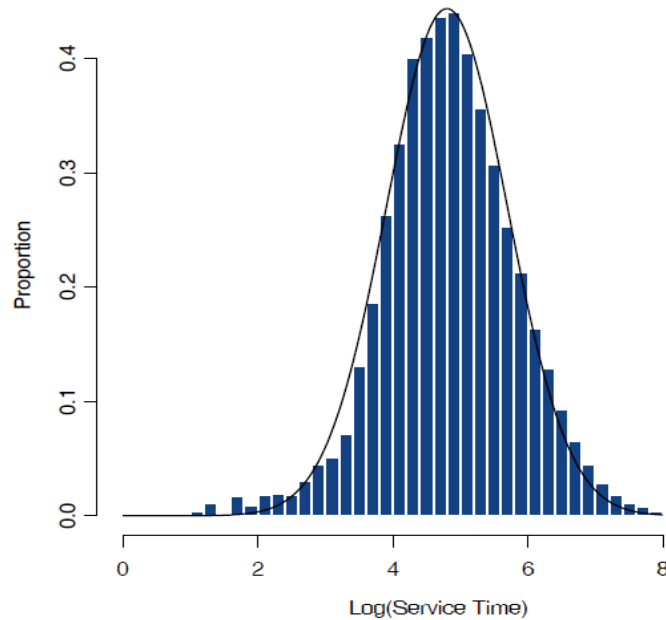


- ▶ **Poisson CV** (Dashed Line) = $1/\sqrt{\text{mean arrival-rate}}$
- ▶ Poisson CV's \ll **Sampled CV's** (Solid) \Rightarrow **Over-Dispersion**

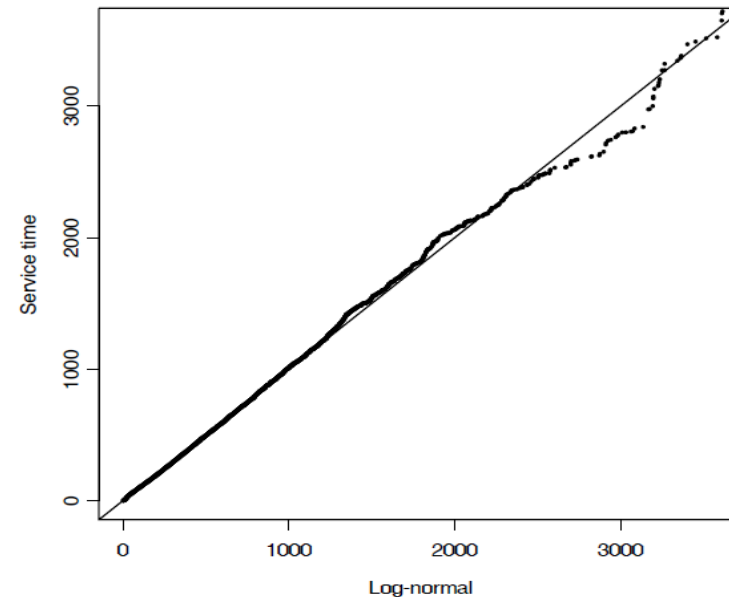
Building-Blocks: Service-Durations e.g. Phone-Calls often LonNormal

Israeli Call Center, Nov–Dec, 1999

Log(Service Times)



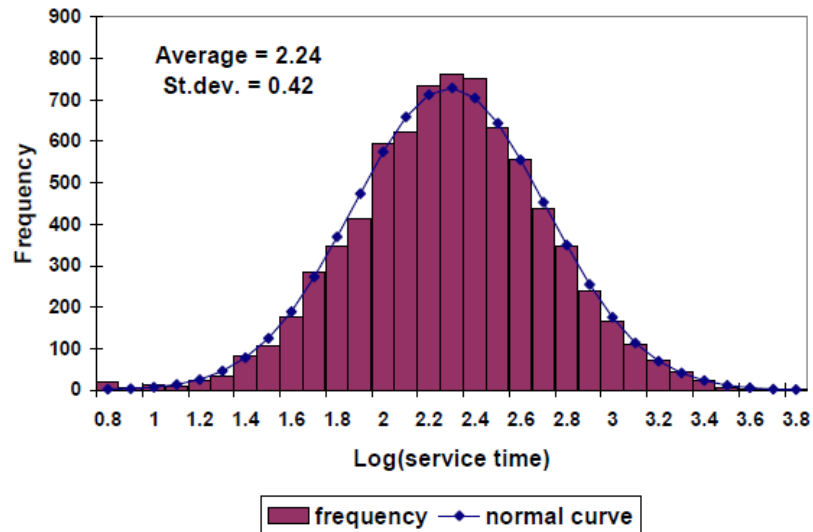
LogNormal QQPlot



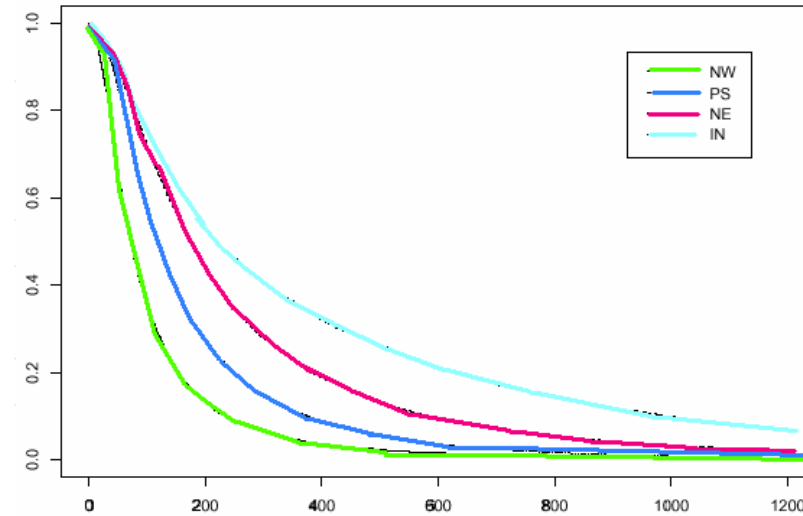
- ▶ **Practically Important:** (mean, std)(log) characterization
- ▶ **Theoretically Intriguing:** Why LogNormal ? Naturally multiplicative but, in fact, also **Infinitely-Divisible** (Generalized Gamma-Convolutions)

Service Durations: LogNormal Prevalent

Israeli Bank Log-Histogram



Service-Classes Survival-Functions



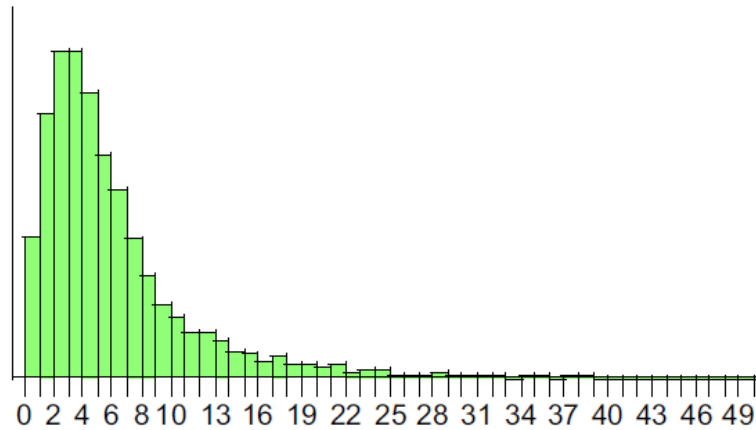
- **New Customers:** 2 min (NW);
- **Regulars:** 3 min (PS);

- **Stock:** 4.5 min (NE);
- **Tech-Support:** 6.5 min (IN).

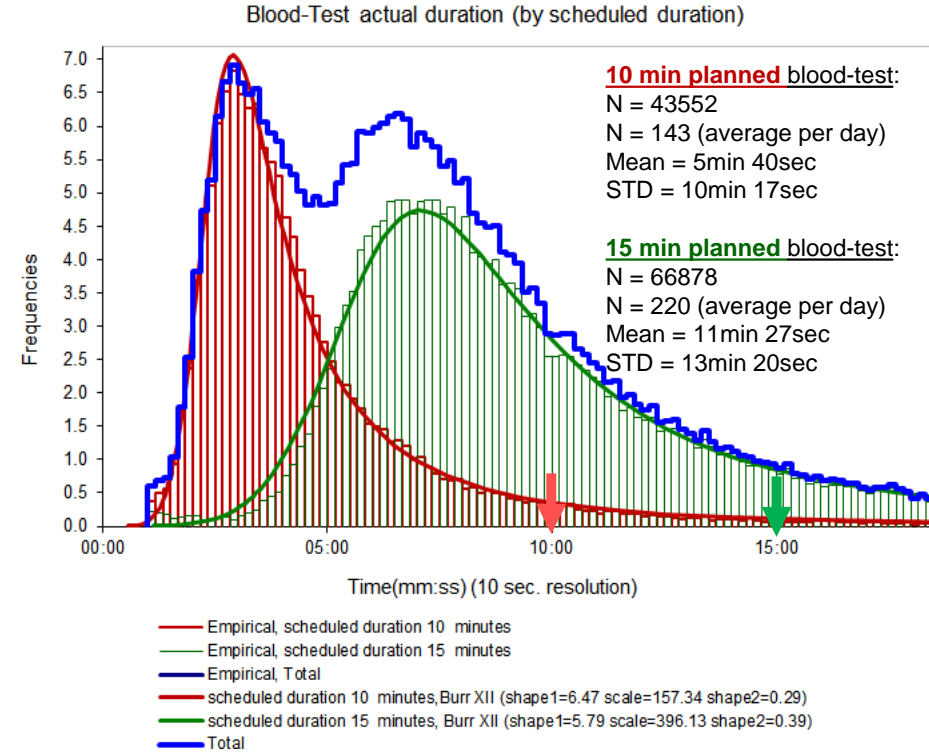
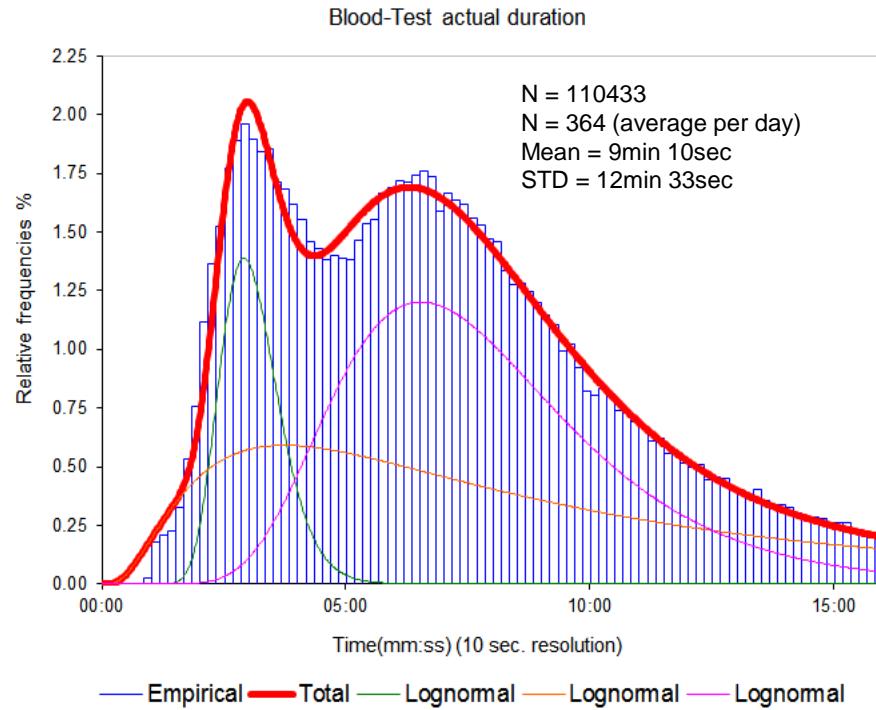
► Service Durations are **LogNormal (LN)** and **Heterogeneous**

Building-Blocks: Length-of-Stay in a Hospital Ward

Israeli Hospital, in Days: LN

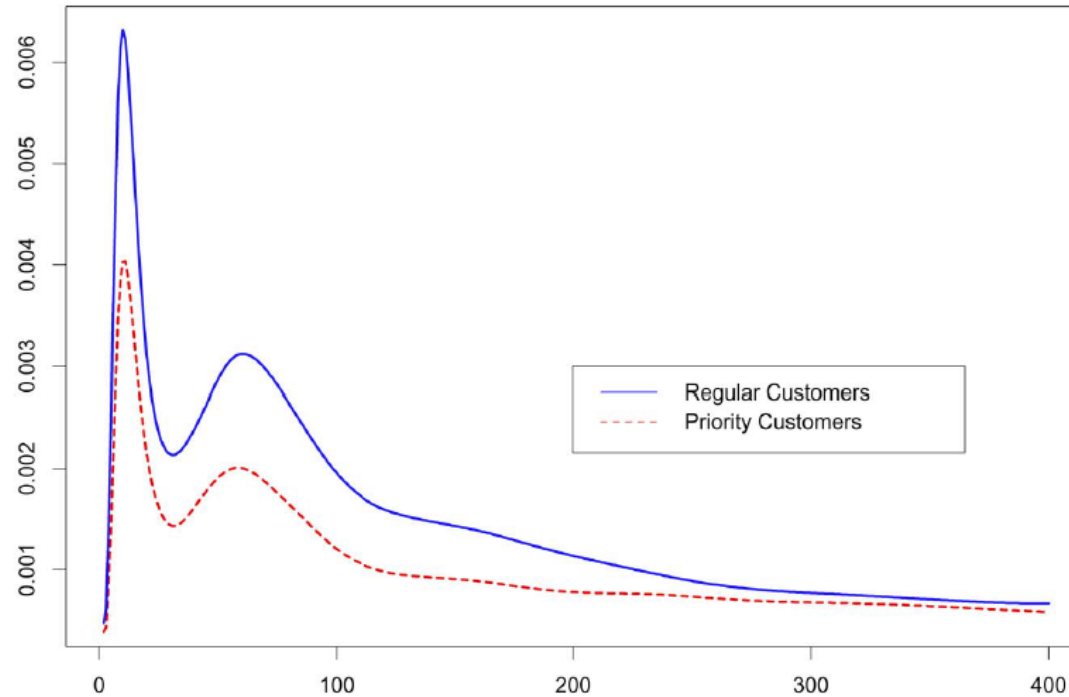


Patients treatment time: Blood-Test - Mixture Fitting and Real Components



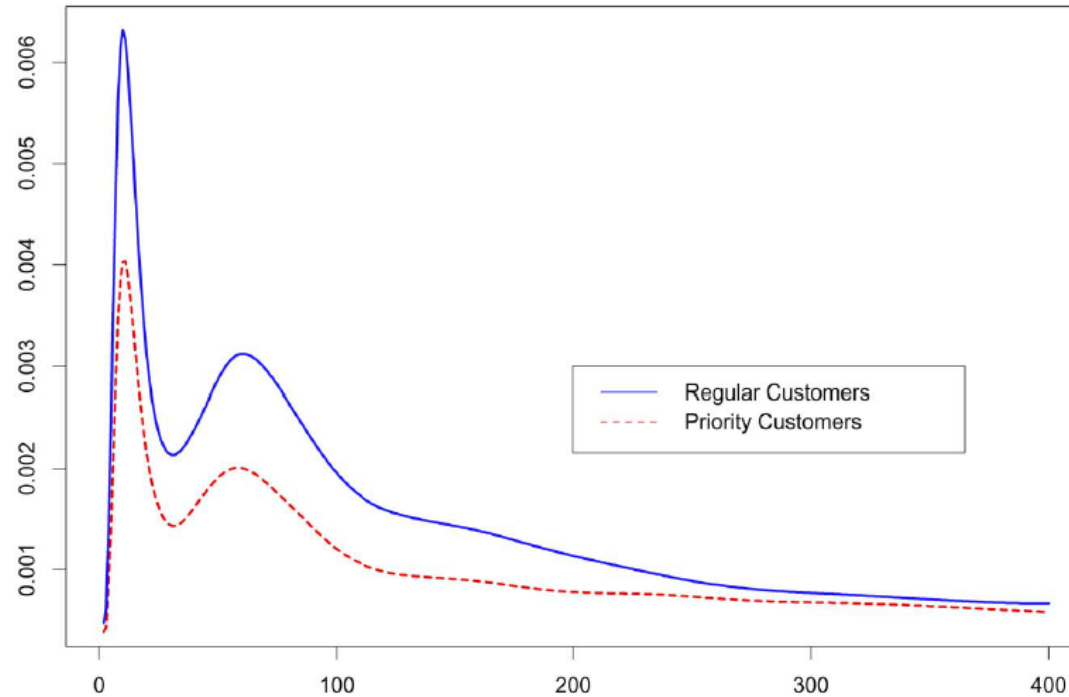
(Im)Patience while Waiting (Palm 1943-53)

Hazard Rate of (Im)Patience Distribution \propto Irritation
Regular over VIP Customers – Israeli Bank



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Regular over VIP Customers – Israeli Bank

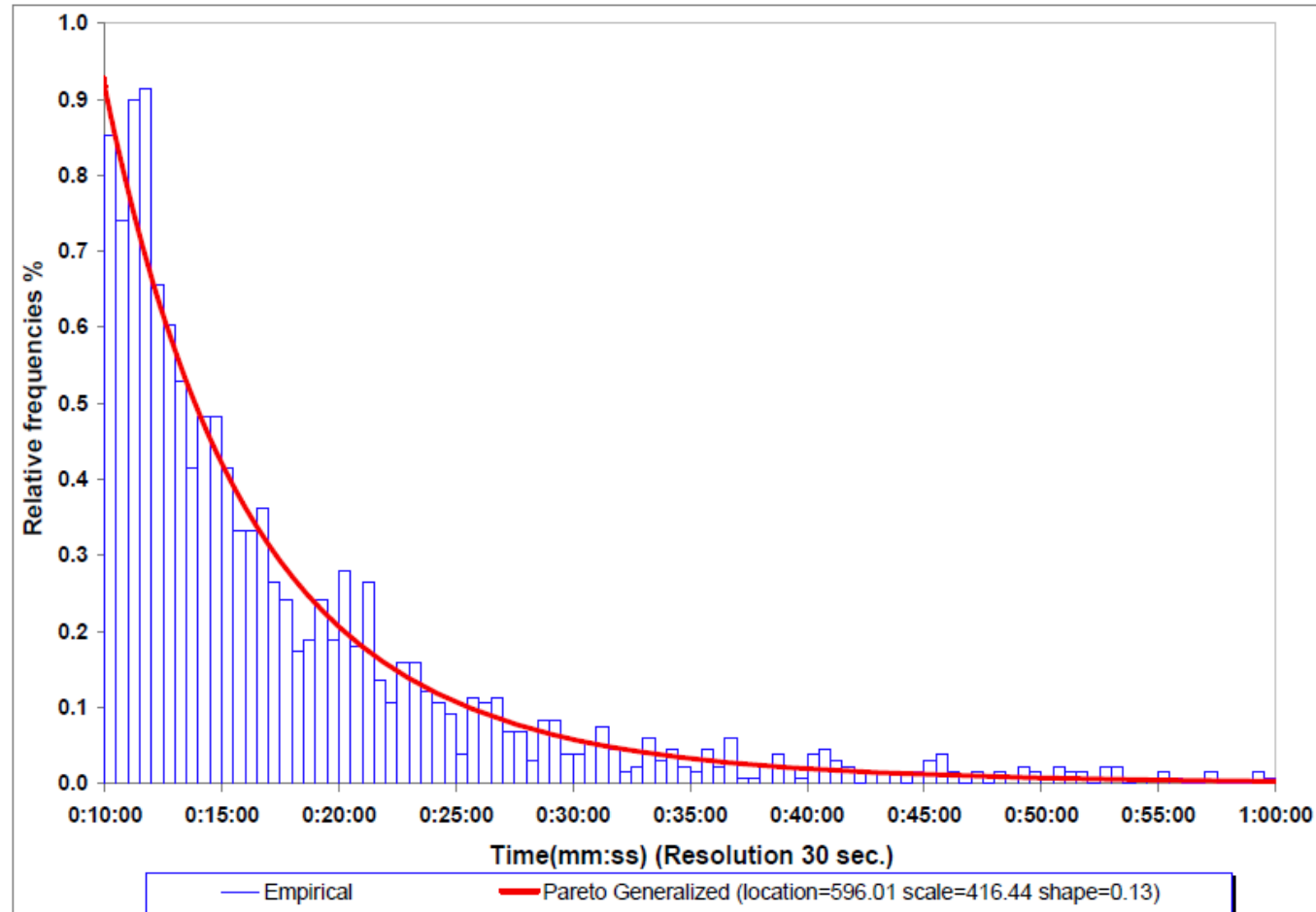


- ▶ **VIP** Customers are **more Patient** (Needy)
- ▶ **Peaks** of abandonment at times of **Announcements**
- ▶ Challenges: **Un-Censoring, Dependence (vs. KP), Smoothing**
- requires **Call-by-Call Data**

Primitives: (Im)Patience

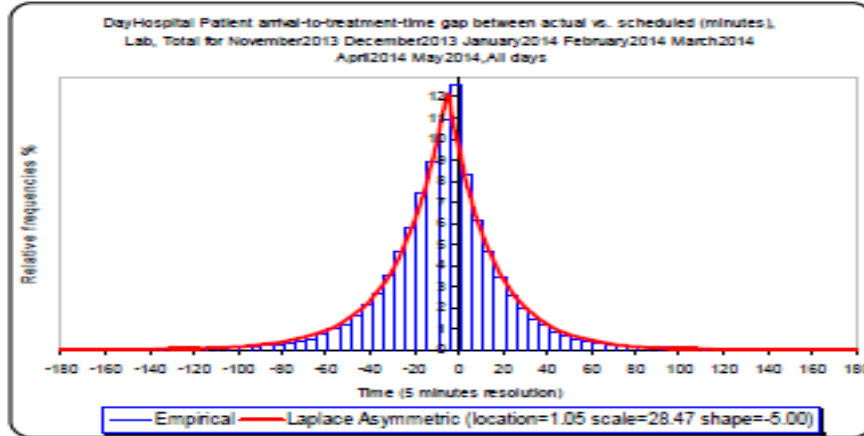
Israeli Bank: Uncensored 13,000 Customers, 24/11/2008

Patience $\geq 10min$: Why Pareto Tail?

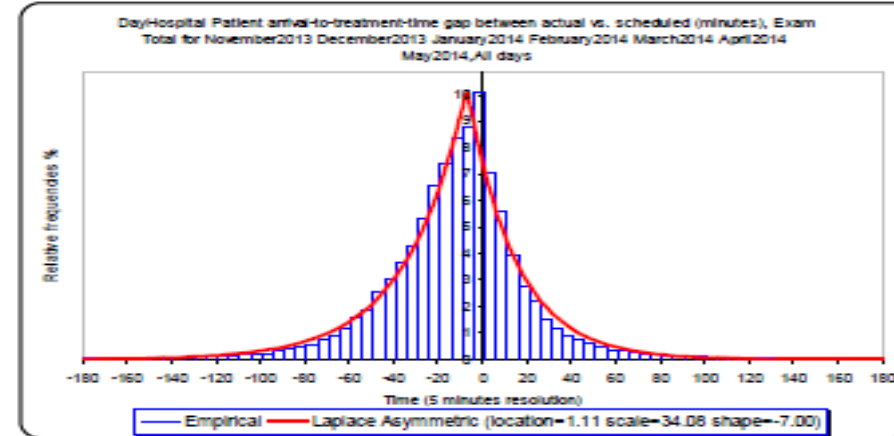


Primitives: Punctuality Planned vs. Actual Arrival to Service @ Stations 1, 2, 3 in a Hospital

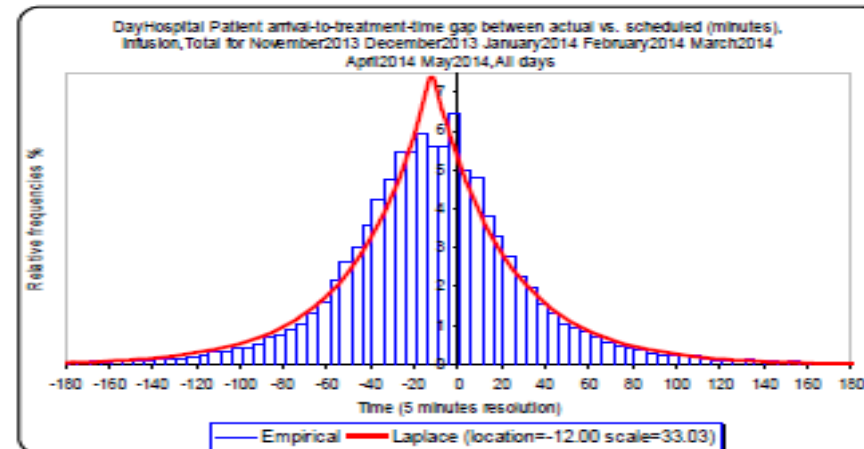
Lab (mean = -7 min)



Exam (mean = -12 min)



Infusion (mean = -12 min)



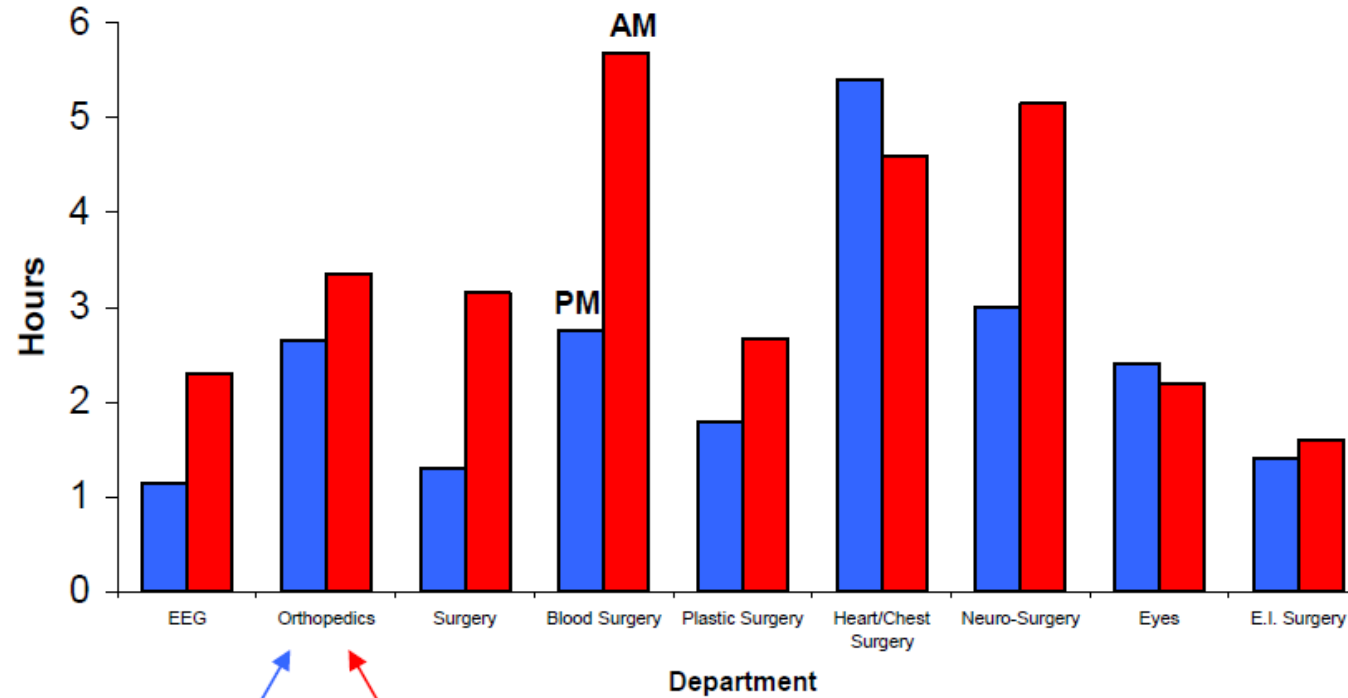
Mining Service Protocols - Examples (Behavioral OR)

- **Incentive-driven** protocols (even averages... , even doctors, ...)
- FCFS (data = heavy-traffic theory)
- Customers: Priorities while waiting
- Servers: “Priorities” while serving
- Management: Discharge from Hospital
- **Data not enough (DFCI Pharmacy: FCFS default, random in peak)**

Research: First steps (with Senderovich; Liberman & Meilijson in TAU)

Interesting Averages: The Human Factor, or Even "Doctors" Can Manage

Operations Time - **Morning (by Hour)** vs. **Afternoon (by Case)**:



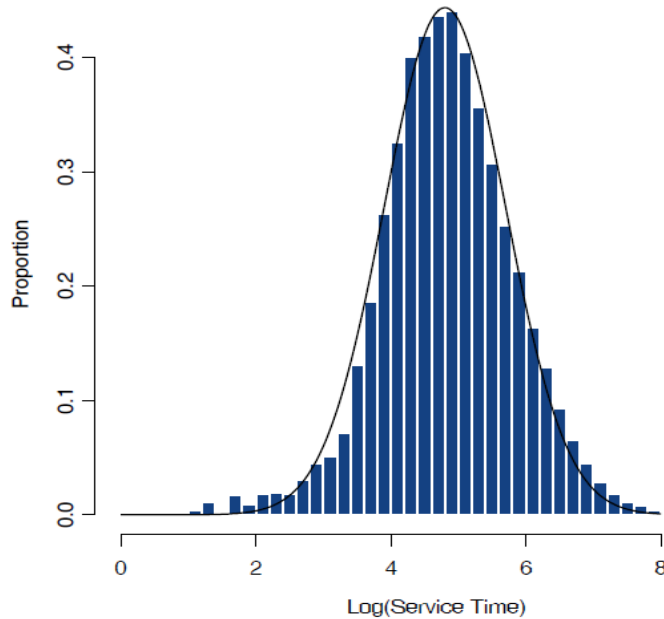
**Afternoon,
by Case**

**Morning,
by Hour**

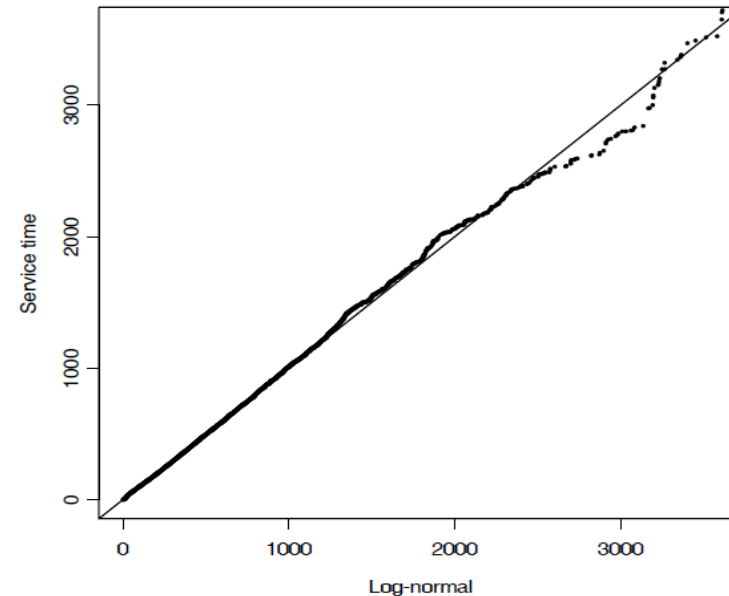
Durations: Phone Calls (2nd Surprise)

Israeli Call Center, Nov–Dec, 1999

Log(Service Times)



LogNormal QQPlot



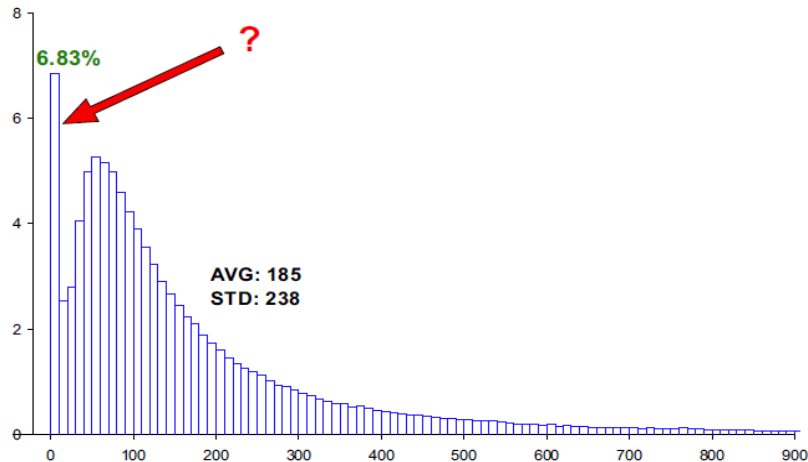
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Building-Blocks: Service-Duration Histograms

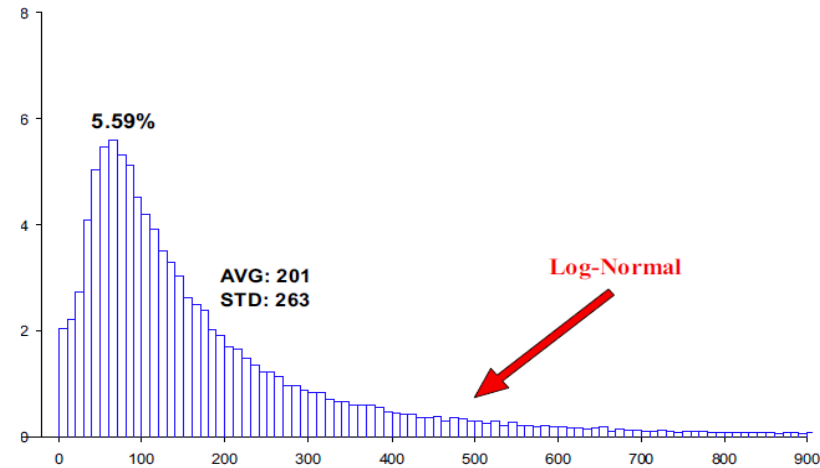
Histogram of Service-Duration in an Israeli Call Center, 1999

Why short services? Why LogNormal?

January-October



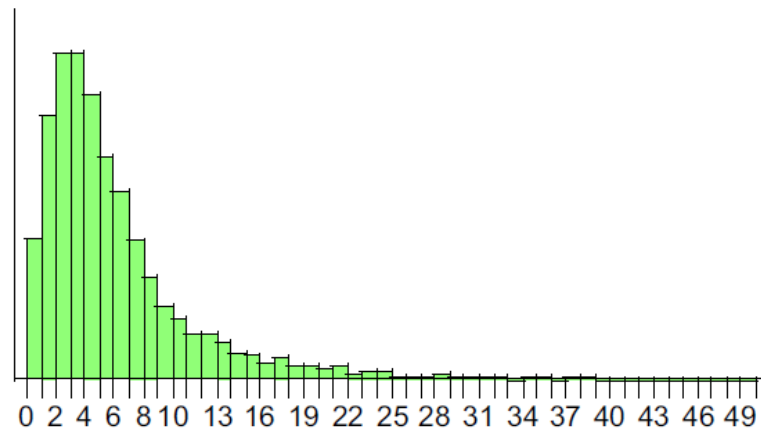
November-December



- ▶ January-October: **6.8% Short-Services** (≤ 10 seconds) ?
- ▶ November-December: **LogNormal** durations (common) ?

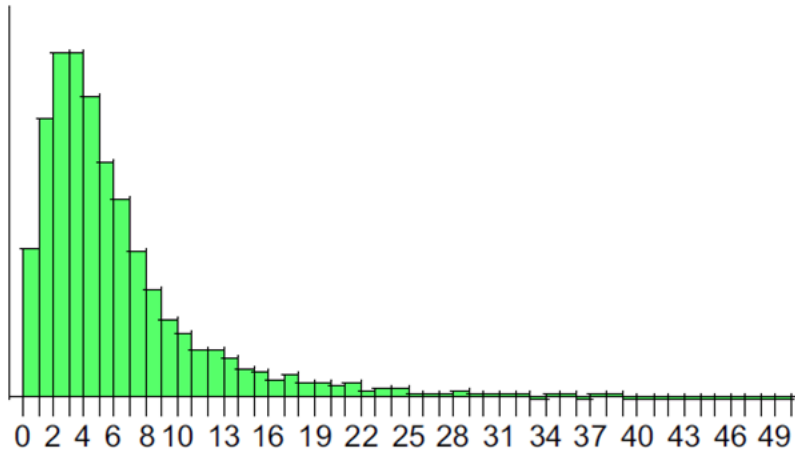
Building-Blocks: Length-of-Stay in a Hospital Ward

Israeli Hospital, in Days: LN

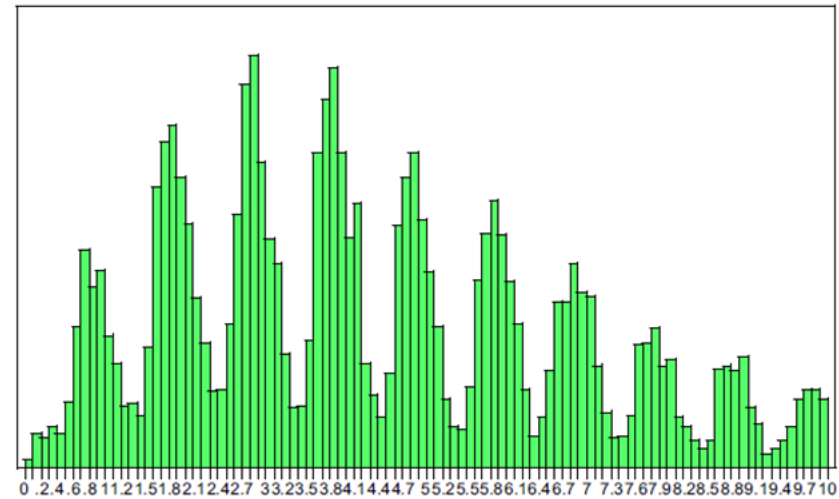


Protocols: LOS in Hospitals - Beyond LogNormal

Israeli Hospital, in Days: LN

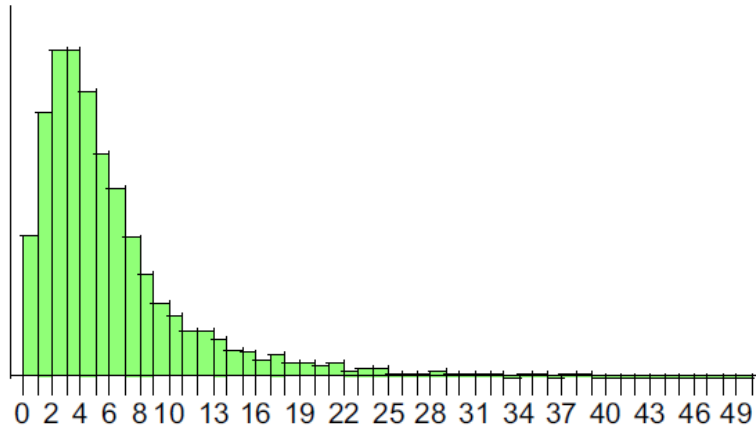


In Hours: 2 Time Scales, Mixture

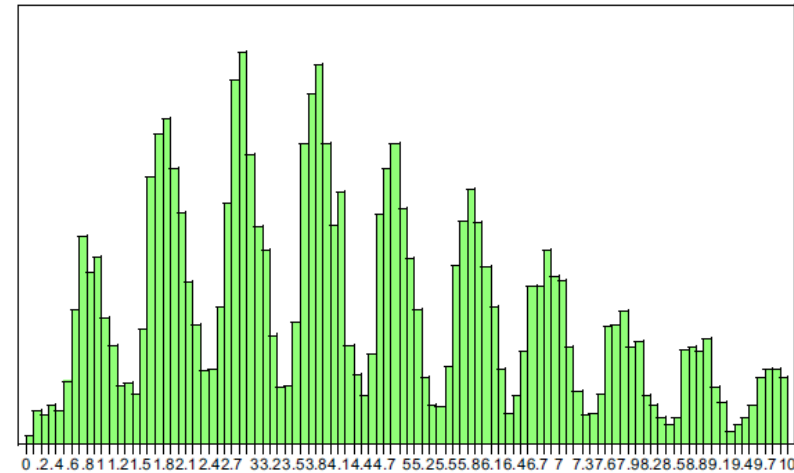


Length-of-Stay in a Hospital: Story in 2 Time-Scales

Israeli Hospital, in Days: LN



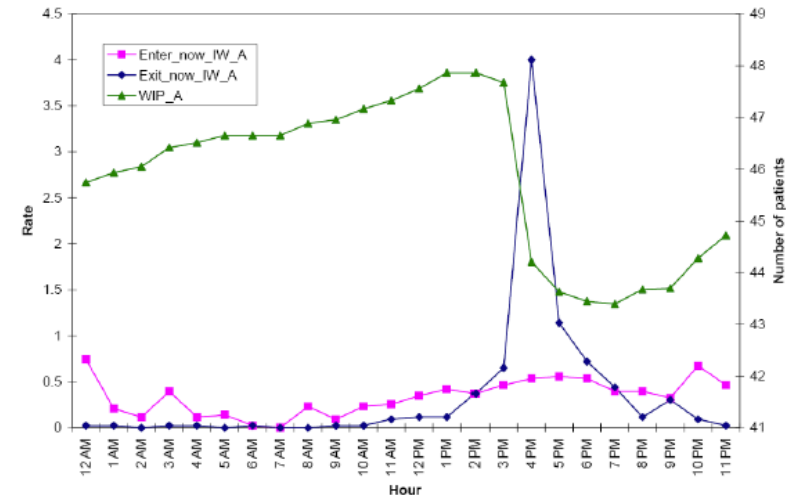
Israeli Hospital, in Hours



“**Explanation**”: Patients released around **3pm** (1pm in Singapore)

Why Bother ?

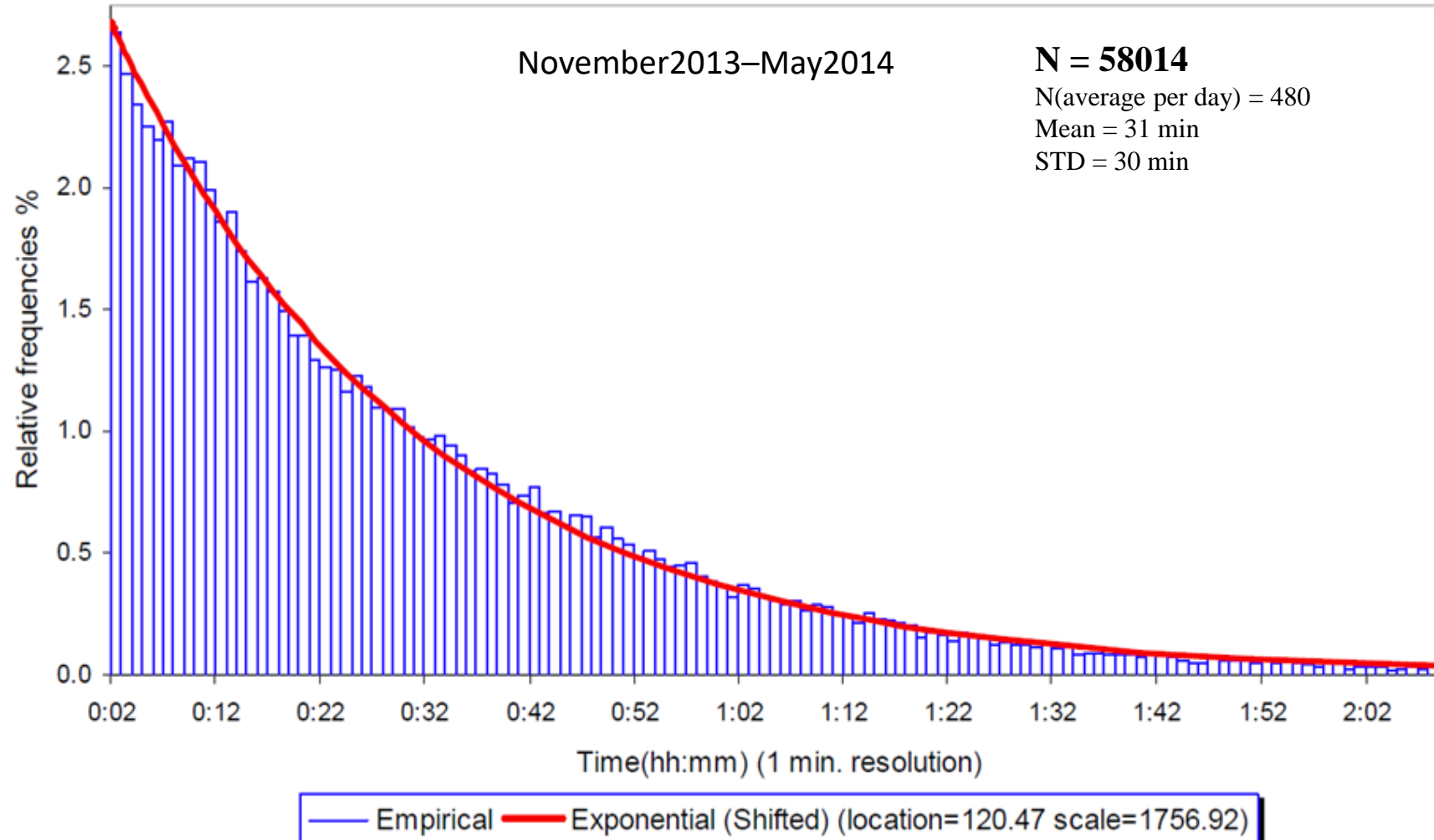
Staffing, Bed Management, ...



Protocols: via Waiting-Time for Physician-Exam

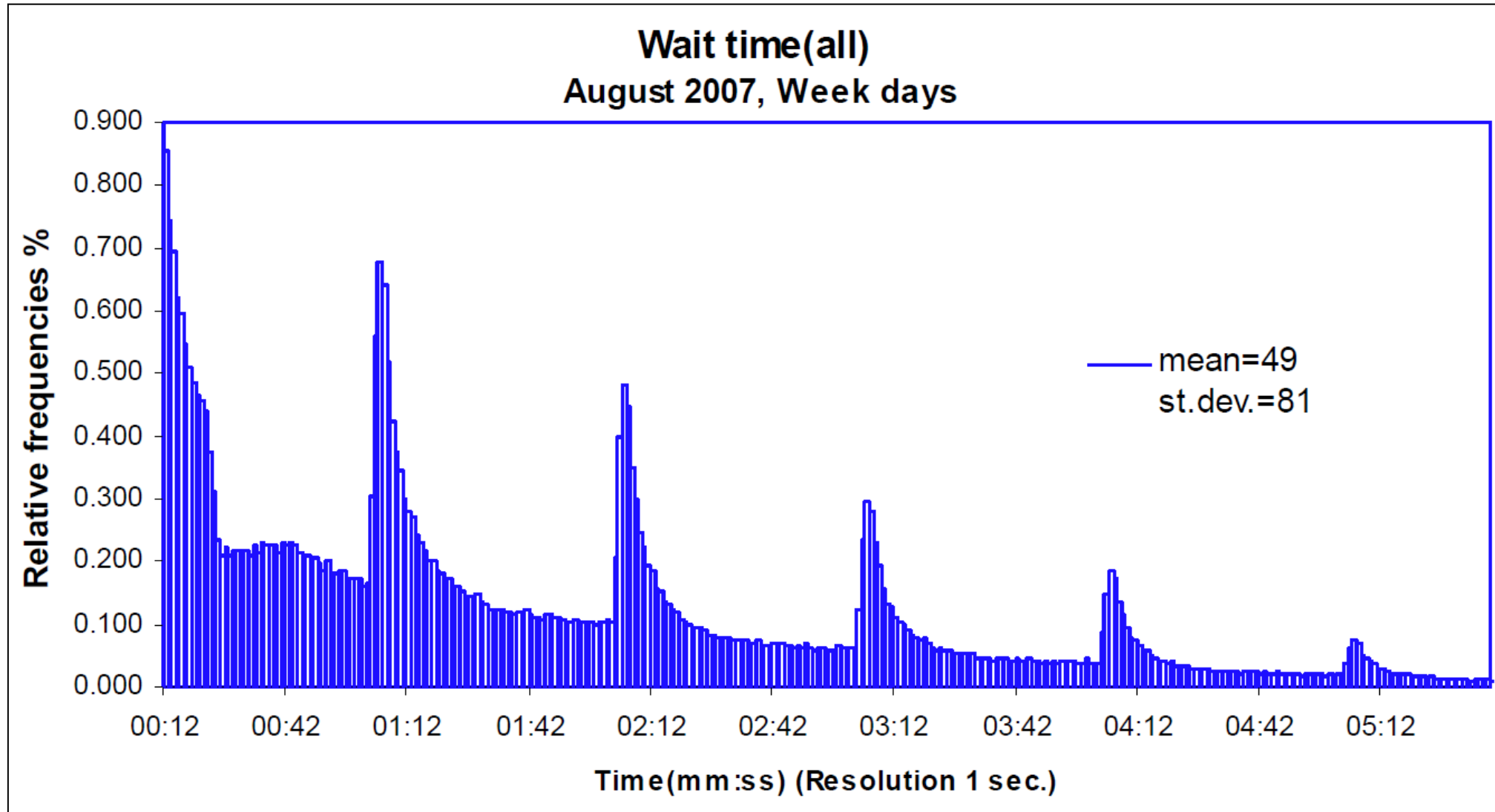
Theory of FCFS Single-Server Queue, in Heavy-Traffic: must be Exponential

(Kingman's Invariance theorem, 60's, under 2nd moments)



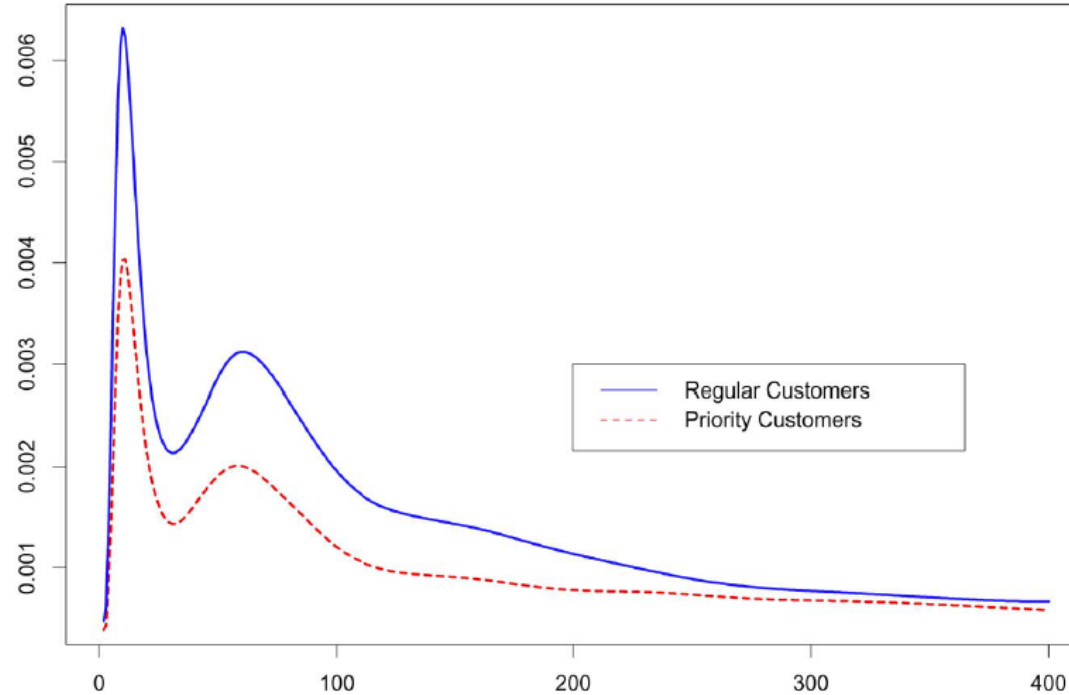
Protocols: via Waiting-Time for Phone Call

Histogram: Peaks every minute, due to Dynamic Priorities



(Im)Patience while Waiting (Palm 1943-53)

Hazard Rate of (Im)Patience Distribution \propto Irritation
Regular over VIP Customers – Israeli Bank



Patience: Customers

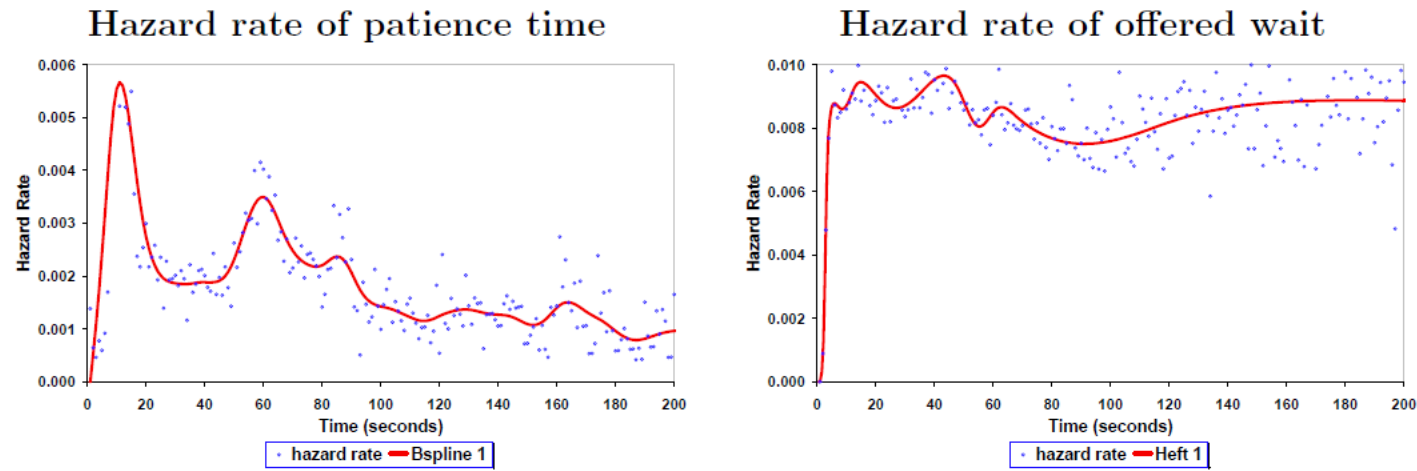


Figure 1: Patience and offered wait in an Israeli call center

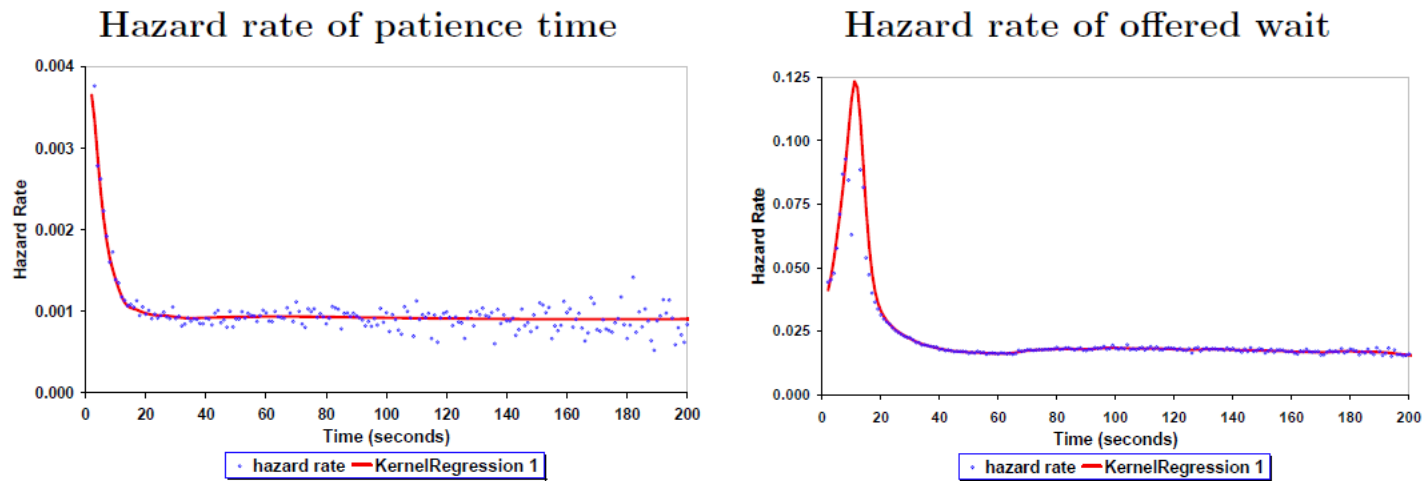


Figure 2: Patience and Offered Wait in a U.S. Call Center

Patience: Customers

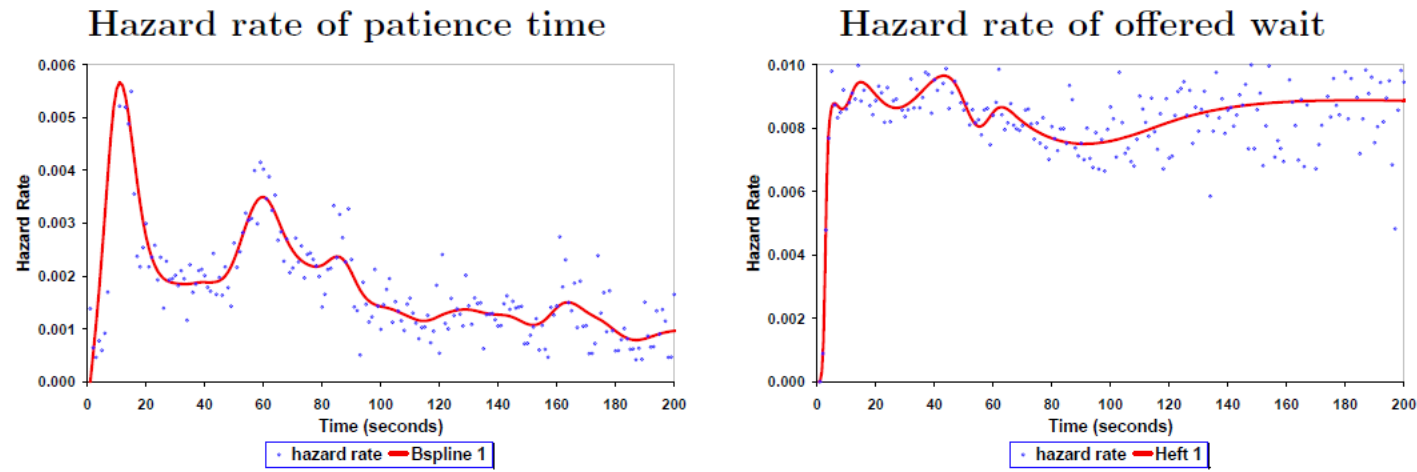


Figure 1: Patience and offered wait in an Israeli call center

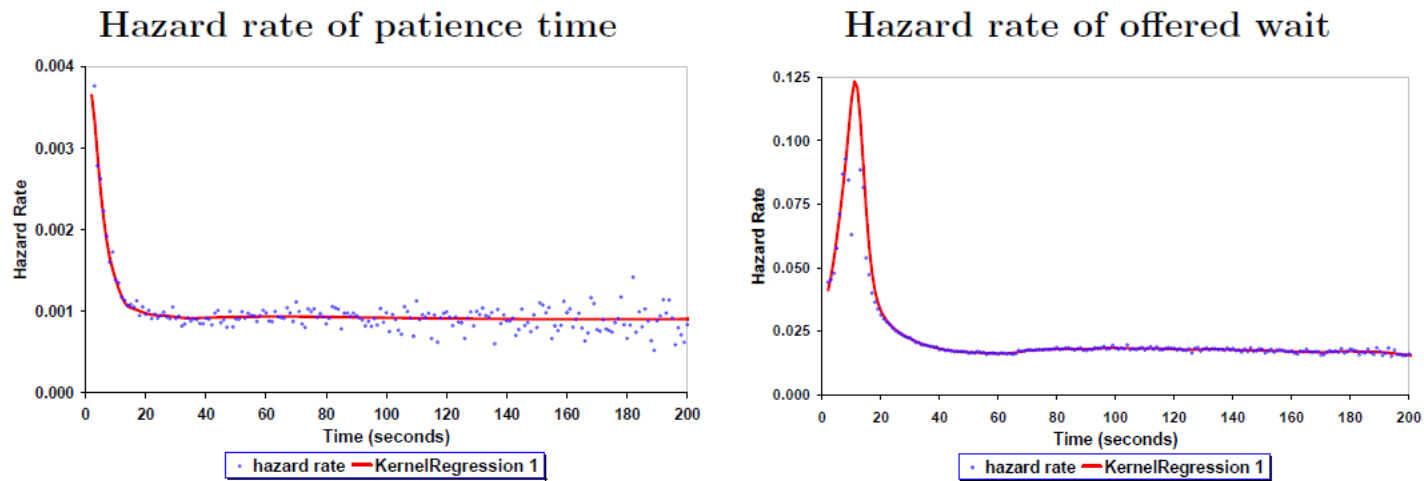
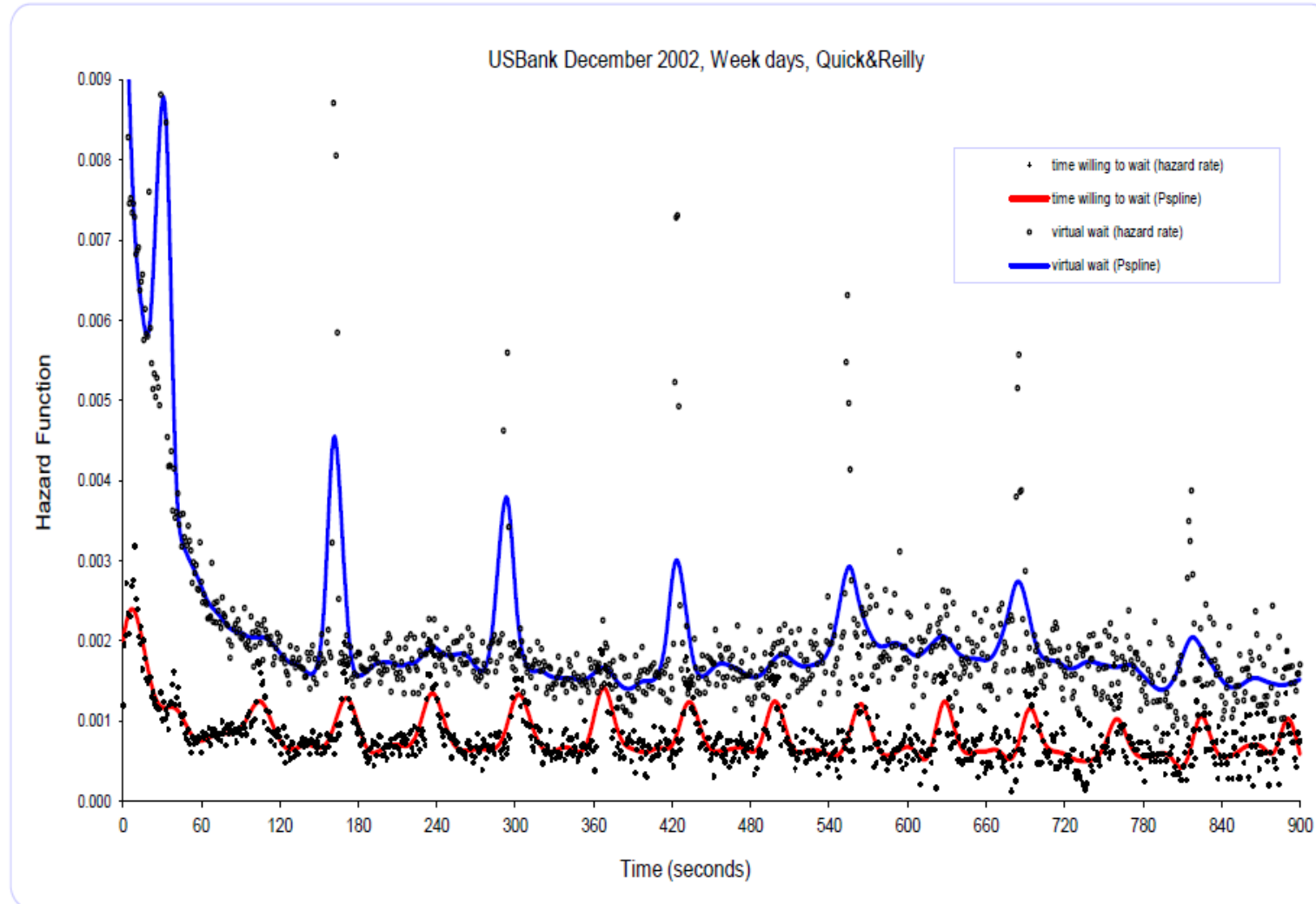


Figure 2: Patience and Offered Wait in a U.S. Call Center

Protocols + Psychology

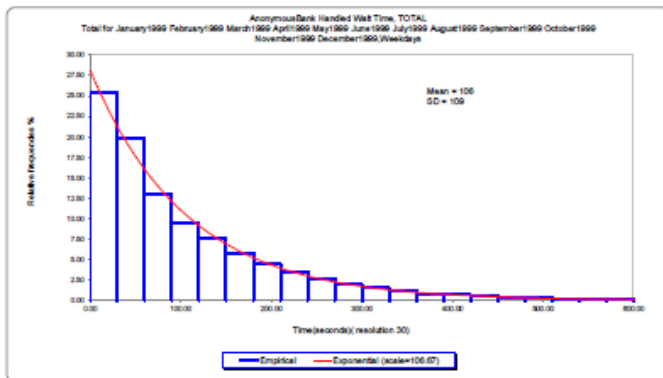
Patient Customers, Announcements, Priority Upgrades



Protocols: Waiting Time in a Call Center

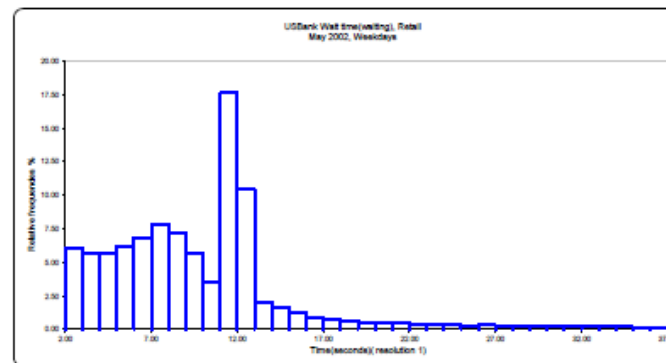
Exponential in Heavy-Traffic (min.)

Small Israeli Bank



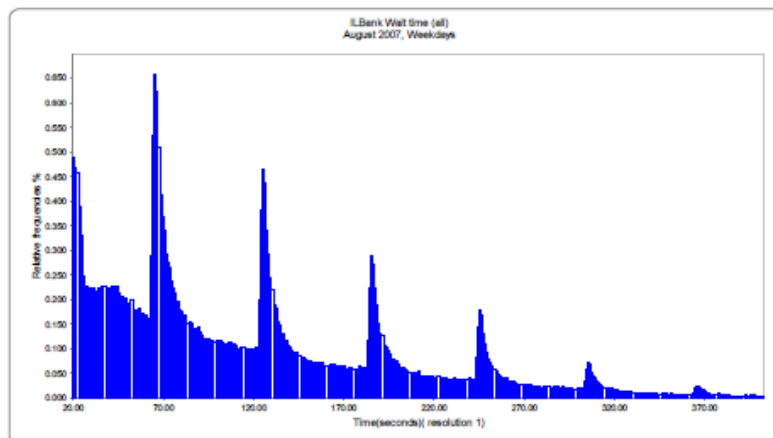
Routing via Thresholds (sec.)

Large U.S. Bank

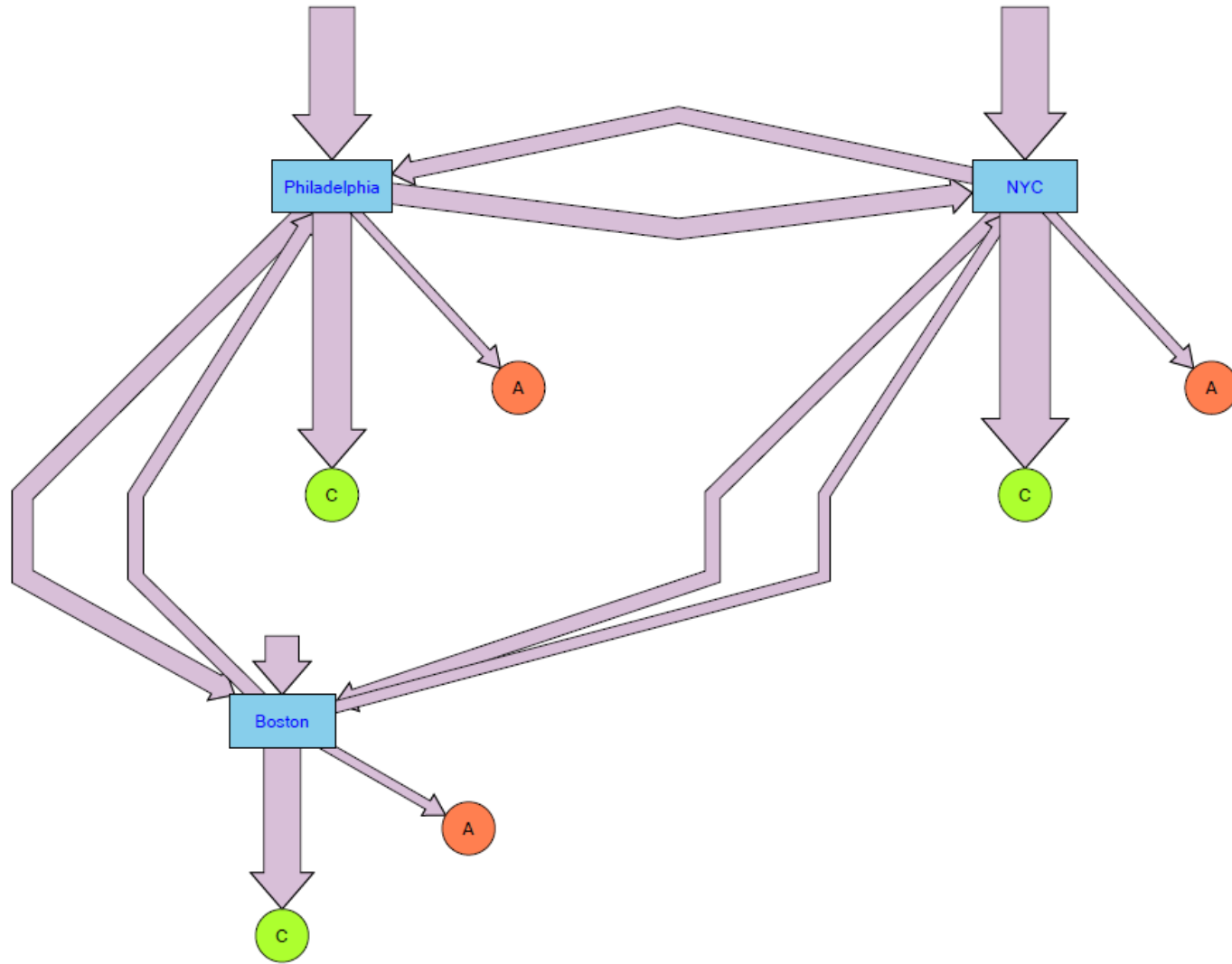


Scheduling Priorities (sec.) [compare Hospital LOS (hours)]

Medium Israeli Bank



Protocol Mining: Snapshots of Connectivity





Ultimate Research Goal

e.g. Specific Emergency-Department, with ample reliable data

- Real-time: **control** of patient-flow (bottlenecks); status **info** and **prediction**
- Short-term: on Monday, set Tuesday's **staffing** levels (or next week's); real **cost** of care for the individual patient (vs. mean/negotiated costs)
- Long-term: **capacity** allocation, facility/triage **design**; **social** network (e.g. correlated w/ outcomes); transformative **changes** (Epic); congestion **laws**



Research Goal (within reach)

e.g. Specific Emergency-Department, with ample reliable data

- Real-time: control of patient-flow (bottlenecks); status info and prediction
- Short-term: on Monday, set Tuesday's staffing levels (or next week's); real cost of care for the individual patient (vs. mean/negotiated costs)
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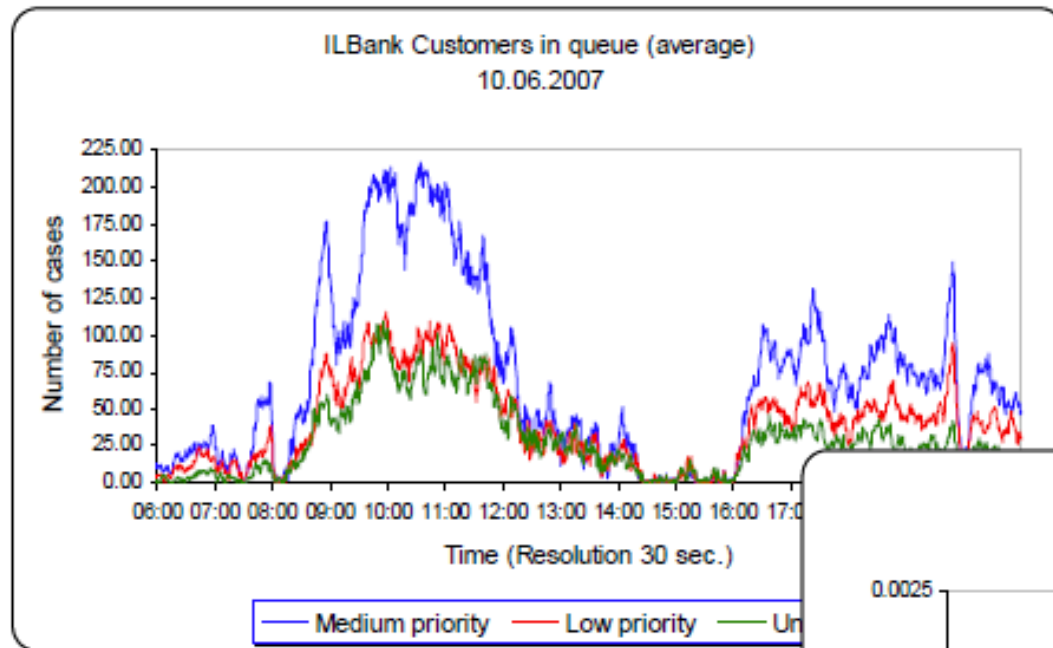
All above will be enabled by parsimonious (robust) models, created in real-time **mining** (semi- or un-supervised) of ED **processes** and **models** (empirical, simulation = **SimNets**, mathematical = **QNets**, **FNets**, **DNets**,...)

2 Prerequisites: Data, Models

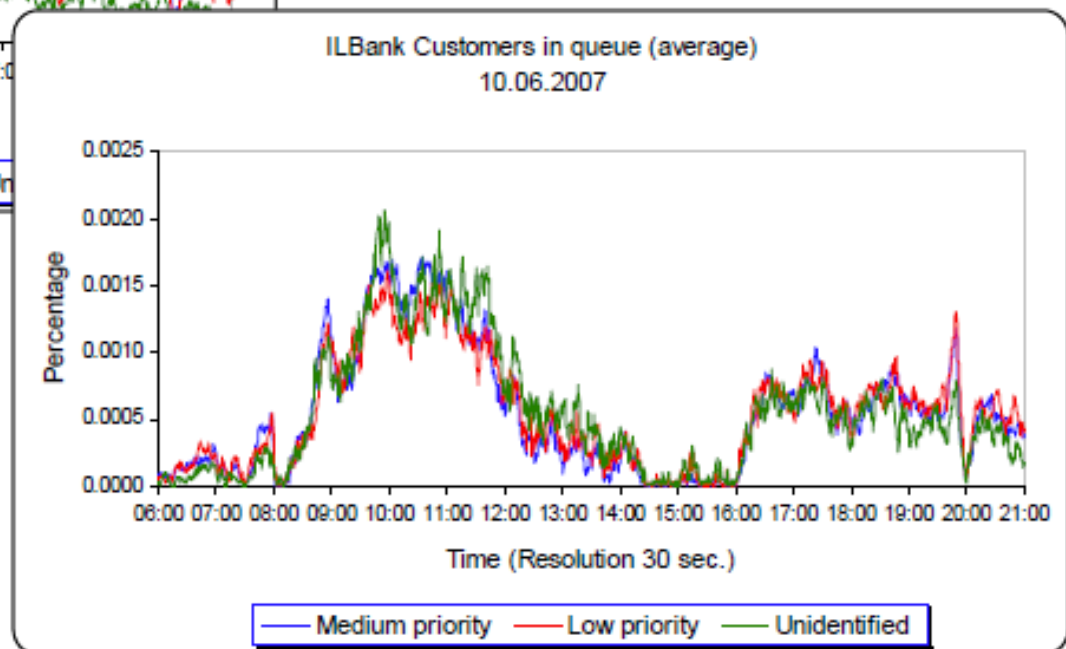
- **Data:** The language of
 - ✓ Multi-disciplinary research (e.g. OR + Psychology + CS/DS)
 - ✓ **Academia-Industry partnerships** (e.g. university & bank, or hospital, or court)
- **Models:** **Simple models** at the service of complex realities (not too simple)
 - ✓ Stochastic networks: Empirical, Analytical (QNets, **FNets**, **DNets**), SimNets
 - ✓ Insights often rooted in deep mathematics (even Little's Law)

Dynamics: Parsimonious Models (Congestion Laws)

3 Queue-Lengths at 30 sec. resolution (ILBank, 10/6/2007)



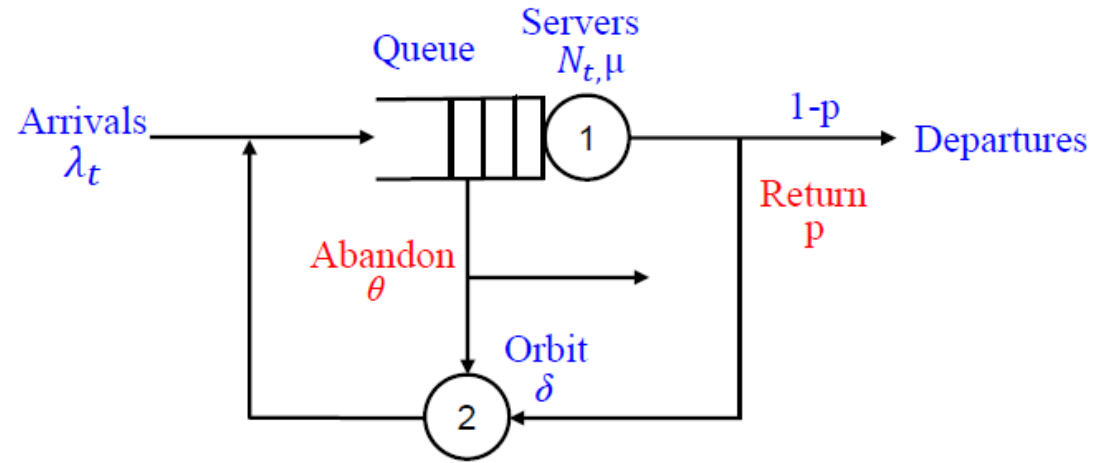
Queue "Shape"



- ▶ Area normalized to 100%
- ▶ **State-Space Collapse**

Model Selection: As Simple as Possible but Not Simpler

Service with Retrials and Abandonment; w/ Massey, Reiman, Stolyar



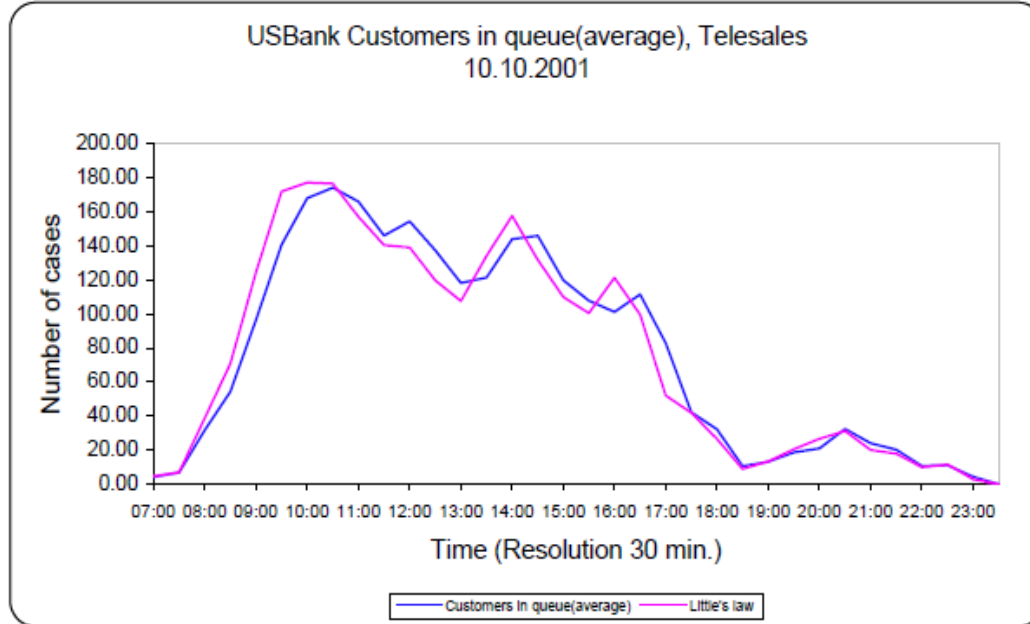
Laws & Models: Data-Based Erlang A/R/S, following B/C

- Little's Law (Steady-State, Transient), State-Space Collapse,...
- Erlang-B (**Blocking**) and Erlang-C (?)
 - **Erlang**, Agner Krarup: Queueing Theory was born in 1909, in his paper "The Theory of Probabilities and **Telephone Conversations**"
- 2. Erlang-A
 - **Abandonment**: While waiting for service, does service-value dominate residual-wait-cost
- 1. Erlang-R
 - **Return/Feedback**: Customers often return to service (positively, negatively, just needing)
- 3. Erlang-S
 - **Servers**: Challenging to manage, and model, no less so than customers

Above: Simple (Parsimonious) models of complex realities, yet not too simple (Robust)

Little's Law $L = \lambda \times W$, in a Time-Varying Environment

Time-Gap: # in System lags behind Little / 30 min

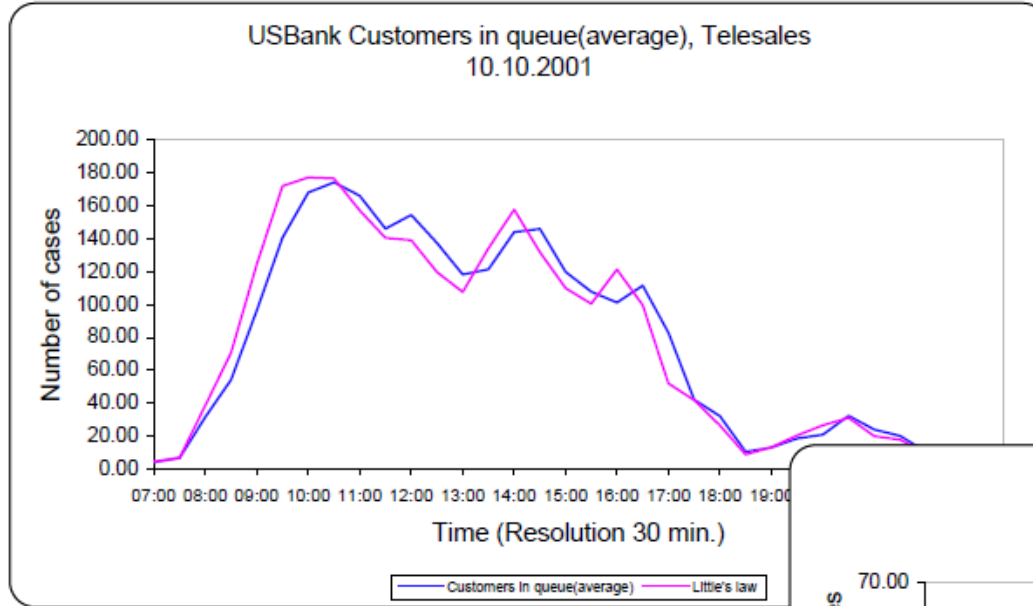


Call Center:

⇒ **Piecewise Steady-State**

Little's Law $L = \lambda \times W$, in a Time-Varying Environment

Time-Gap: # in System lags behind **Little** / 30 min



Call Center:

⇒ **Piecewise Steady-State**

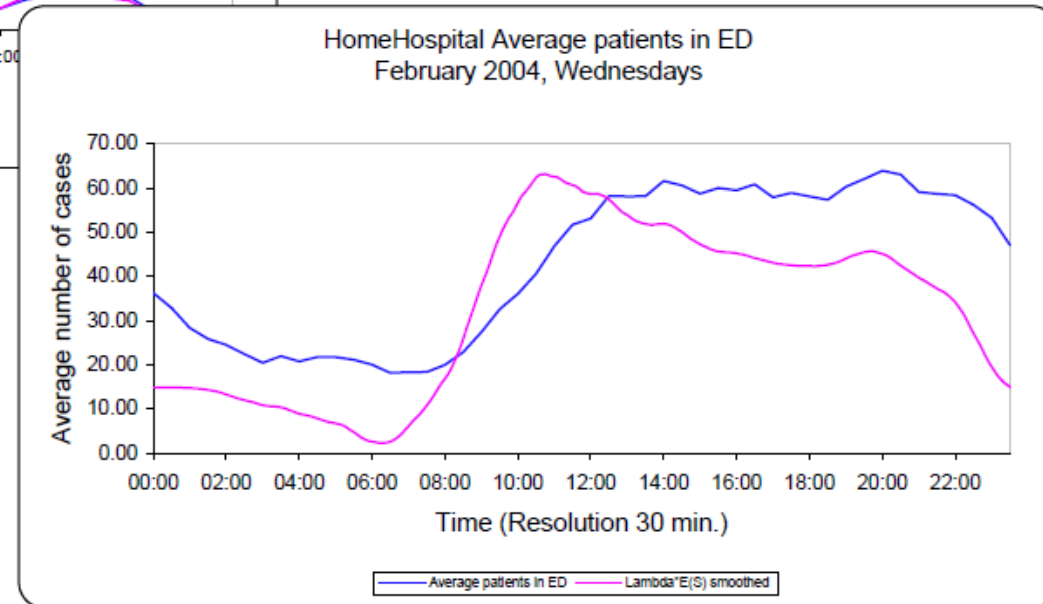
Emergency Dept:

⇒ **Time-Varying Transient**

$$EL(t) = \tilde{\lambda}(t) \times EW,$$

$$\tilde{\lambda}(t) = E\lambda(t - W_e).$$

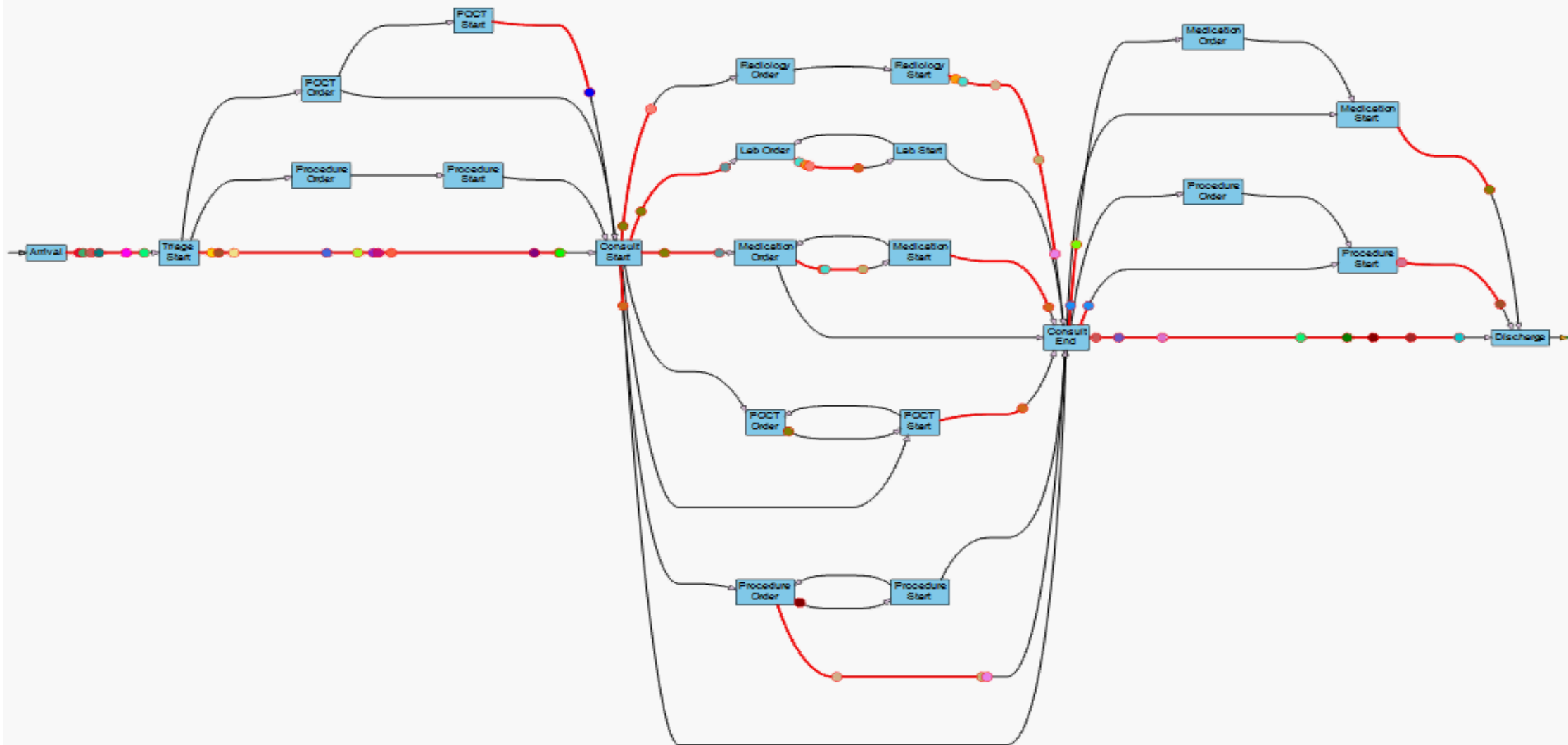
(Bertsimas, Mourtzinou;
Fralix, Riano, Serfozo)



Ideally for each model:

1. Motivating **Phenomenon**, via Data
2. Informal **Description** of a Model that captures the phenomenon
3. Example of **Application(s)**
4. Model **Expressiveness** (Strength)
5. Insights, **Extensions**

Patients flow (XYHospital)
October 2012

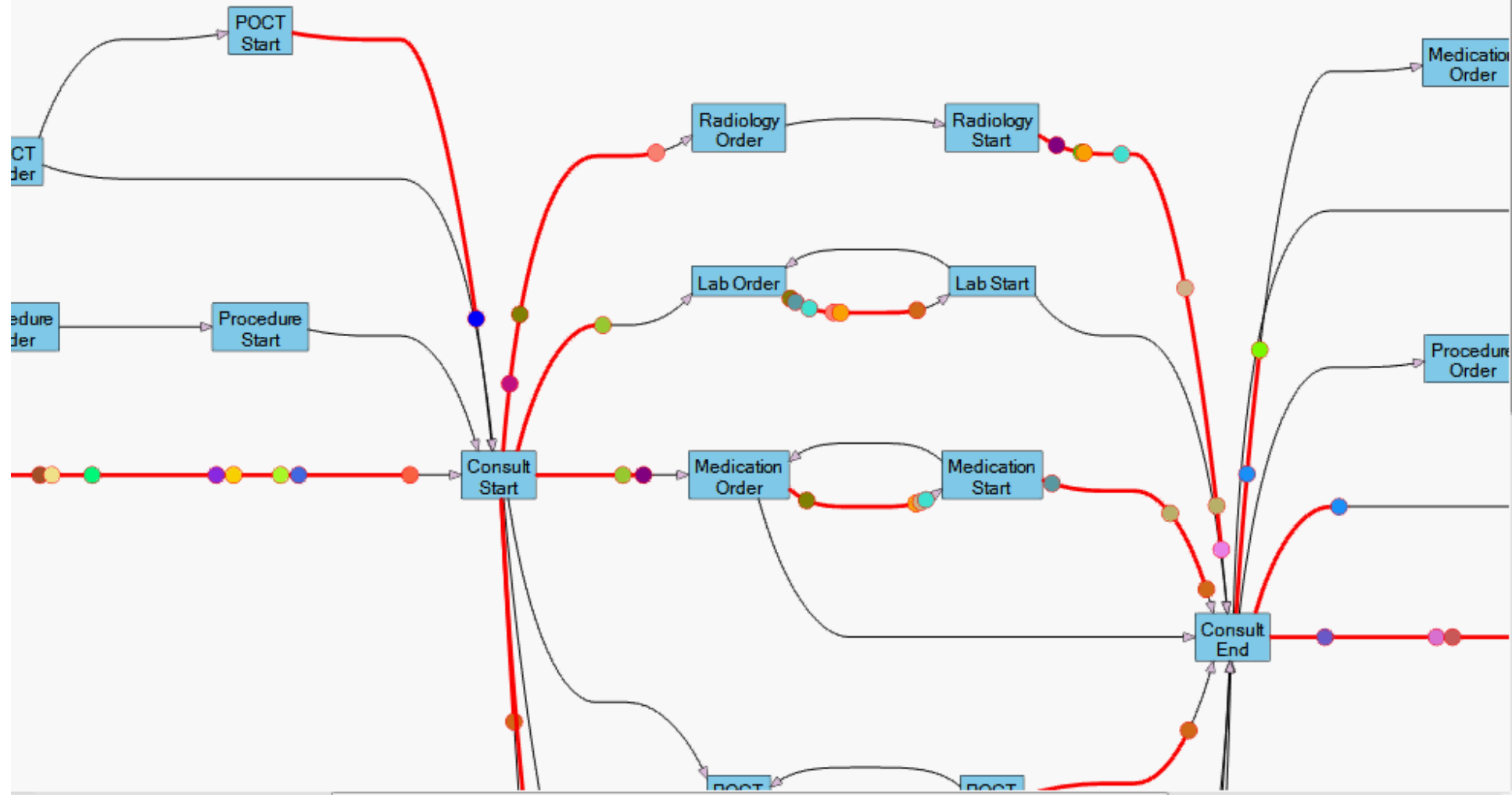


PAUSE 16 : 42 : 10 2 October 2012

Sampling time interval (sec.) 30

Display time interval (millisec.) 100

Patients flow (XYHospital) October 2012

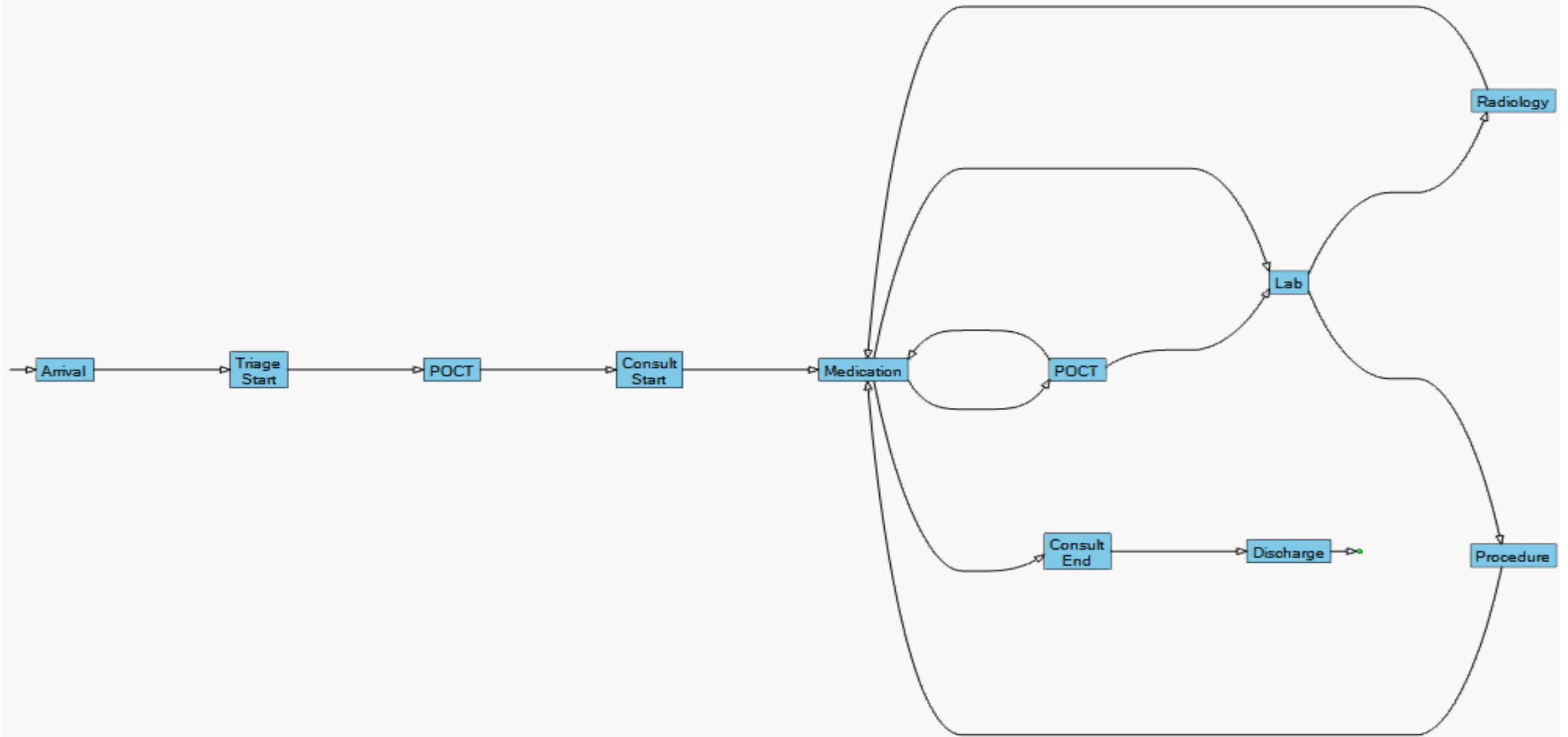


PAUSE 16 : 57 : 40 2 October 2012

Sampling time interval (sec.) 30

Display time interval (millisec.) 100

Patient flow (XYHospital)
October 2012



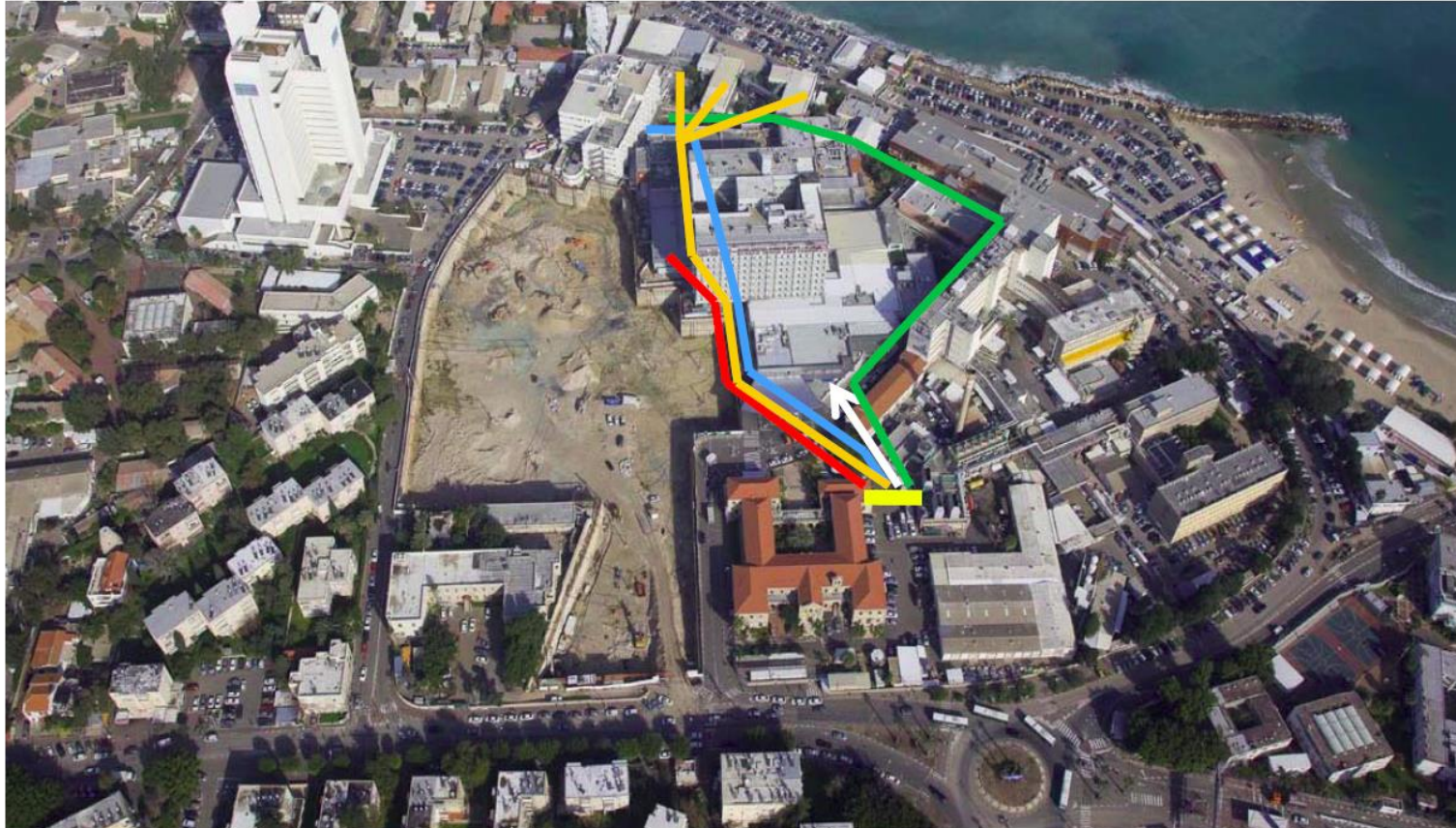
PLAY **17 : 01 : 52** 12 October 2012

Sampling time interval (sec.) 180

Display time interval (millisec.) 100

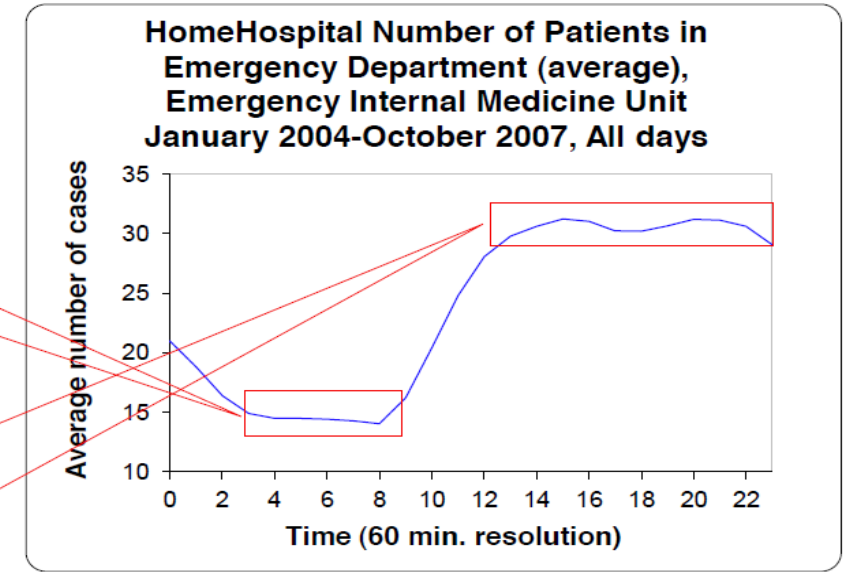
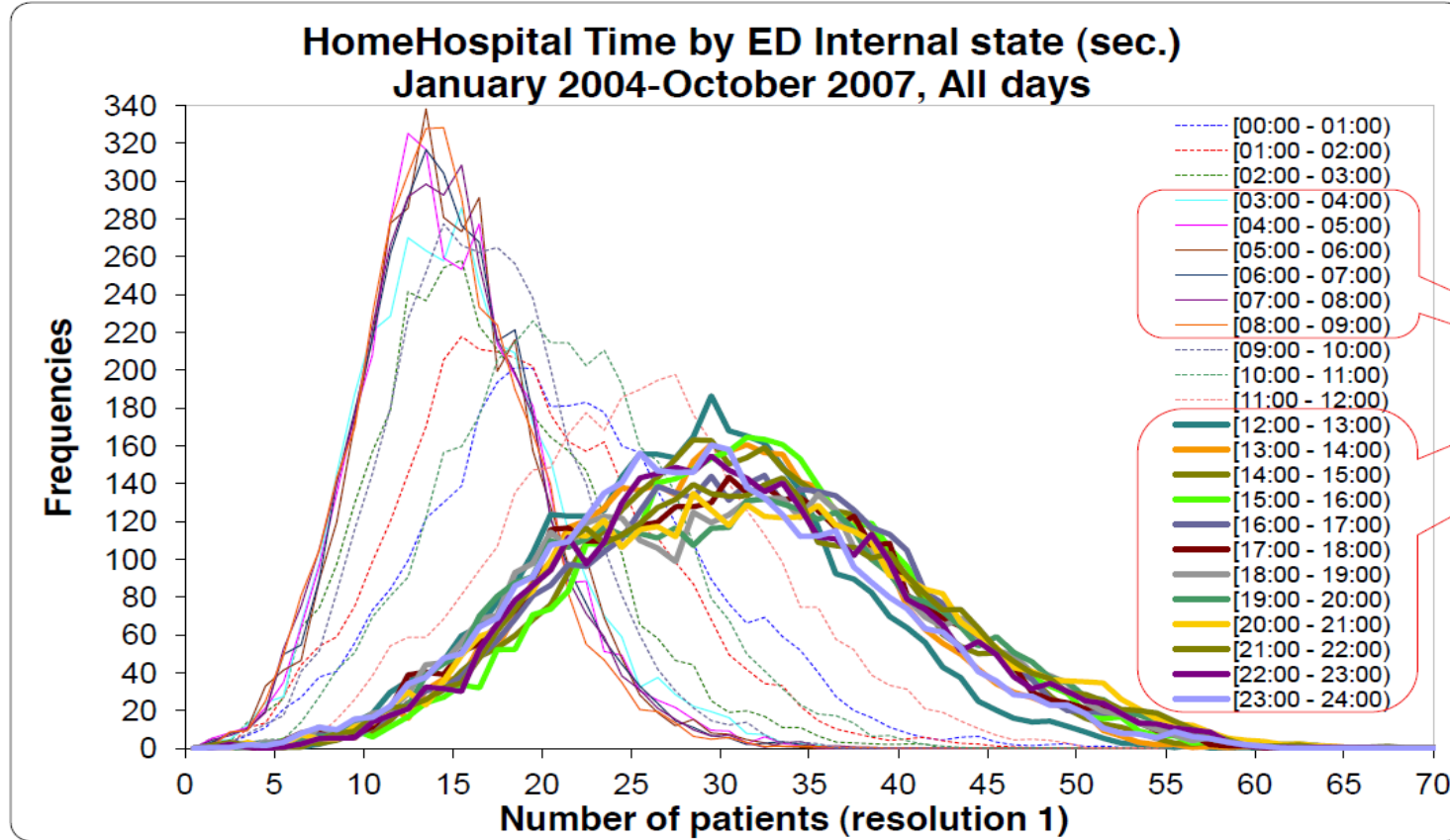
eg. RFID-Based Data: Mass Casualty Event (MCE)

Drill: Chemical MCE, Rambam Hospital, May 2010



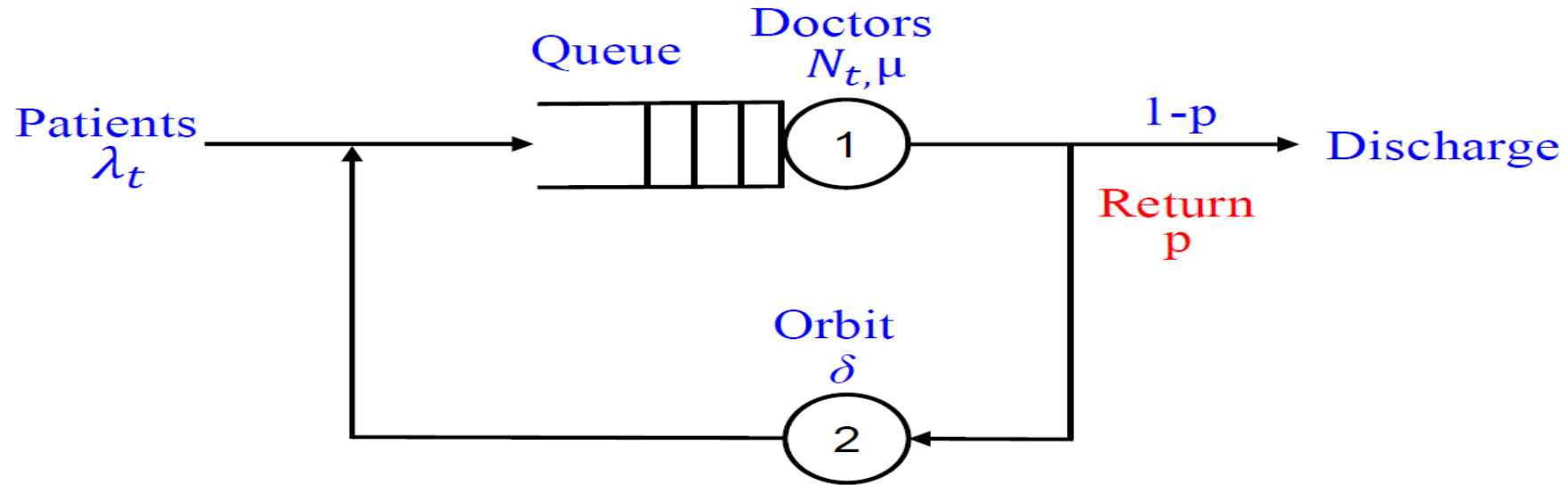
Focus on **severely wounded** casualties (≈ 40 in drill)

Note: 20 observers support real-time control (helps validation)



Internal ED Occupancy histogram (left) and Average Census (right),
by hour of the day

Erlang-R \leftrightarrow Fluid Model, w/ Galit Yom-Tov



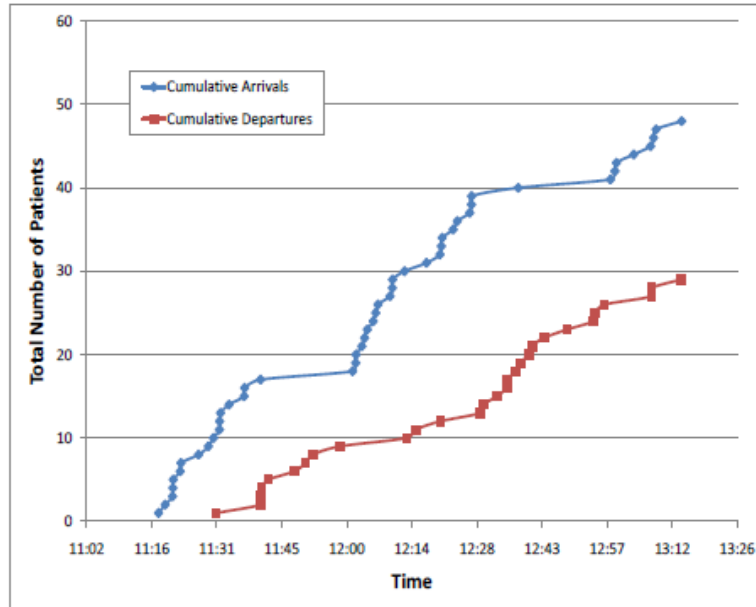
Functional Strong Law of Large Numbers, for a 2-station QNet. BUT **FNet** = ODE: derived **directly** (no QNet), spreadsheet “solution”

$$\begin{aligned}\frac{d}{dt} q_t^1 &= \lambda_t - \mu \cdot (q_t^1 \wedge N_t) + \delta \cdot q_t^2 \\ \frac{d}{dt} q_t^2 &= p \cdot \mu \cdot (q_t^1 \wedge N_t) - \delta \cdot q_t^2\end{aligned}$$

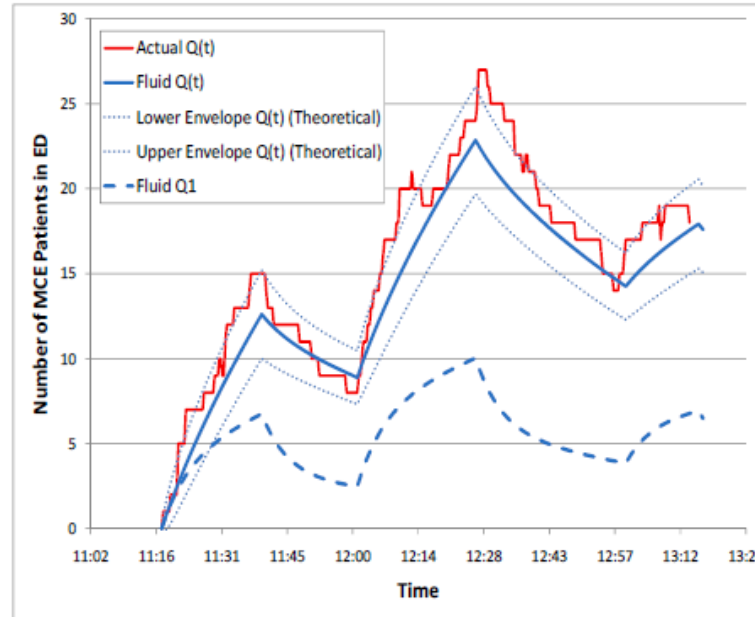
Erlang-R Value: FNet vs. Data

Chemical MCE Drill (Israel, May 2010, 11:00-13:00)

Arrivals & Departures (RFID)



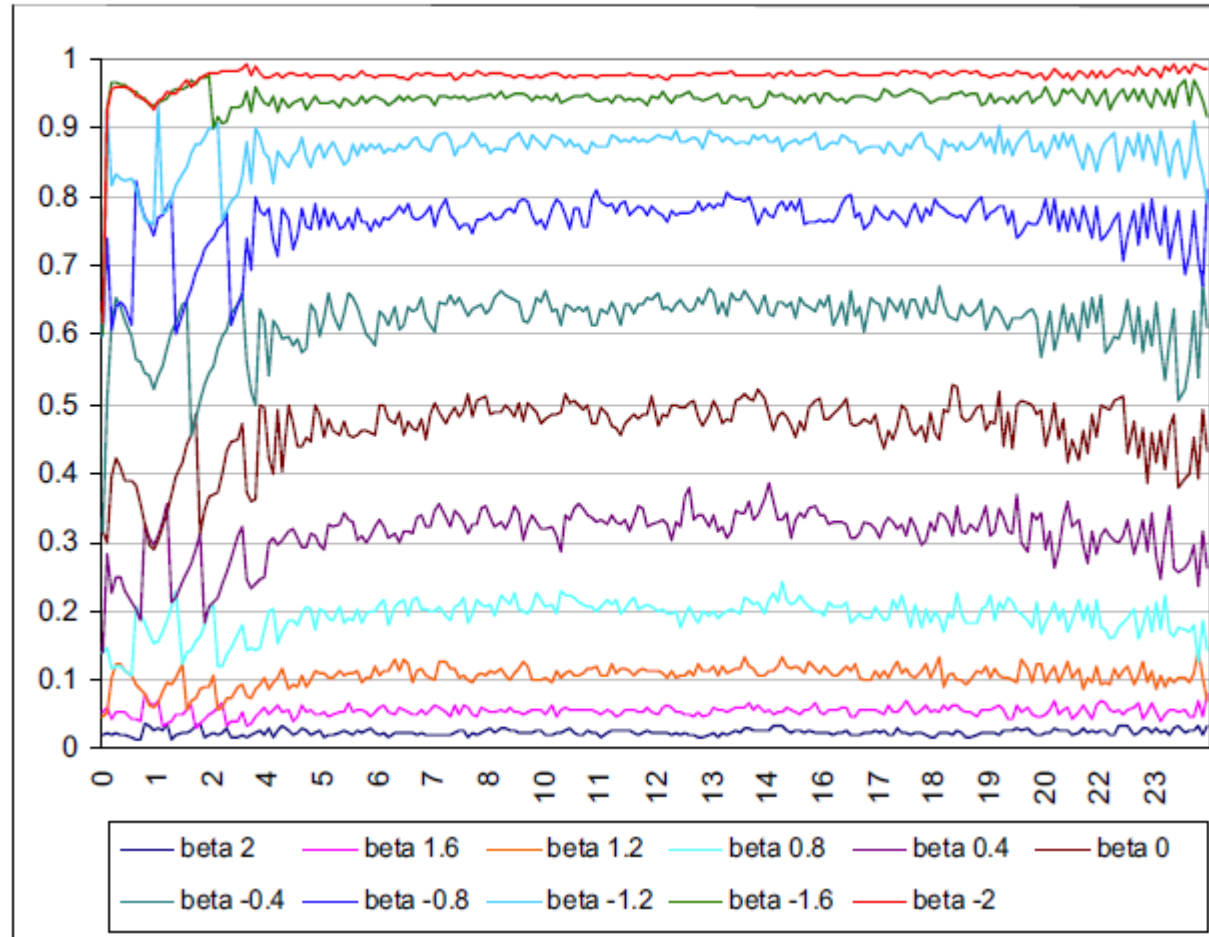
Erlang-R (Fluid, Diffusion)



- ▶ **Recurrent/Repeated** services in Chemical MCE: injection every 15/30/60 min
- ▶ **Fluid** = ODE
- ▶ **Diffusion** (confidence band), via F. Central Limit Theorem: Usefully narrow

Time-Stable Performance of Time-Varying Systems

Delay Probability = As in the Stationary Erlang-A / R



A Data-Based Framework, or “Erlang-R in the ED”

System = e.g. Emergency Department

- ▶ **QNet** = Erlang-R (time-varying 2-station Jackson; w/ Yom-Tov)
- ▶ **FNets** = 2-dim dynamical system (Massey & Whitt)
- ▶ **DNets** = 2-dim Markovian Service Net (w/ Massey and Reiman)
- ▶ **SimNet** = Customized ED-Simulator (Marmor & Sinreich)

A Data-Based Framework, or “Erlang-R in the ED”

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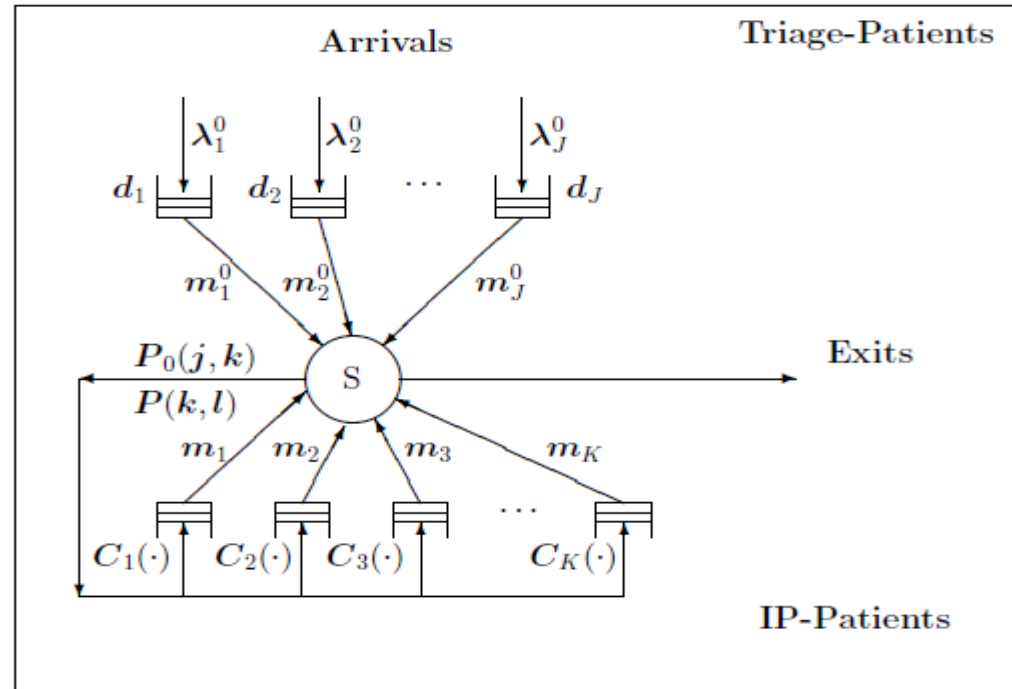
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- ▶ **DNets** = 2-dim Markovian Service Net (w/ Massey and Reiman)
- ▶ **SimNet** = Customized ED-Simulator (Marmor & Sinreich)

Framework: Mining (all) ServNets from Data

- ▶ **MCE ED:** FNet \Rightarrow Census, DNet = Confidence band
Performance Analysis, Prediction
Validated against Data
- ▶ **Normal ED:** FNet \Rightarrow Physician offered-load \Rightarrow $\sqrt{\cdot}$ -Staffing
Staffing to stabilize operational performance
Validated against SimNet

ED Patient Flow: The Physicians View

with J. Huang, B. Carmeli, S. Israelit

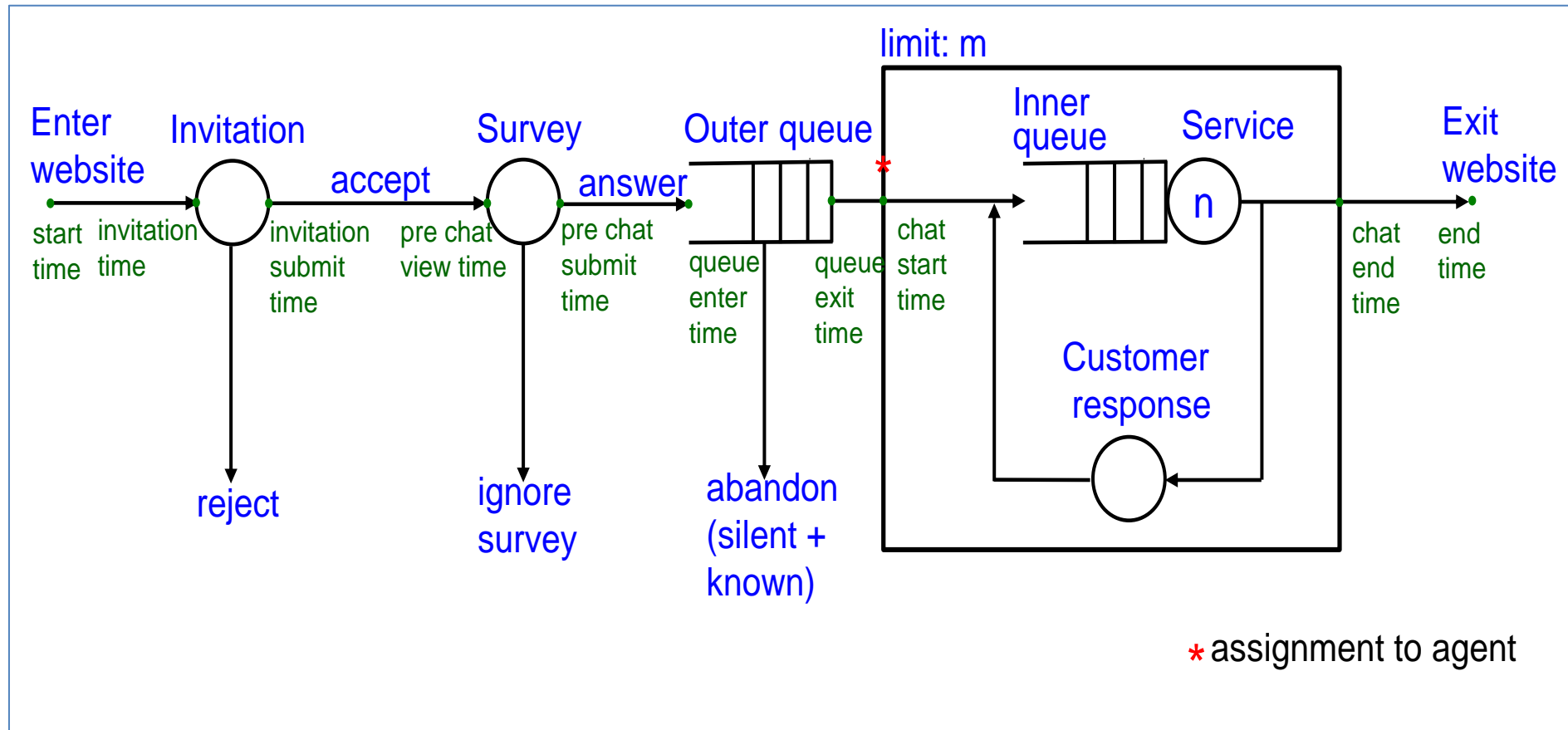


Goal: Adhere to **Triage-Constraints**, then **release In-Process** Patients

Following **Plambeck, Kumar, Harrison (2001): Throughput-time constraints**

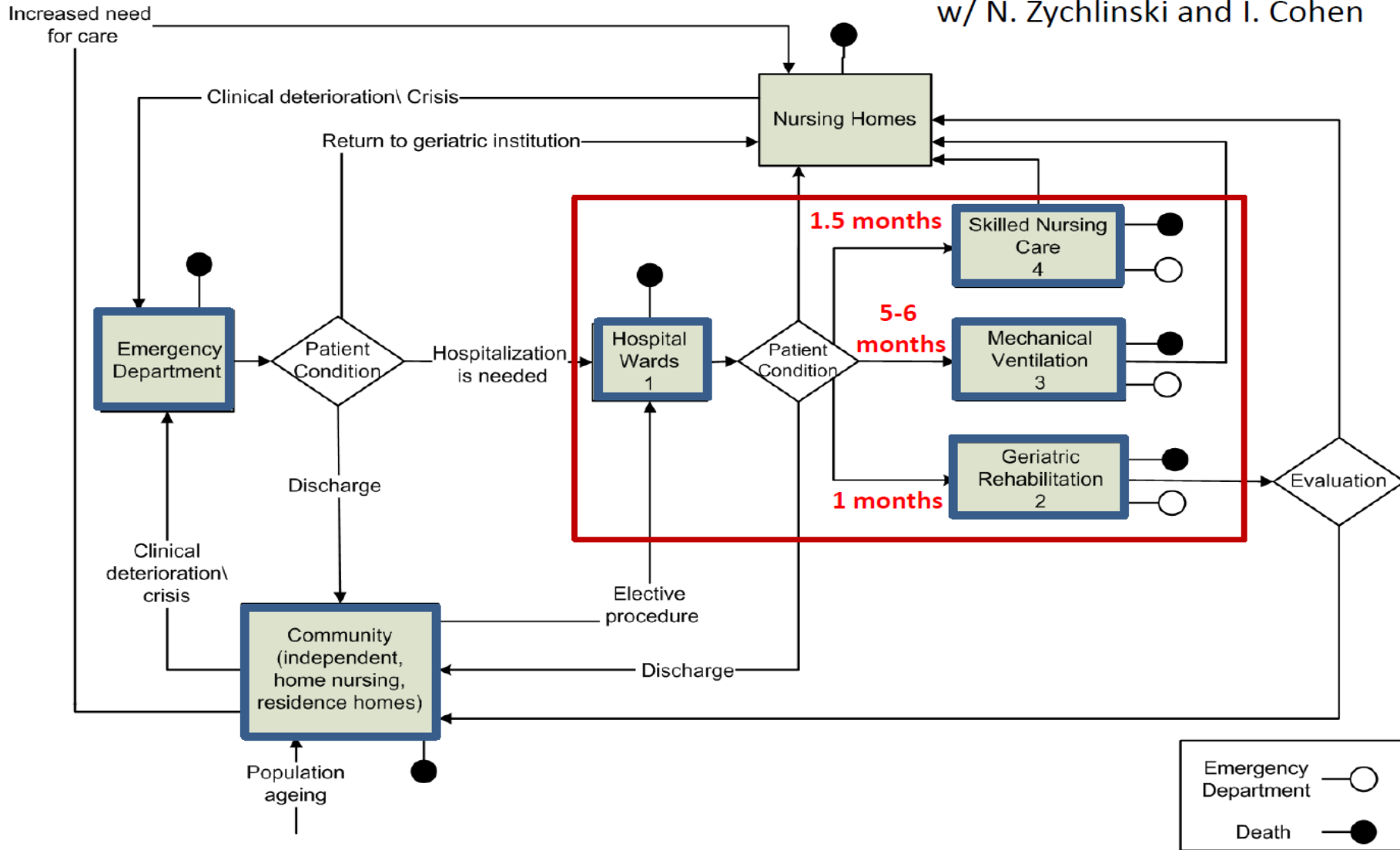
Online Chats: from Internet-Prompts to Chat-Sessions

Server with Multiple Returning Customers
Galit Yom-Tov, Anat Rafaeli, graduate (OB) students



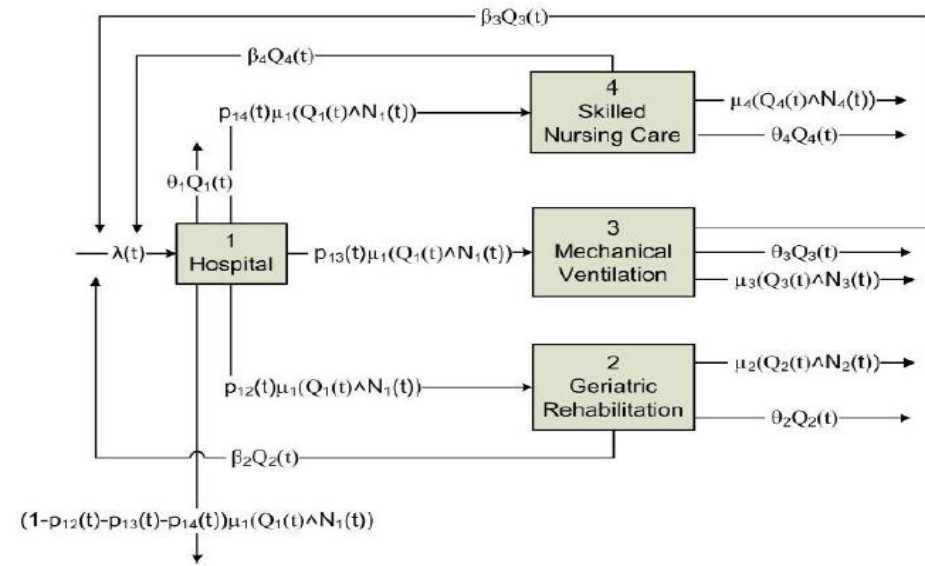
Patient Flow: ED to Wards to Nursing/Geriatric Institutions

w/ N. Zychlinski and I. Cohen



A Fluid Model

Stations 2,3 and 4

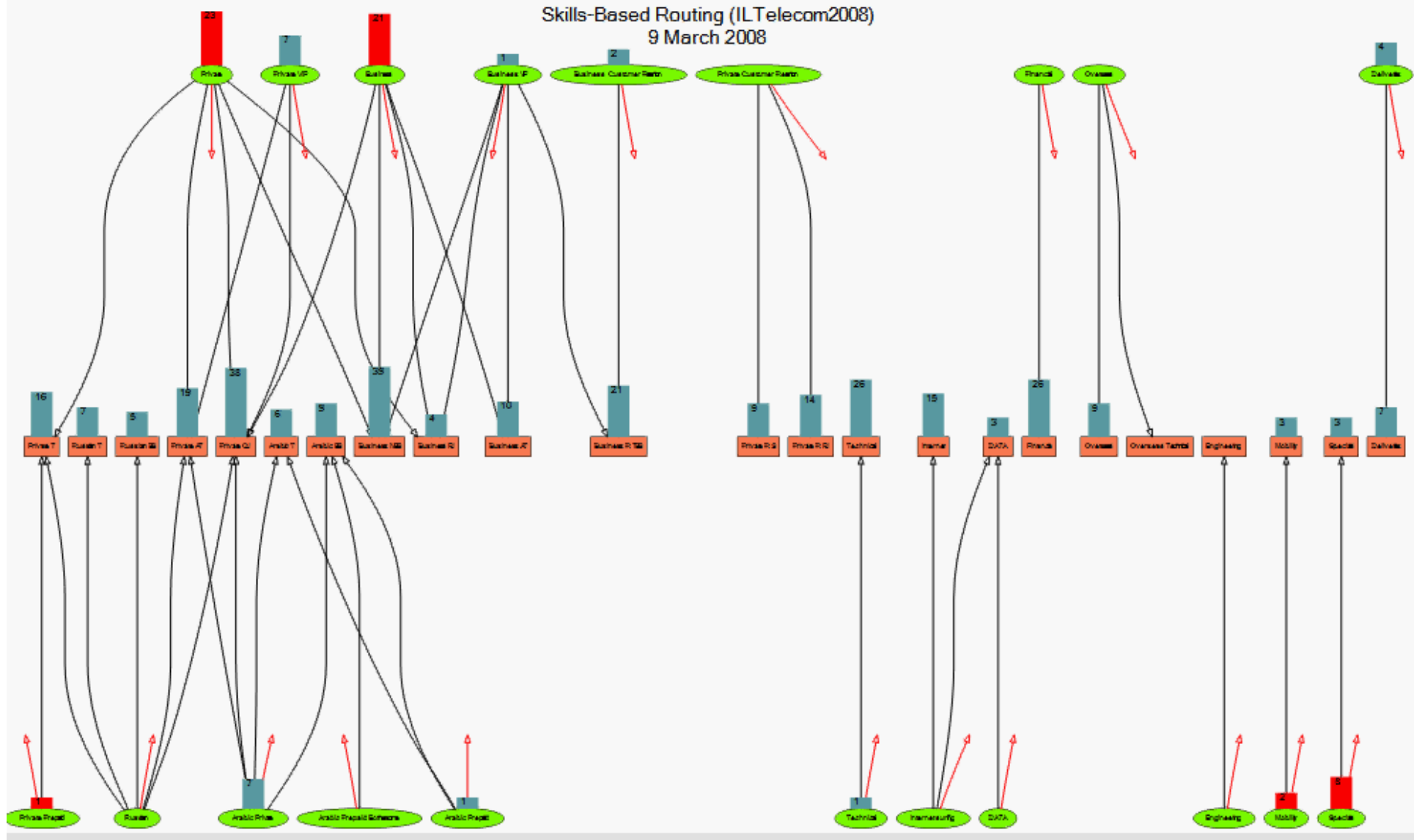


$$\dot{Q}_2(t) = (p_{12}(t)\mu_1 \cdot (Q_1(t) \wedge N_1)) \wedge (N_2 - Q_2(t))^+ - \theta_2 Q_2(t) - \beta_2 Q_2(t) - \mu_2 \cdot (Q_2(t) \wedge N_2)$$

$$\dot{Q}_3(t) = (p_{13}(t)\mu_1 \cdot (Q_1(t) \wedge N_1)) \wedge (N_3 - Q_3(t))^+ - \theta_3 Q_3(t) - \beta_3 Q_3(t) - \mu_3 \cdot (Q_3(t) \wedge N_3)$$

$$\dot{Q}_4(t) = (p_{14}(t)\mu_1 \cdot (Q_1(t) \wedge N_1)) \wedge (N_4 - Q_4(t))^+ - \theta_4 Q_4(t) - \beta_4 Q_4(t) - \mu_4 \cdot (Q_4(t) \wedge N_4).$$

Skills-Based Routing (ILTecom2008)
9 March 2008



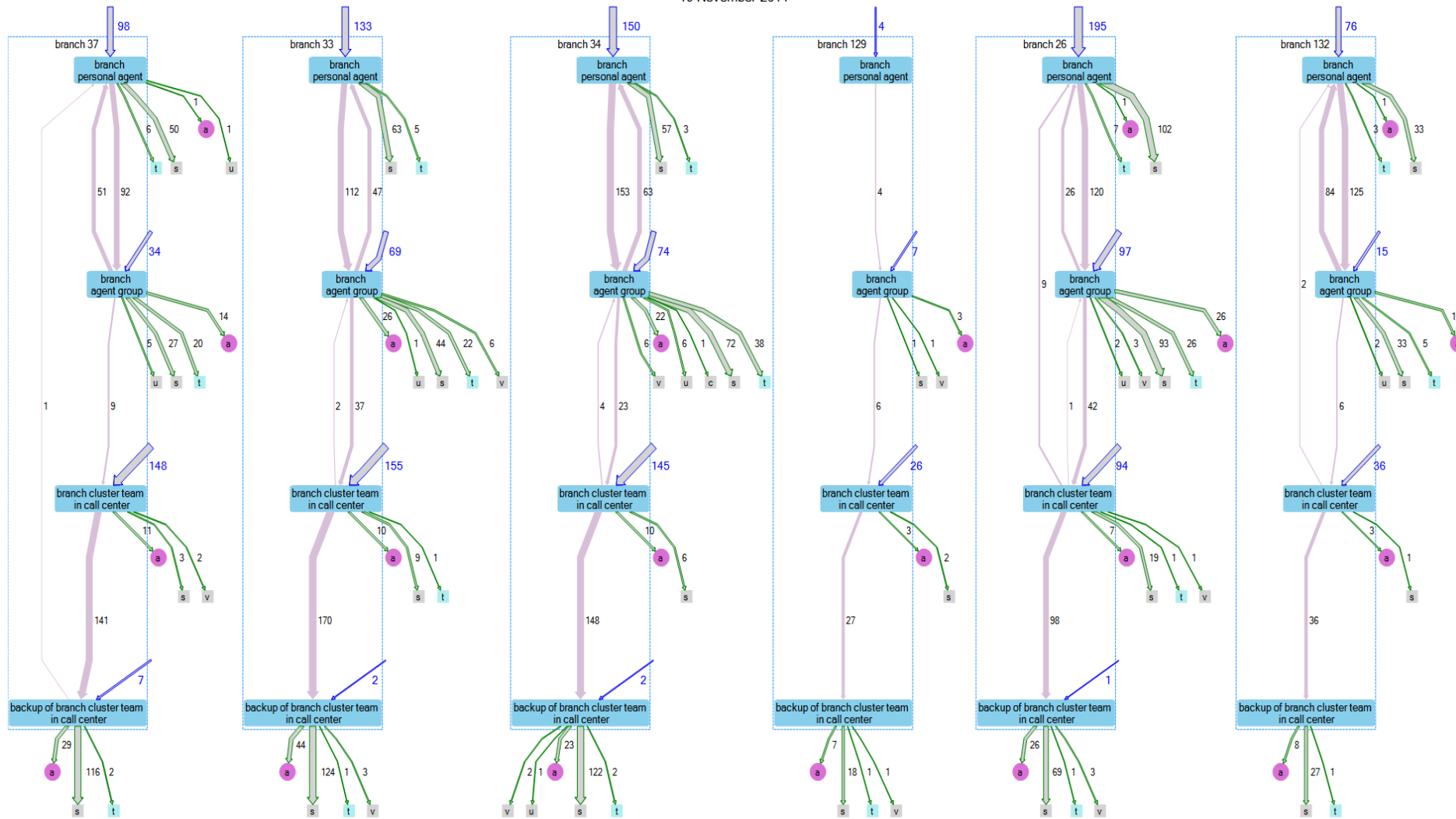
PAUSE 10 : 13 : 00 9 March 2008

Sampling time interval (sec.) 300

Display time interval (millisc.) 300

ILDU Banking: Single Branch-Group-10 Virtual Queues - note the Abandonments

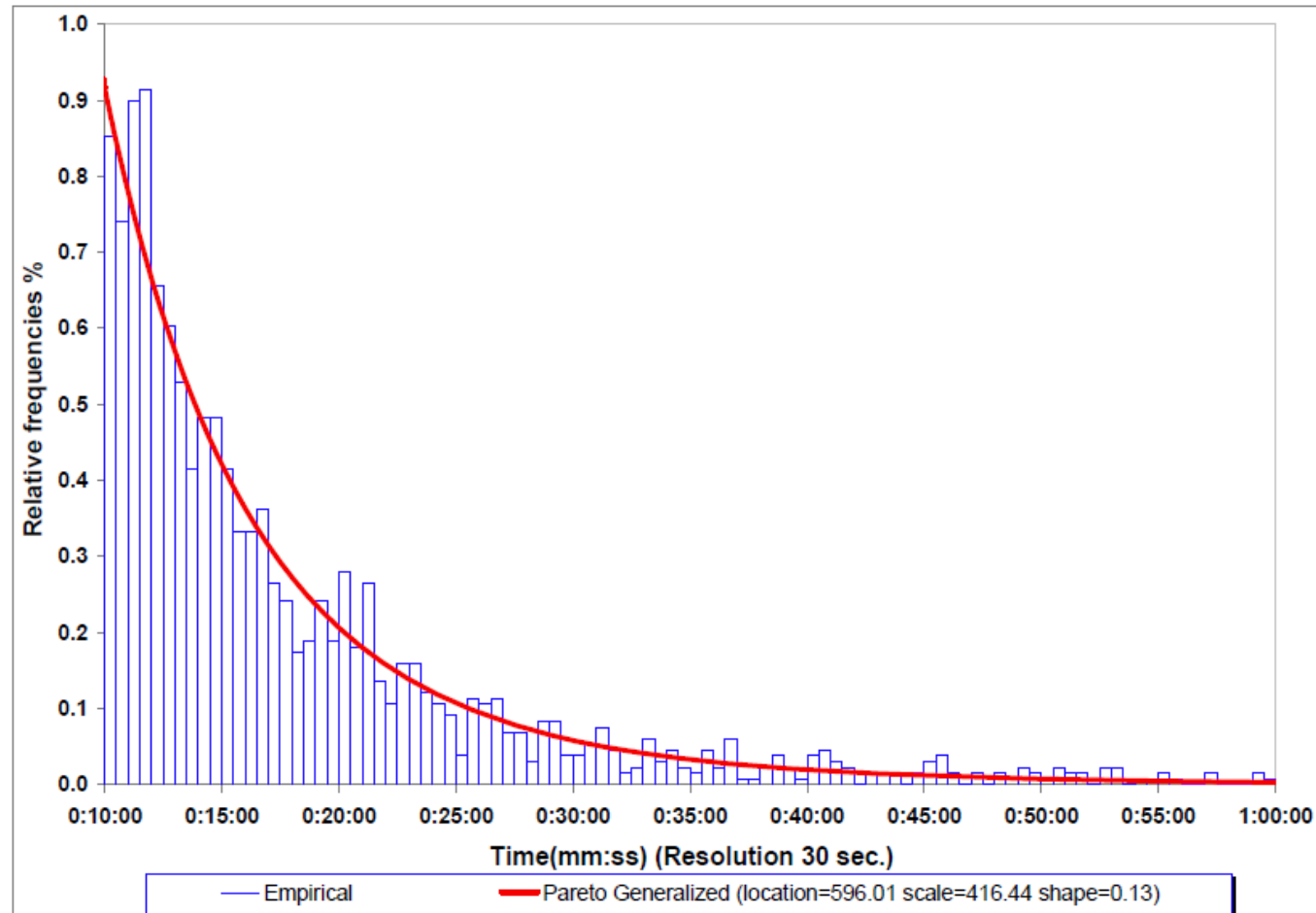
Virtual queues of bank branch-group-10
16 November 2014



Primitives: (Im)Patience

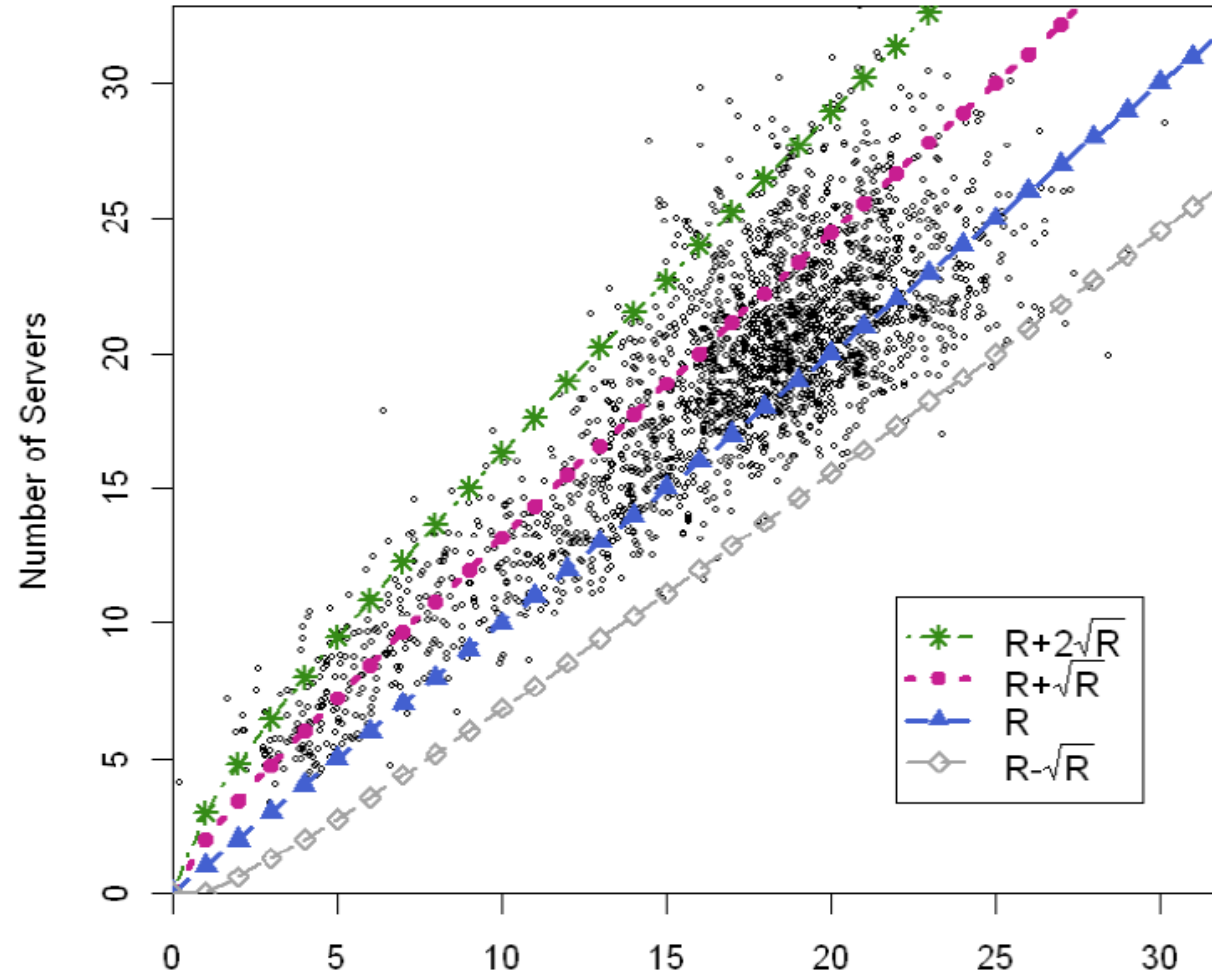
Israeli Bank: Uncensored 13,000 Customers, 24/11/2008

Patience $\geq 10min$: Why Pareto Tail?



Beyond Fluid: #Agents vs. Offered-Load ($N \approx R + \beta\sqrt{R}$)

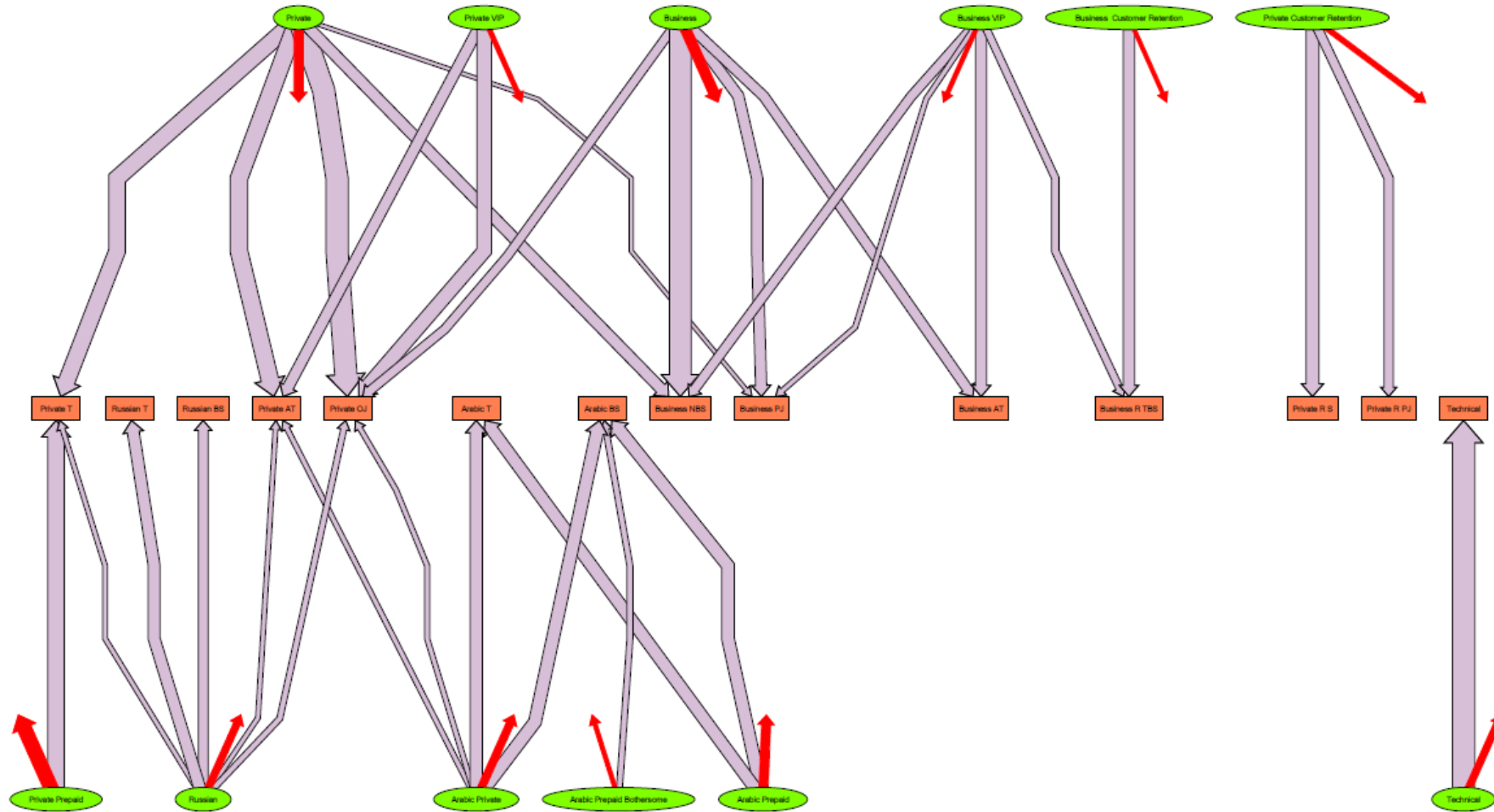
IL Telecom; June-September, 2004 (2205 30min intervals, over 13 weeks, week-days)



e.g. Offered-load $R \stackrel{avg}{=} 5 \text{ calls per min} \times 3.2 \text{ min per call} = 16 \text{ Erlangs}$

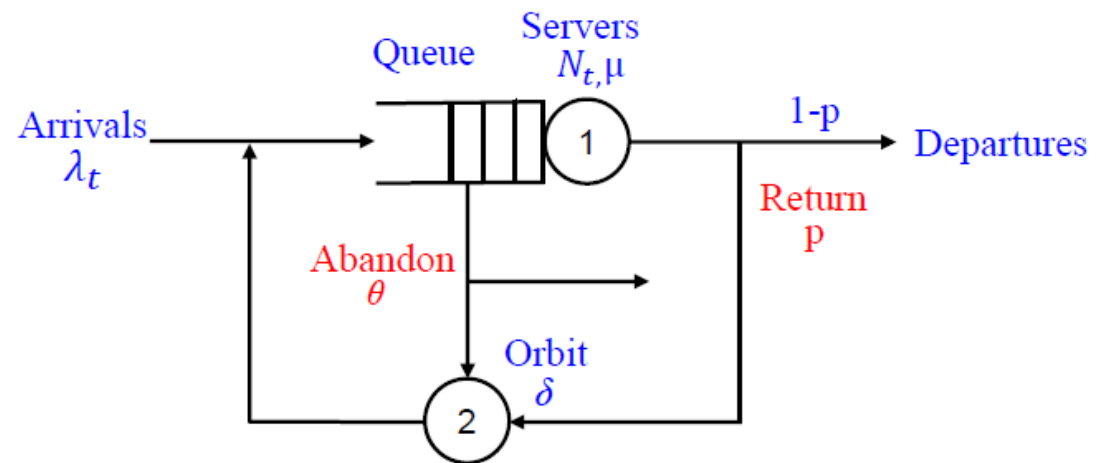
Impatient Customers - Isolate or Aggregate

ILTelecom 9/3/2008



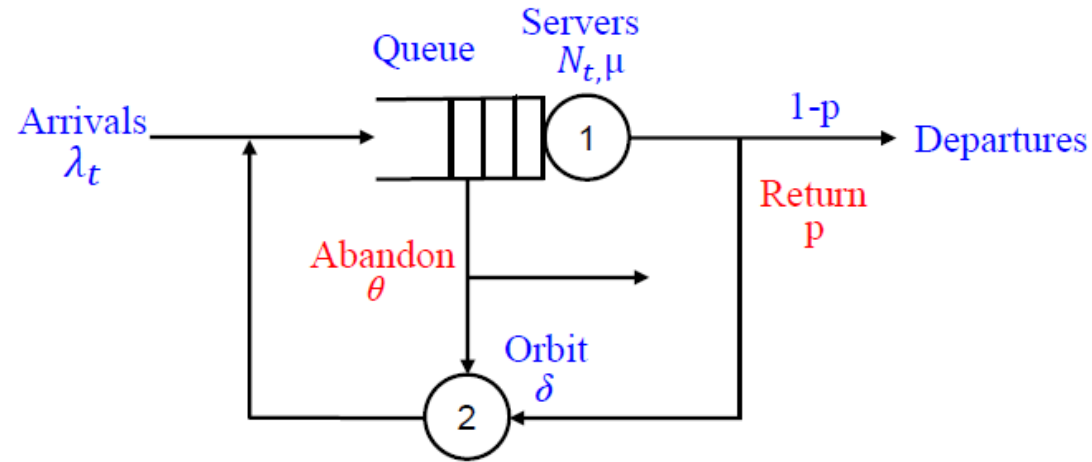
Model Selection: As Simple as Possible but Not Simpler

Service with Retrials and Abandonment; w/ Massey, Reiman, Stolyar



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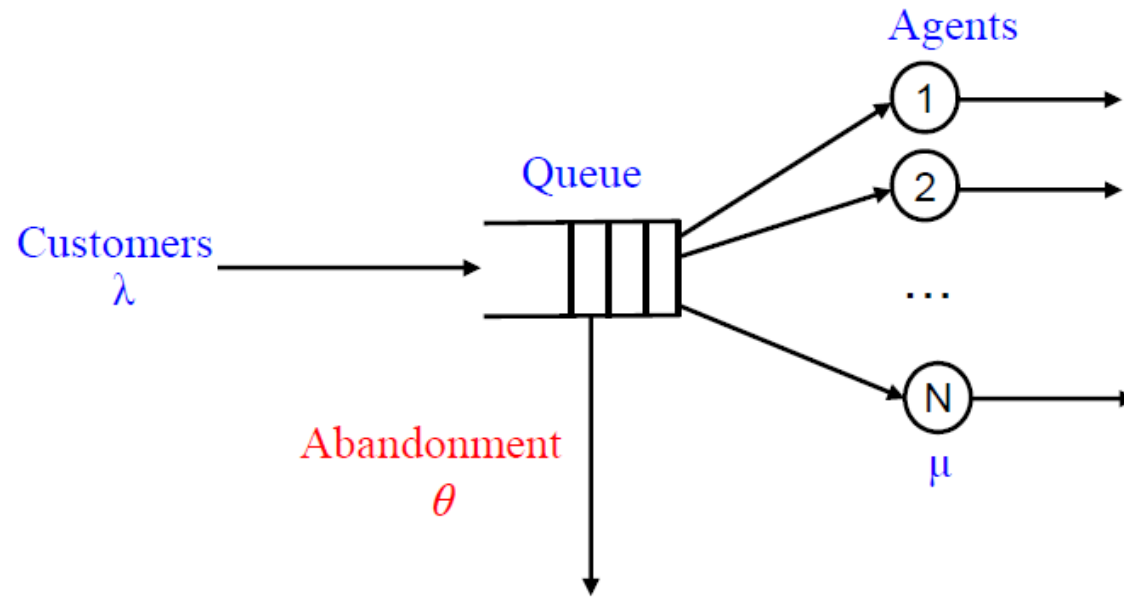


- ▶ Call centers: Visit durations naturally measured in minutes
 - ▶ Arrival rates are "constant" during visit
 - ▶ Returns occur hours after visit

⇒ "Select" Base Model (of 1/2 hour):

Stationary, Abandonment

A Basic Staffing Model: Erlang-A



w/ O. Garnett

“**Birth & Death**” Queue = M/M/N + M (Palm 1940’s):

- ▶ λ – **Arrival** rate (Poisson)
- ▶ μ – **Service** rate (Exponential; $E[S] = \frac{1}{\mu}$)
- ▶ θ – **Patience** rate (Exponential, $E[\text{Patience}] = \frac{1}{\theta}$)
- ▶ N – Number of **Servers** (Agents).

Erlang-A: Is it Relevant?

Experience:

- ▶ Arrival process **not pure Poisson** (time-varying, σ^2 too large)
- ▶ Service times **not Exponential** (typically close to LogNormal)
- ▶ Patience times **not Exponential** (behavior-dependent).

- ▶ Building Blocks need **not be independent** (eg. long wait associated with long service; w/ **M. Reich & Y. Ritov**)
- ▶ Customers and Servers **not homogeneous** (classes, skills):
w/ **R. Atar, G. Shaikhet; R. Atar, I. Gurvich, ...**
- ▶ Customers return for service (after busy, abandonment; dependently:
P. Khudiakov, R. Ghebali, M. Gorfine, P. Feigin)
- ▶ ..., and more.

Question: **Is Erlang-A Relevant?**

Erlang-A: Is it Relevant?

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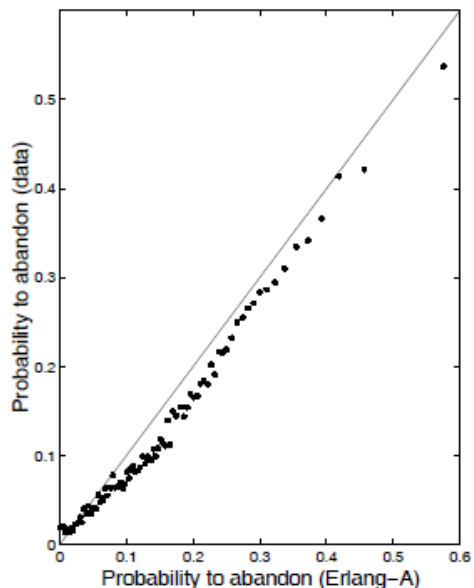
Question: **Is Erlang-A Relevant? Robust enough? YES !**

- ▶ **Practice**: Staffing engine of Work-Force Management software
- ▶ **Theory**: Theoretical engine of Operational Regimes
QD, ED, QED

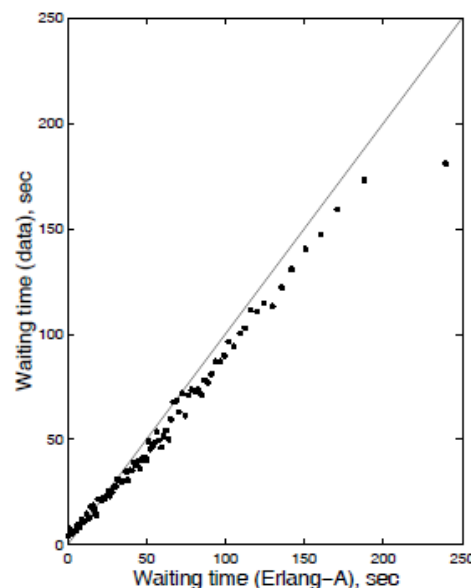
Erlang-A: Fitting a Simple Model to a Complex Reality

Hourly Performance vs. Erlang-A Predictions (1 year)

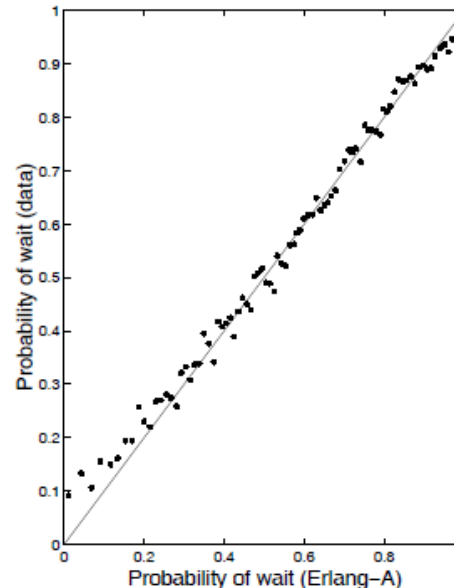
% Abandon



E[Wait]



%{Wait > 0}



- ▶ Empirically-Based & Theoretically-Supported Estimation of (Im)Patience: $\hat{\theta} = P\{Ab\} / E[W_q]$
- ▶ Small Israeli Bank (more examples in progress)
- ▶ Hourly performance vs. Erlang-A predictions, 1 year: aggregated groups of 40 similar hours

Example of a Theorem (QED)

Prerequisite II: Models (Diffusion/QED's Q's)

Traditional Queueing Theory predicts that **Service-Quality** and **Servers' Efficiency** must be traded off against each other.

For example, **M/M/1** (single-server queue): **91%** server's utilization goes with

$$\text{Congestion Index} = \frac{E[\text{Wait}]}{E[\text{Service}]} = \mathbf{10},$$

and only **9%** of the customers are served immediately upon arrival.

Prerequisite II: Models (Diffusion/QED's Q's)

Traditional Queueing Theory predicts that **Service-Quality** and **Servers' Efficiency** must be traded off against each other.

For example, **M/M/1** (single-server queue): **91%** server's utilization goes with

$$\text{Congestion Index} = \frac{E[\text{Wait}]}{E[\text{Service}]} = \mathbf{10},$$

and only **9%** of the customers are served immediately upon arrival.

Yet, heavily-loaded queueing systems with **Congestion Index = 0.1** (Waiting one order of magnitude less than Service) are prevalent:

- ▶ **Call Centers:** Wait "**seconds**" for **minutes** service;
- ▶ **Transportation:** Search "**minutes**" for **hours** parking;

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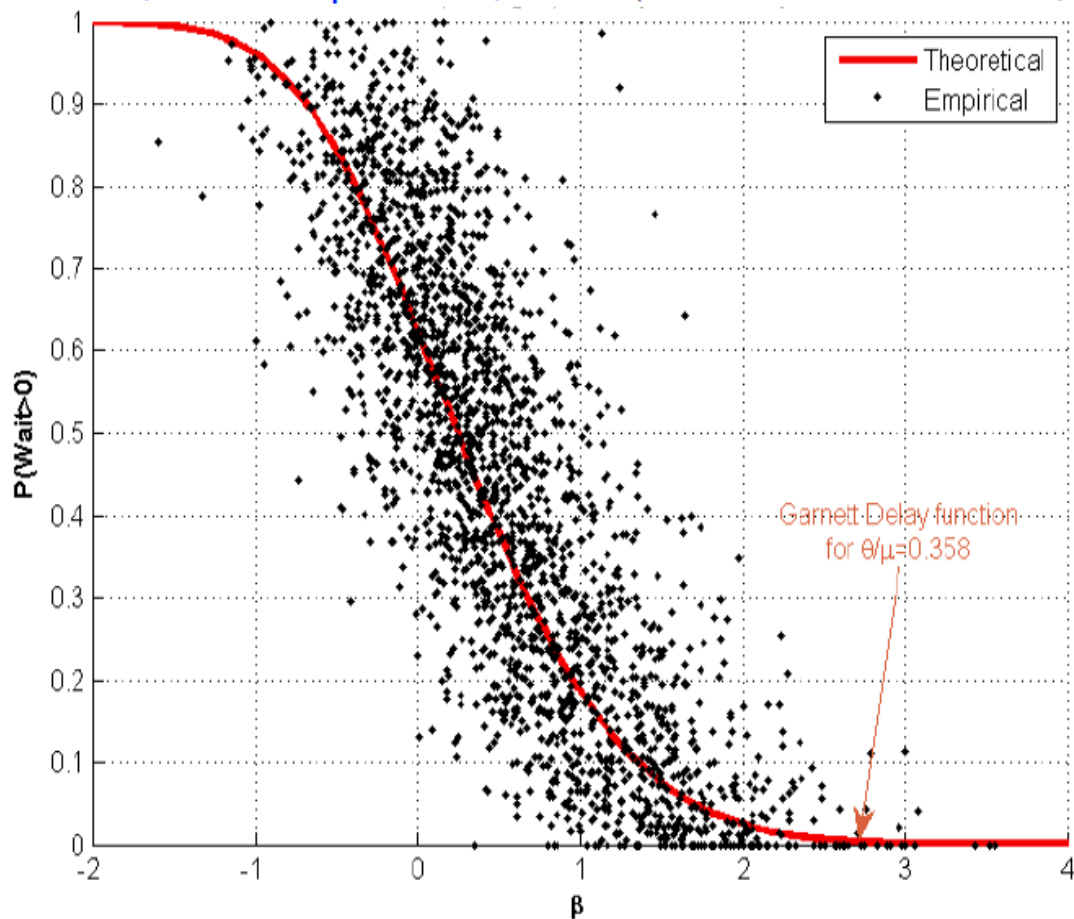
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- ▶ **Call Centers:** Wait "**seconds**" for **minutes** service;
- ▶ **Transportation:** Search "**minutes**" for **hours** parking;
- ▶ **Hospitals:** Wait "**hours**" in ED for **days** hospitalization in IW's;

and, moreover, a significant fraction are not delayed in queue. (For example, in well-run call-centers, **50%** served "immediately", along with over **90%** agents' utilization, is not uncommon) **?** **QED**

Erlang-A Value: DNet $P(W_q > 0)$ vs. Data

IL Telecom; June-September, 2004 (2205 30min intervals, weekdays)



- Approximations, w/ Patience $\approx 3 \times$ Service-Duration ($\mu/\theta \approx 3$)

QED Theory (Erlang '13; Halfin & Whitt '81; w/Garnett & Reiman '02)

Consider a sequence of **steady-state** M/M/ N + M queues, $N = 1, 2, 3, \dots$
Then the following points of view are **equivalent**, as $N \uparrow \infty$:

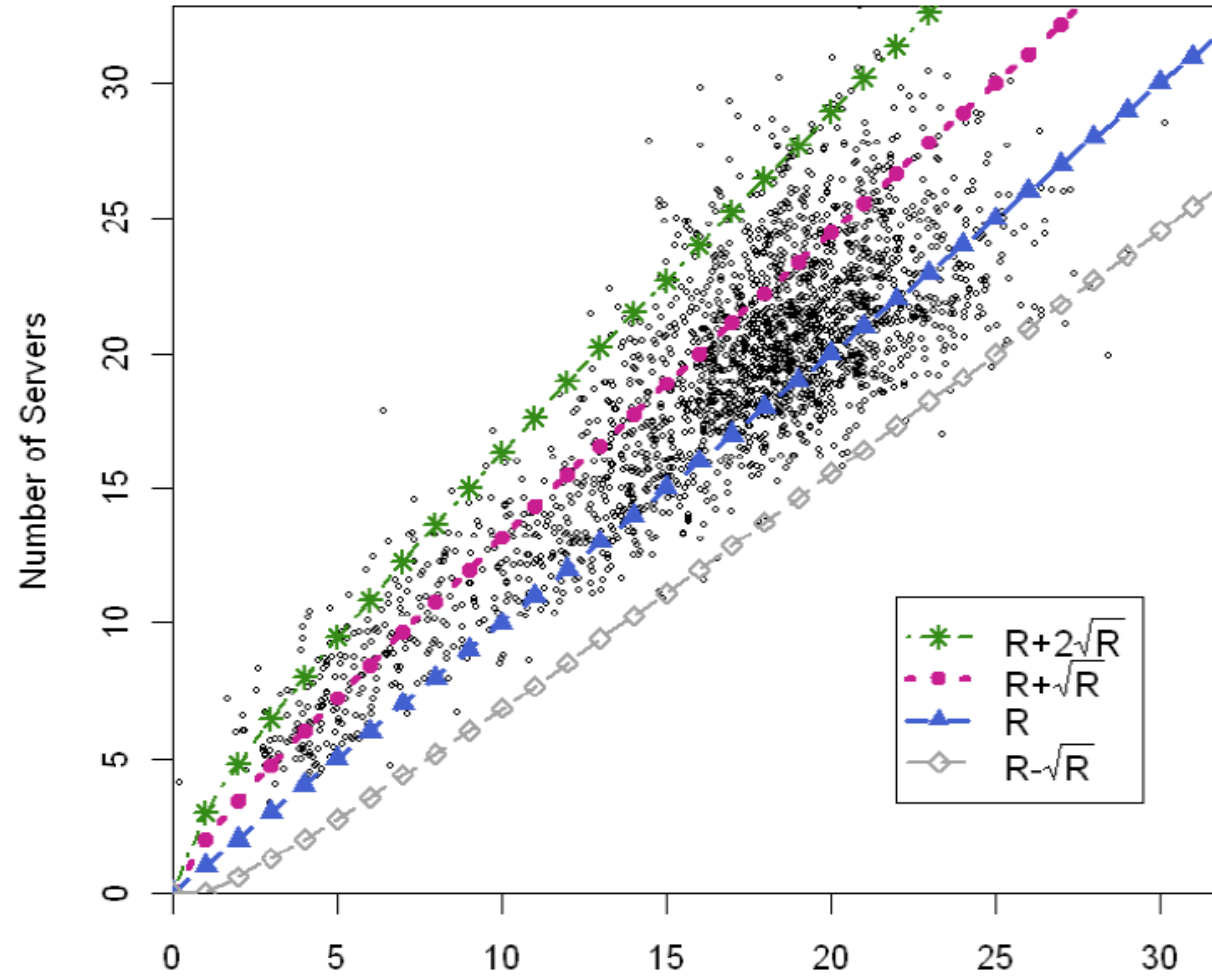
- **QED** $\% \{ \text{Cust Wait} > 0 \} \approx \alpha, \quad 0 < \alpha < 1;$
 or $\% \{ \text{Serv Idle} > 0 \} \approx 1 - \alpha$
- **Customers** $\{ \text{Abandon} \} \approx \frac{\gamma}{\sqrt{N}}, \quad 0 < \gamma;$
- **Agents** $\text{OCC} \approx 1 - \frac{\beta + \gamma}{\sqrt{N}} \quad -\infty < \beta < \infty;$
- **Managers** $N \approx R + \beta \sqrt{R}, \quad R = \lambda \times E(S) \quad \text{not small};$

Here $R = \text{Offered Load}$

eg. $R = 25 \text{ call/min.} \times 4 \text{ min./call} = 100$

Beyond Fluid: #Agents vs. Offered-Load ($N \approx R + \beta\sqrt{R}$)

IL Telecom; June-September, 2004 (2205 30min intervals, over 13 weeks, week-days)



e.g. Offered-load $R \stackrel{avg}{=} 5 \text{ calls per min} \times 3.2 \text{ min per call} = 16 \text{ Erlangs}$

Erlang-A: QED Approximations (Examples)

Assume **Offered Load** R not small ($\lambda \rightarrow \infty$).

Let $\hat{\beta} = \beta \sqrt{\frac{\mu}{\theta}}$, $h(\cdot) = \frac{\phi(\cdot)}{1 - \Phi(\cdot)}$ = hazard rate of $\mathcal{N}(0, 1)$.

► **Delay Probability:**

$$P\{W_q > 0\} \approx \left[1 + \sqrt{\frac{\theta}{\mu}} \cdot \frac{h(\hat{\beta})}{h(-\beta)} \right]^{-1}.$$

► **Probability to Abandon:**

$$P\{\text{Ab} | W_q > 0\} \approx \frac{1}{\sqrt{N}} \cdot \sqrt{\frac{\theta}{\mu}} \cdot [h(\hat{\beta}) - \hat{\beta}].$$

► **$P\{\text{Ab}\} \propto E[W_q]$** , both order $\frac{1}{\sqrt{N}}$:

$$\frac{P\{\text{Ab}\}}{E[W_q]} = \theta \quad (\approx g(0) > 0).$$

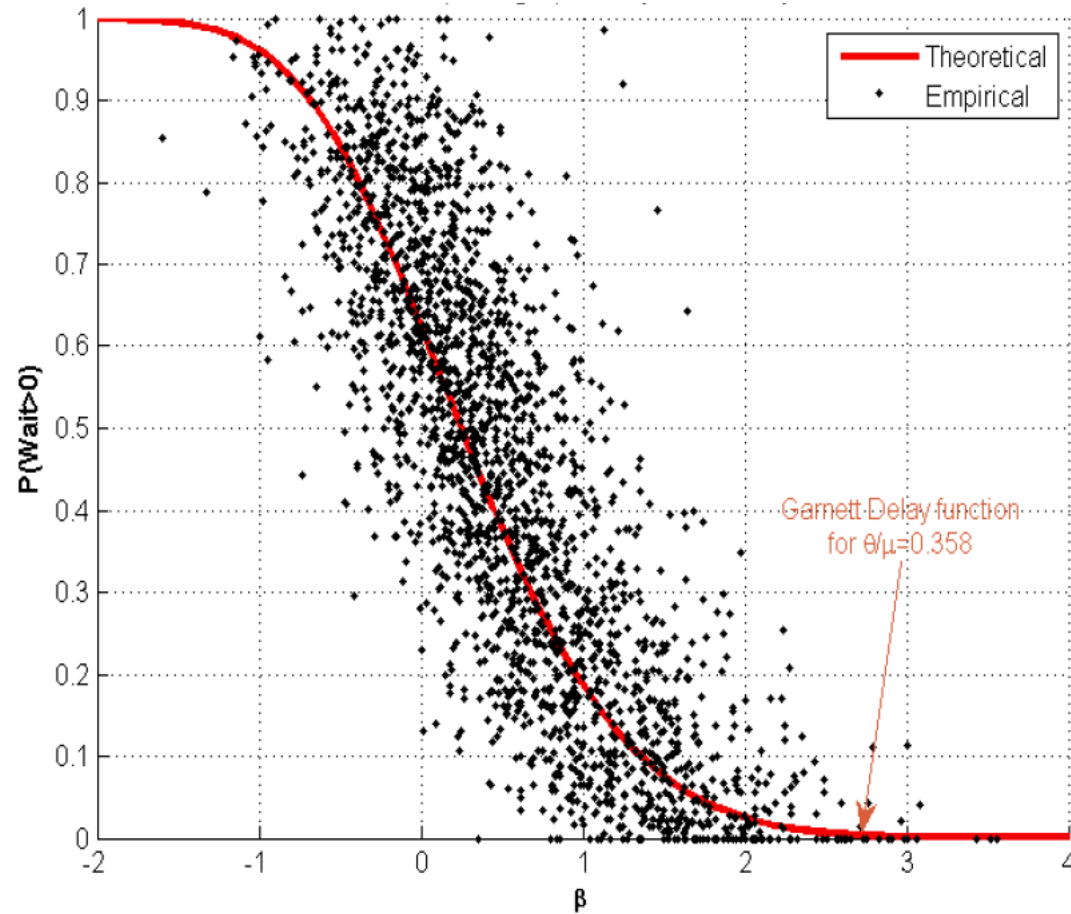
Asymptotic Landscape: 9 Operational Regimes, and then some Erlang-A, w/ I. Gurvich & J. Huang

Erlang-A μ & θ fixed	Conventional scaling			Many-Server scaling			NDS scaling		
	Sub	Critical	Over	QD	QED	ED	Sub	Critical	Over
Offered load per server	$\frac{1}{1+\delta}$	$1 - \frac{\beta}{\sqrt{n}}$	$\frac{1}{1-\gamma}$	$\frac{1}{1+\delta}$	$1 - \frac{\beta}{\sqrt{n}}$	$\frac{1}{1-\gamma}$	$\frac{1}{1+\delta}$	$1 - \frac{\beta}{n}$	$\frac{1}{1-\gamma}$
Arrival rate λ	$\frac{\mu}{1+\delta}$	$\mu - \frac{\beta}{\sqrt{n}}\mu$	$\frac{\mu}{1-\gamma}$	$\frac{n\mu}{1+\delta}$	$n\mu - \beta\mu\sqrt{n}$	$\frac{n\mu}{1-\gamma}$	$\frac{n\mu}{1+\delta}$	$n\mu - \beta\mu$	$\frac{n\mu}{1-\gamma}$
# servers	1			n			n		
Time-scale	n			1			n		
Impatience rate	θ/n			θ			θ/n		
Staffing level	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu}(1 + \frac{\beta}{\sqrt{n}})$	$\frac{\lambda}{\mu}(1-\gamma)$	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu} + \beta\sqrt{\frac{\lambda}{\mu}}$	$\frac{\lambda}{\mu}(1-\gamma)$	$\frac{\lambda}{\mu}(1+\delta)$	$\frac{\lambda}{\mu} + \beta$	$\frac{\lambda}{\mu}(1-\gamma)$
Utilization	$\frac{1}{1+\delta}$	$1 - \sqrt{\frac{\theta}{\mu}} \frac{h(\hat{\beta})}{\sqrt{n}}$	1	$\frac{1}{1+\delta}$	$1 - \sqrt{\frac{\theta}{\mu}} \frac{h(\hat{\beta})}{\sqrt{n}}$	1	$\frac{1}{1+\delta}$	$1 - \sqrt{\frac{\theta}{\mu}} \frac{h(\hat{\beta})}{n}$	1
$E(Q)$	$\frac{1}{\delta(1+\delta)}$	$\sqrt{n}g(\hat{\beta})$	$\frac{n\mu\gamma}{\theta(1-\gamma)}$	$\frac{1}{\delta}\varrho_n$	$\sqrt{n}g(\hat{\beta})\alpha$	$\frac{n\mu\gamma}{\theta(1-\gamma)}$	$o(1)$	$ng(\hat{\beta})$	$\frac{n^2\mu\gamma}{\theta(1-\gamma)}$
$P(Ab)$	$\frac{1}{n} \frac{1}{\delta} \frac{\theta}{\mu}$	$\frac{\theta}{\sqrt{n\mu}}g(\hat{\beta})$	γ	$\frac{1}{n} \frac{(1+\delta)}{\delta} \frac{\theta}{\mu} \varrho_n$	$\frac{\theta}{\sqrt{n\mu}}g(\hat{\beta})\alpha$	γ	$o(\frac{1}{n^2})$	$\frac{\theta}{n\mu}g(\hat{\beta})$	γ
$P(W_q > 0)$	$\frac{1}{1+\delta}$	≈ 1		ϱ_n	$\alpha \in (0, 1)$	≈ 1	≈ 0	≈ 1	
$P(W_q > T)$	$\frac{1}{1+\delta} e^{-\frac{\delta}{1+\delta}\mu T}$	$1 + O(\frac{1}{\sqrt{n}})$	$1 + O(\frac{1}{n})$	≈ 0		$f(T)$	≈ 0	$\frac{\bar{\Phi}(\hat{\beta} + \sqrt{\theta\mu}T)}{\bar{\Phi}(\hat{\beta})}$	$1 + O(\frac{1}{n})$
Congestion $\frac{EW_q}{ES}$	$\frac{1}{\delta}$	$\sqrt{n}g(\hat{\beta})$	$n\mu\gamma/\theta$	$\frac{1}{n} \frac{(1+\delta)}{\delta} \varrho_n$	$\frac{\alpha}{\sqrt{n}}g(\hat{\beta})$	$\frac{\mu\gamma}{\theta}$	$o(\frac{1}{n})$	$g(\hat{\beta})$	$n\mu\gamma/\theta$

- ▶ Conventional: Ward & Glynn (03, $G/G/1 + G$)
- ▶ Many-Server:
 - ▶ QED: Halfin-Whitt (81), w/ Garnett & Reiman (02)
 - ▶ ED: Whitt (04)
 - ▶ NDS: Atar (12)
- ▶ “Missing”: ED+QED; Hazard-rate scaling (M/M/N+G); Time-Varying, Non-Parametric; Moderate- and Large-Deviation; Networks (multi-regimes)

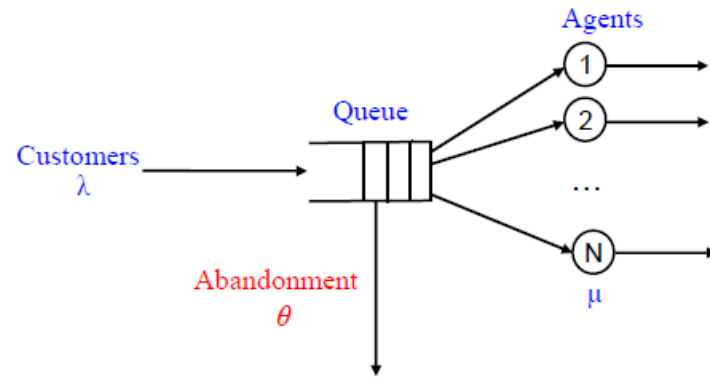
$P(W_q > 0)$, or "Universalizing" the QED regime

IL Telecom; June-September, 2004



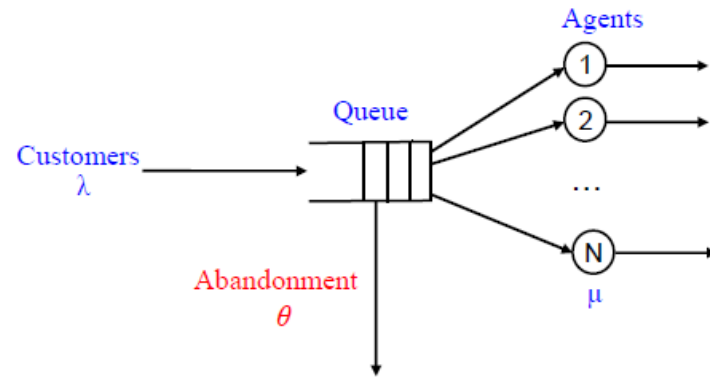
- ▶ 2205 half-hour intervals (13 summer weeks, week-days)
- ▶ Erlang-A approximations for the appropriate $\mu/\theta \approx 3$

Universal Approximations: Erlang-A (M/M/N + M)



w/ I. Gurvich & J. Huang

Universal Approximations: Erlang-A (M/M/N + M)

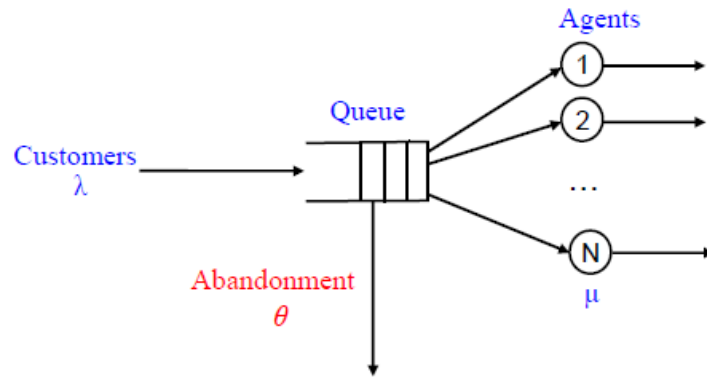


w/ I. Gurvich & J. Huang

- **QNet:** Birth & Death Queue, with B - D rates

$$F(q) = \lambda - \mu \cdot (q \wedge n) - \theta \cdot (q - n)^+, \quad q = 0, 1, \dots$$

Universal Approximations: Erlang-A (M/M/N + M)



w/ I. Gurvich & J. Huang

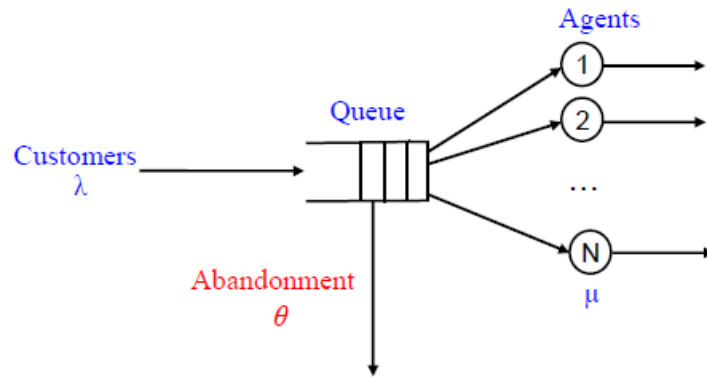
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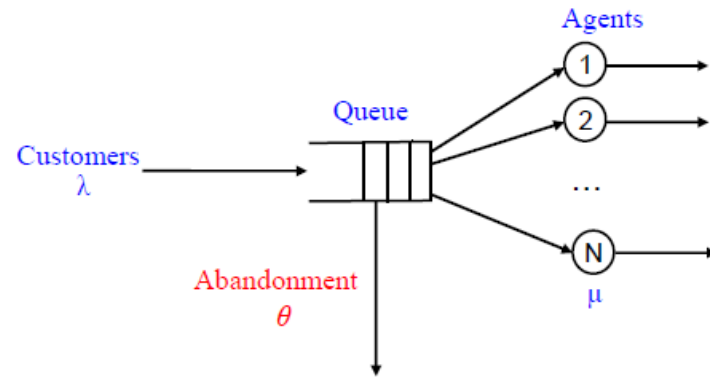
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Universal Approximations: Erlang-A (M/M/N + M)



w/ I. Gurvich & J. Huang

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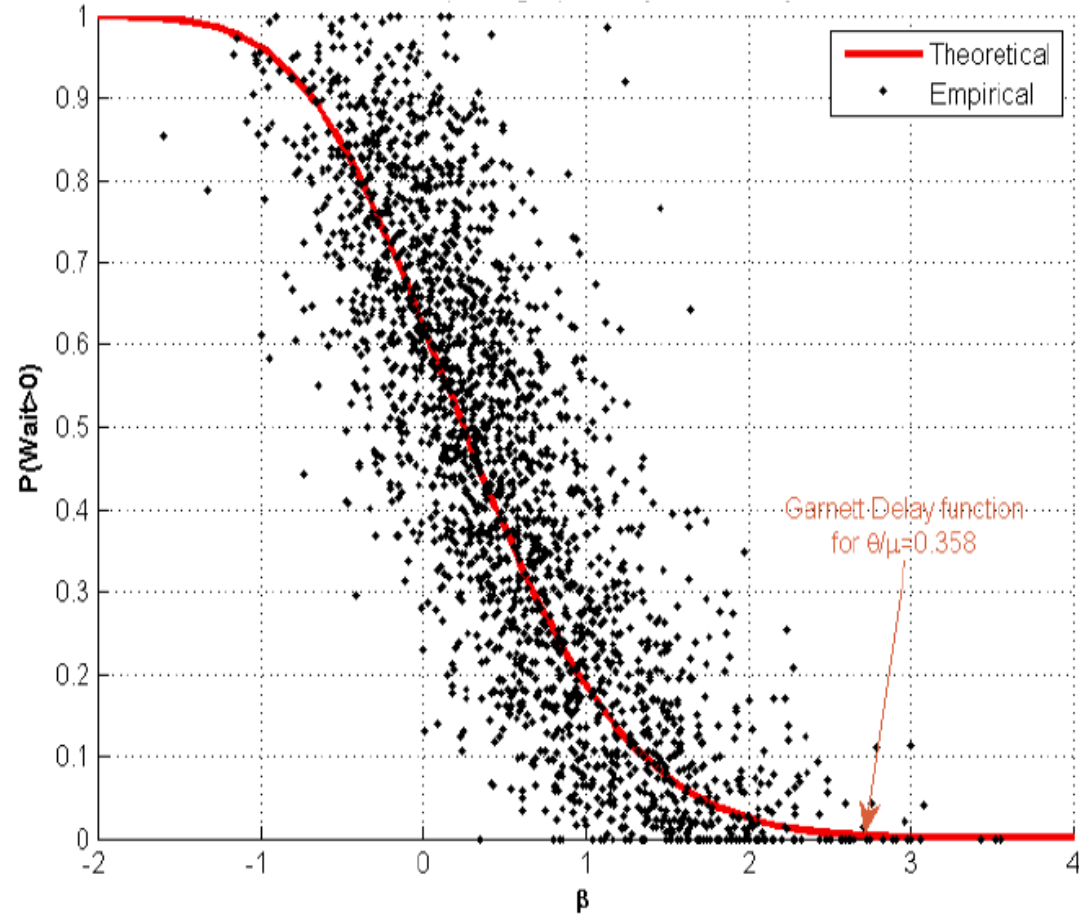
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eg. $\mu = \theta$: $\dot{x} = \lambda - \mu \cdot x$, $Y = \text{OU process}$

Parsimonious (Tractable, Robust), Accurate, Valuable

Erlang-A Value: DNet $P(W_q > 0)$ vs. Data

IL Telecom; June-September, 2004 (2205 30min intervals, weekdays)



- Approximations, w/ Patience $\approx 3 \times$ Service-Duration ($\mu/\theta \approx 3$)

Accuracy: DNet vs. QNet

- ▶ Δ^λ is the “balancing” state, obtained by solving

$$\lambda = \mu(n \wedge \Delta^\lambda) + \theta(\Delta^\lambda - n)^+.$$

Solution: $\Delta^\lambda = \frac{\lambda}{\mu} - \left(\frac{\lambda}{\mu} - n\right)^+ \left(1 - \frac{\mu}{\theta}\right).$

Specifically: **QD** = $\frac{\lambda}{\mu}$; **ED** = $n + \frac{1}{\theta}(\lambda - n\mu)$; **QED** = $n + \mathcal{O}(\sqrt{\lambda})$

- ▶ Centered processes (excursions):

$$\tilde{Q}^\lambda(\cdot) = Q(\cdot) - \Delta^\lambda, \quad \tilde{Y}^\lambda(\cdot) = Y(\cdot) - \Delta^\lambda.$$

Theorem: For f bounded by an m -degree polynomial ($m \geq 0$):

$$\mathbb{E}f(\tilde{Q}^\lambda(\infty)) - \mathbb{E}f(\tilde{Y}^\lambda(\infty)) = \mathcal{O}(\sqrt{\lambda}^{m-1}).$$

- ▶ **Accurate:** more than heavy-traffic *limits*

Simplicity: Why 2λ ?

- ▶ Semi-martingale representation of the B&D process:
Fluid + Martingale
- ▶ Predictable quadratic variation:

$$\int_0^t [\lambda + \mu(Q_s \wedge n) + \theta(Q_s - n)^+] ds$$

- ▶ In **steady-state**, arrival rate \equiv departure rate:

$$\lambda = \mathbb{E}[\mu(Q_s \wedge n) + \theta(Q_s - n)^+]$$

- ▶ Expectation of the predictable quadratic variation:

$$\mathbb{E} \int_0^t [\lambda + \mu(Q_s \wedge n) + \theta(Q_s - n)^+] ds = 2\lambda t$$

- ▶ **Simple** \Rightarrow **Tractable, Robust**: $d\text{Martingale}_t \approx \sqrt{2\lambda} \cdot d\text{Brownian}_t$

Reconciling Time-Varying and Steady-State Models

- ▶ **Rigid** (fixed) staffing level during a time-varying shift:
Doomed to alternate between overloading and underloading
- ▶ **Flexible** staffing:
Can design **time-varying staffing** that achieves, **at all times,**
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via Square-Root Staffing (Modified Offered-Load)

Reconciling Time-Varying and Steady-State Models

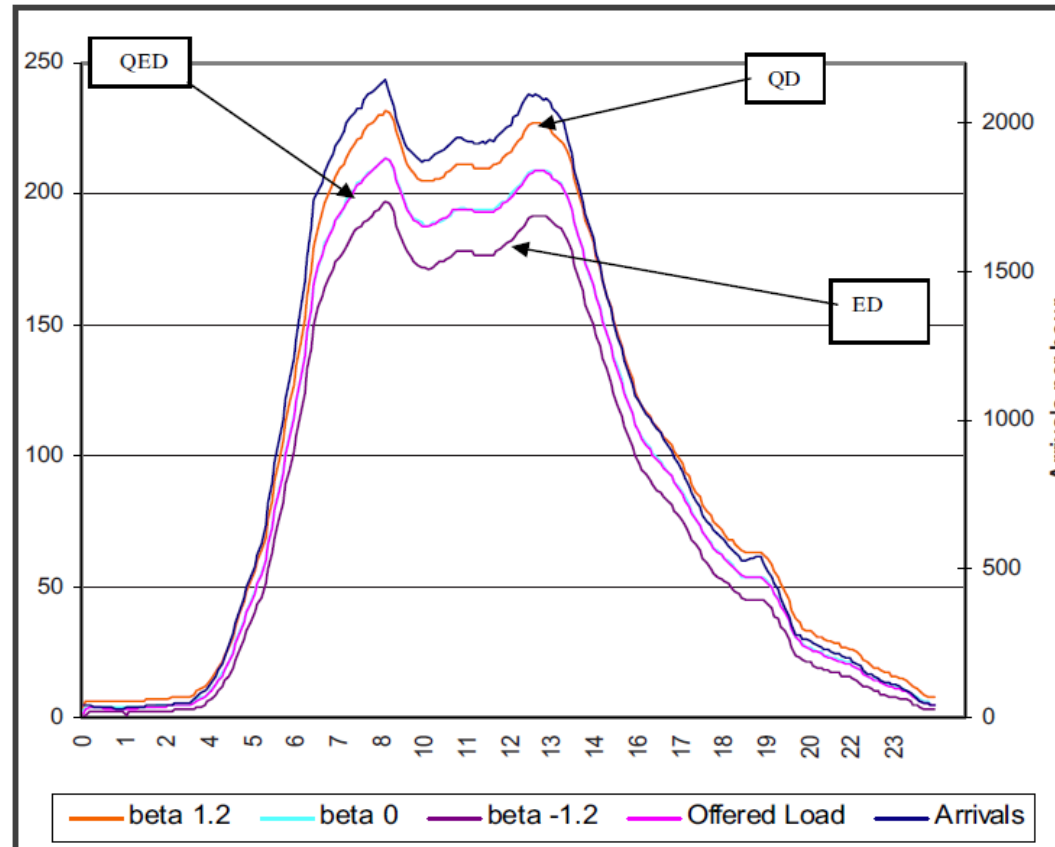
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via Square-Root Staffing (Modified Offered-Load)
- ▶ **History:**
 - ▶ Jennings, M., Reiman, Whitt (1996): Emergence of the phenomenon, with infinite-server heuristics
 - ▶ Feldman, M., Massey, Whitt (2008): Stabilize delay probability with QED staffing, with little theory
 - ▶ Liu and Whitt (2012): Stabilize abandonment probability, with ED theory
 - ▶ w/ Huang, Gurvich (ongoing): QED theory

Why Does Erlang-A Work? Time-Varying Arrival Rates

Square-Root Staffing: $N_t = R_t + \beta\sqrt{R_t}$, $-\infty < \beta < \infty$

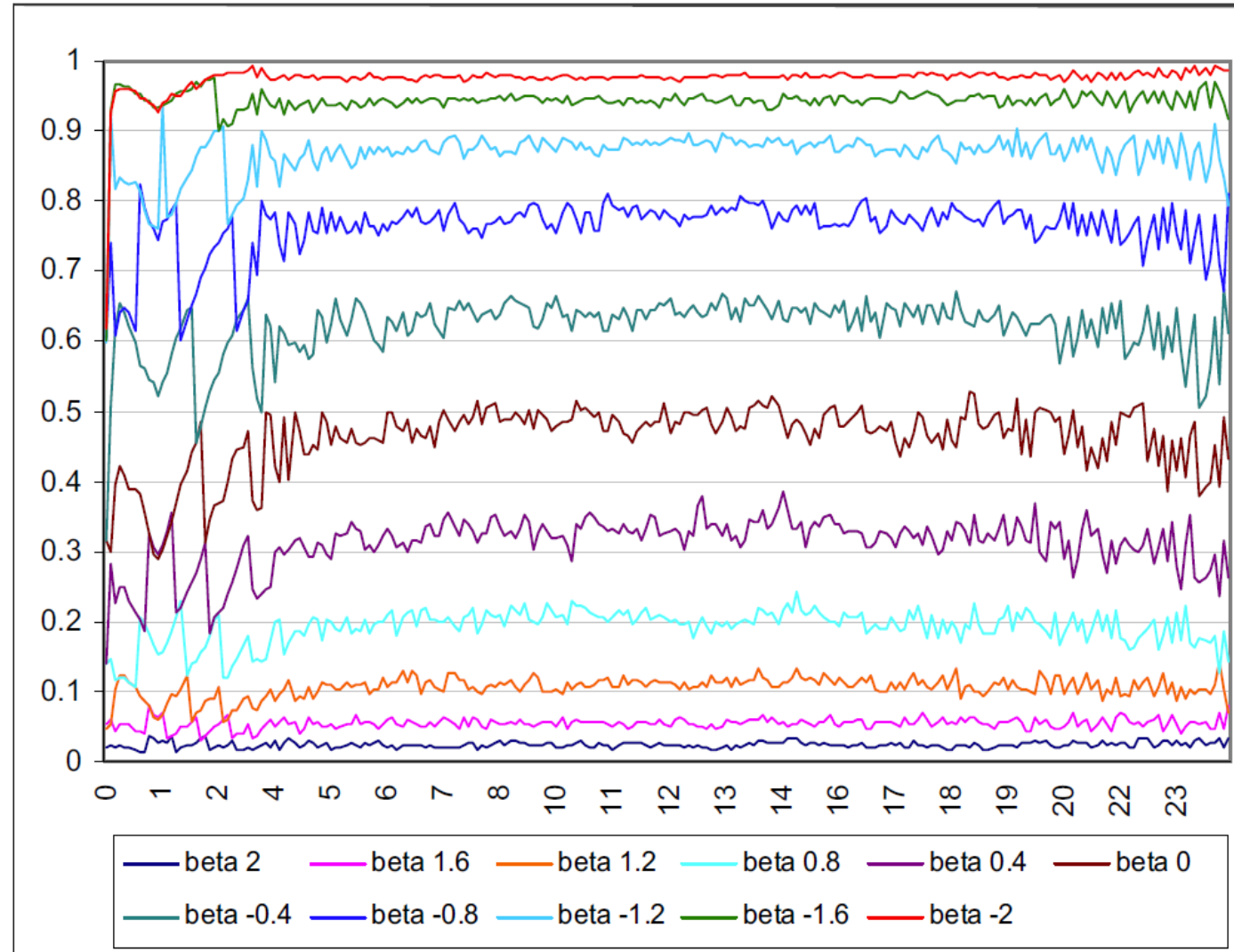
What is R_t , the **Offered-Load** at time t ? ($R_t \neq \lambda_t \times E[S]$)

Arrivals, Offered-Load and Staffing



Time-Stable Performance of Time-Varying Systems

Delay Probability = As in the **Stationary Erlang-A** (Garnett)



Calculating the Offered-Load $R(t)$, Theoretically

- ▶ Offered-Load Process: $L(\cdot) =$ **Least** number of **servers** that guarantees **no delay**.
- ▶ **Offered-Load** Function $R(t) = E[L(t)]$, $t \geq 0$.
Think $M_t/G/N_t^? + G$ vs. $M_t/G/\infty$: **Ample-Servers**.

Four (all useful) representations, capturing “**workload before t**”:

$$\begin{aligned} R(t) = E[L(t)] &= \int_{-\infty}^t \lambda(u) \cdot P(S > t - u) du = E \left[A(t) - A(t - S) \right] = \\ &= E \left[\int_{t-S}^t \lambda(u) du \right] = E[\lambda(t - S_e)] \cdot E[S] \approx \dots \end{aligned}$$

- ▶ $\{A(t), t \geq 0\}$ Arrival-Process, rate $\lambda(\cdot)$;
- ▶ S (S_e) generic Service-Time (Residual Service-Time).
- ▶ Relating L, λ, S (“ W ”): **Time-Varying Little’s Formula**.
Stationary models: $\lambda(t) \equiv \lambda$ then $R(t) \equiv \lambda \times E[S]$.

Abandonment: Further Applications

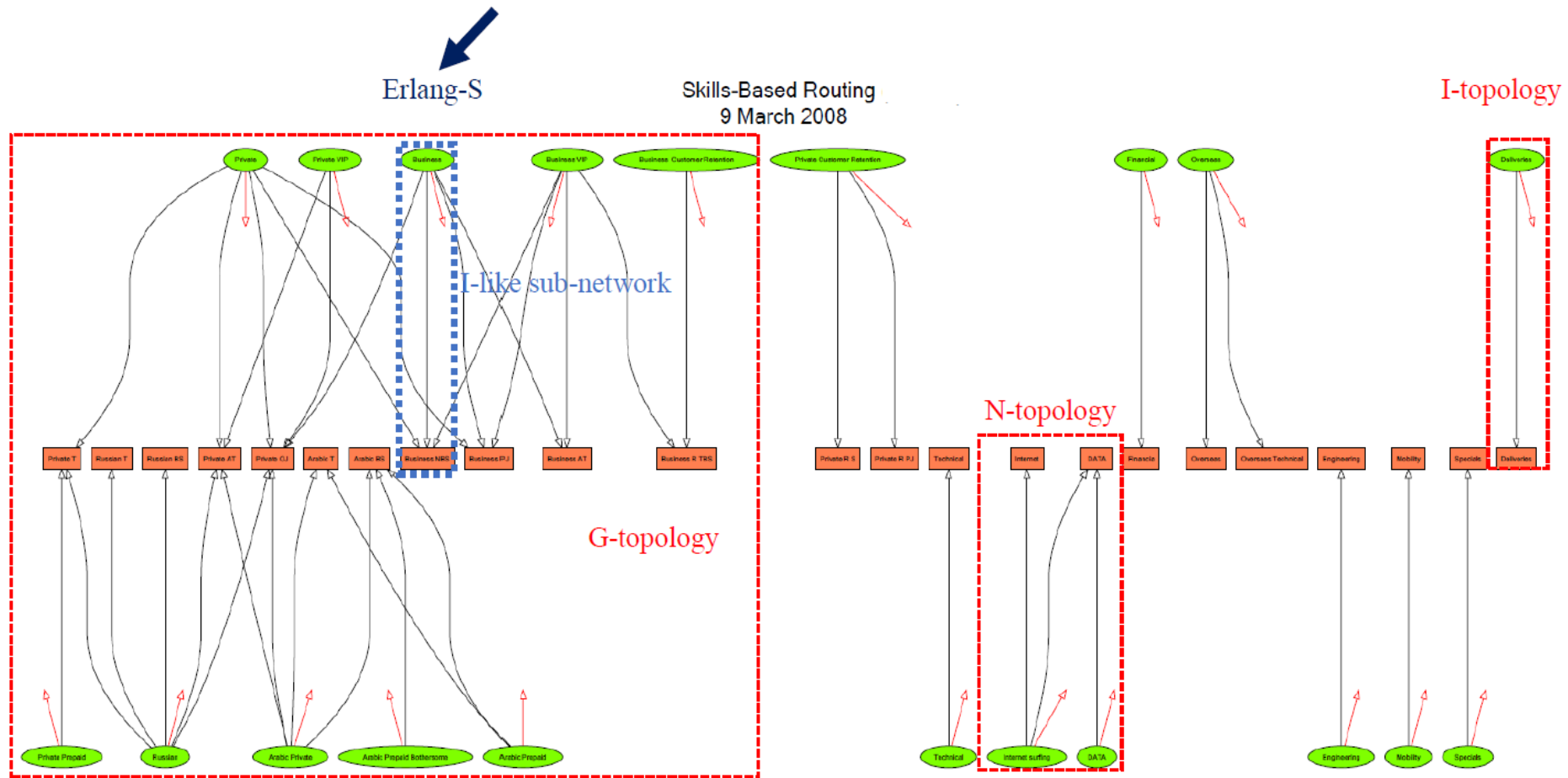
- **Personalized Queueing:** e.g. when Impatience has been estimated (personalized), exactly or approximately, choosing who to serve next will benefit from taking this information into account (e.g. Shortest-Patience-First); **w/ P. Momcilovic**
- **CCs:** Control of congestion via announcement / information that monitors the queue, **w/ Junfei Huang, Hanqin Zhang, Jiheng Zhang**
- **IVRs** (Self-Services): identify via Mixture Fitting, **w/ N. Carmeli, H. Kaspi**
- **EDs:** Left-Without-Being-Seen. Theory exists if All-Unknown – Current-Status Data. Otherwise, namely when having both Right-Censoring and Current-Status data, theory developed **w/ Y. Yefenof, Y. Goldberg, Y. Ritov**
- **Chats:** Before or within session (registered vs. silent abandonment)

Laws & Models: Data-Based Erlang A/R/S, following B/C

- Little's Law (Steady-State, Transient), State-Space Collapse,...
- Erlang-B (**Blocking**) and Erlang-C (?)
 - **Erlang**, Agner Krarup: Queueing Theory was born in 1909, in his paper "The Theory of Probabilities and **Telephone Conversations**"
- 2. Erlang-A
 - **Abandonment**: While waiting for service, does service-value dominate residual-wait-cost
- 1. Erlang-R
 - **Return/Feedback**: Customers often return to service (positively, negatively, just needing)
- 3. Erlang-S
 - **Servers**: Challenging to manage, and model, no less so than customers

Above: Simple (Parsimonious) models of complex realities, yet not too simple (Robust)

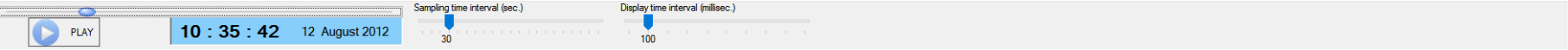
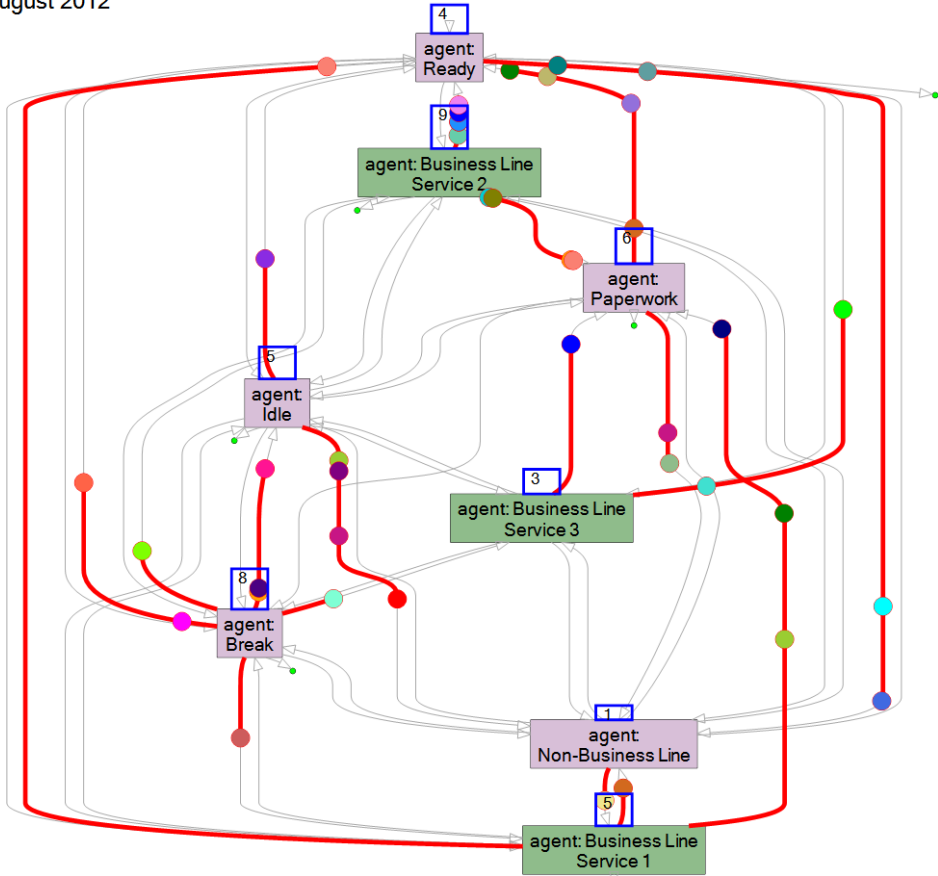
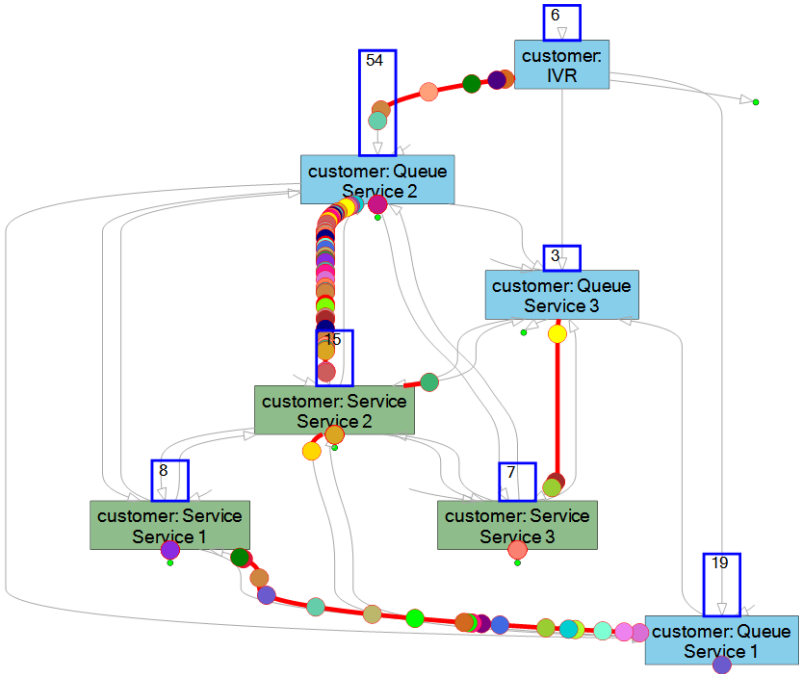
Erlang-S: Servers in Qnets – Aggregate or Zoom w/ D. Azriel, P. Feigin



Topology of a call center:
 Server-queues are in the **rectangles** and customer-queues are in the **ovals**

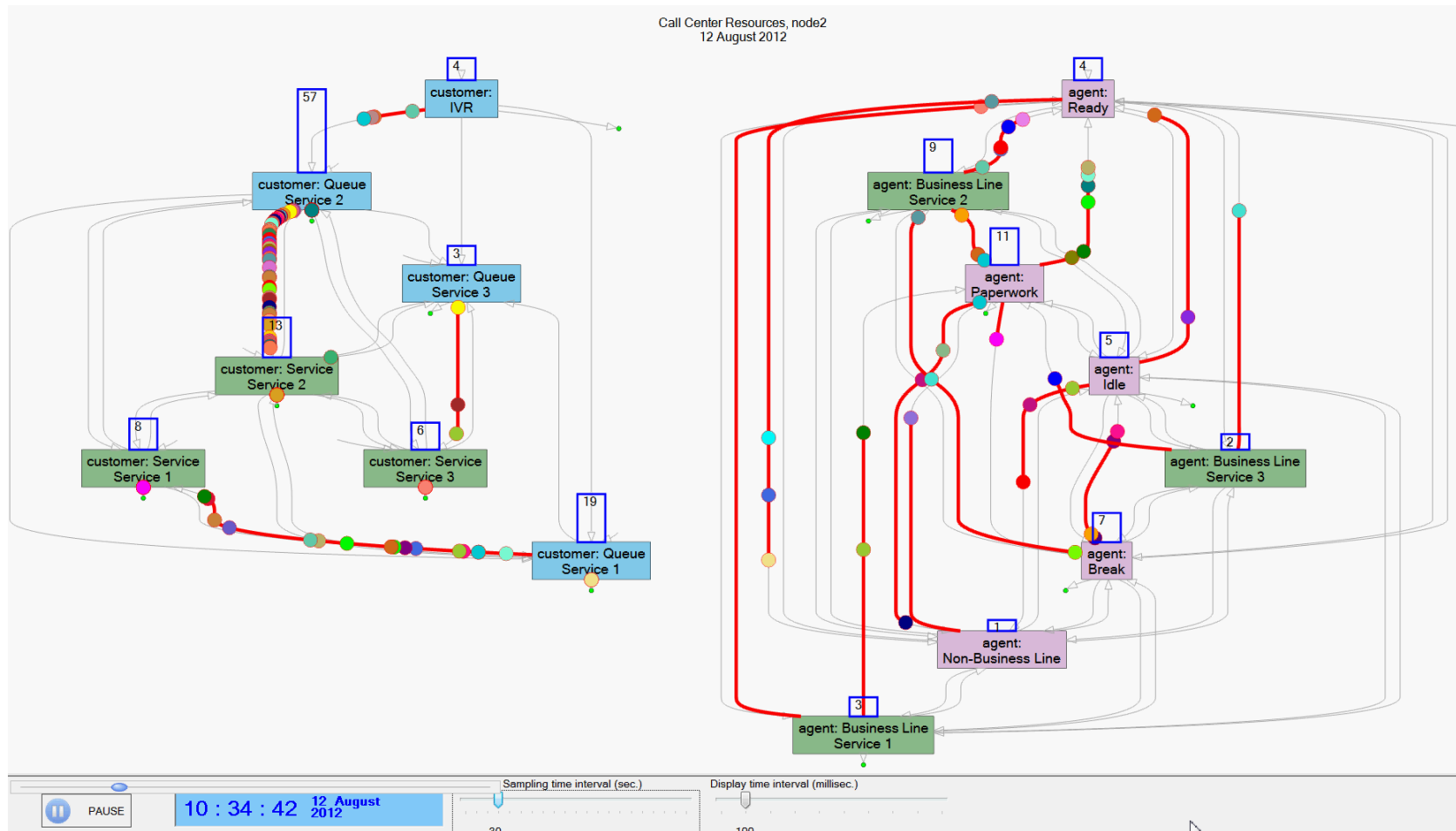
Telephone Services: Customers and Servers - Symmetric Viewpoint

Call Center Resources
12 August 2012

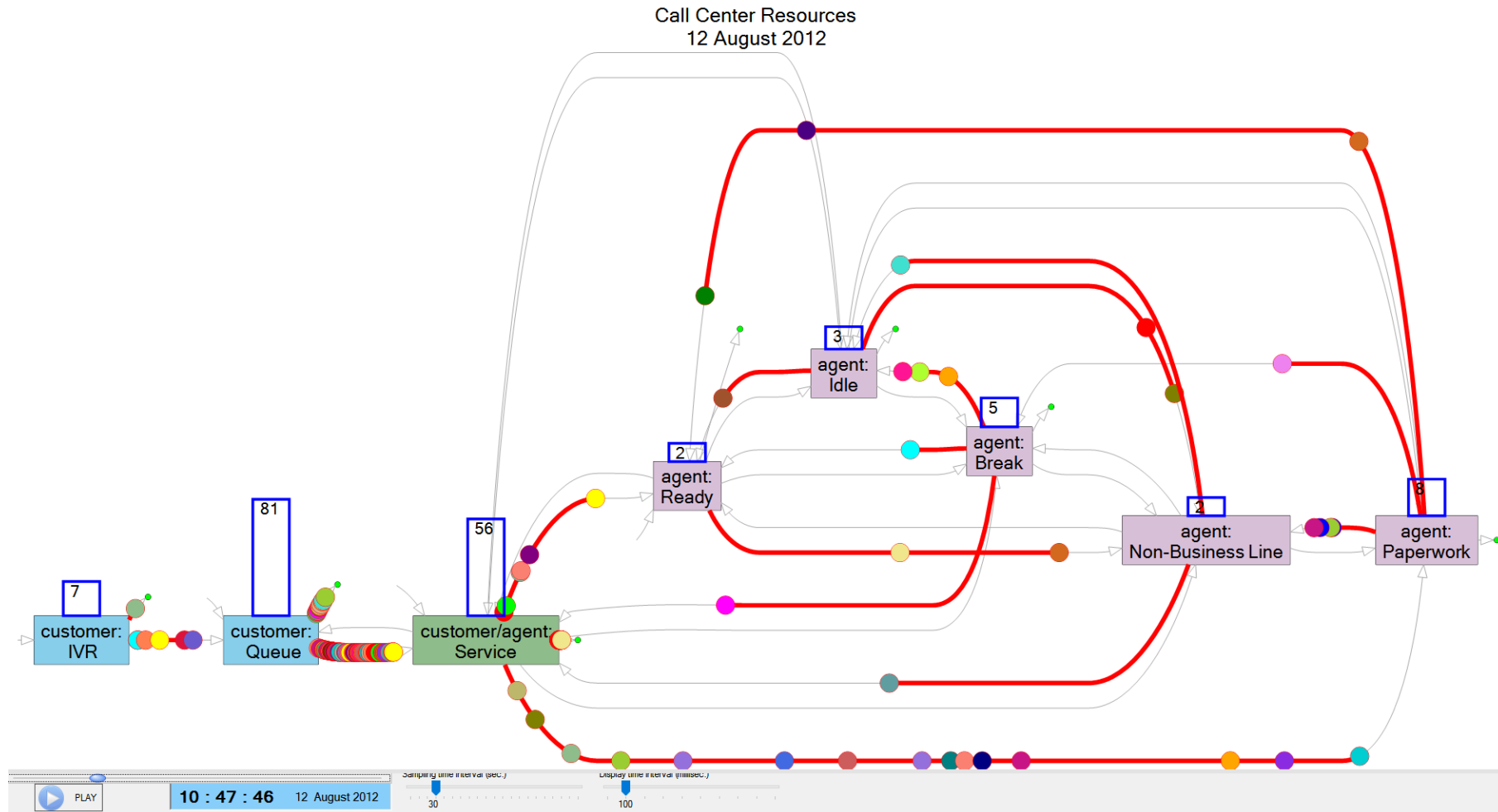


Telephone Services: Customers and Servers

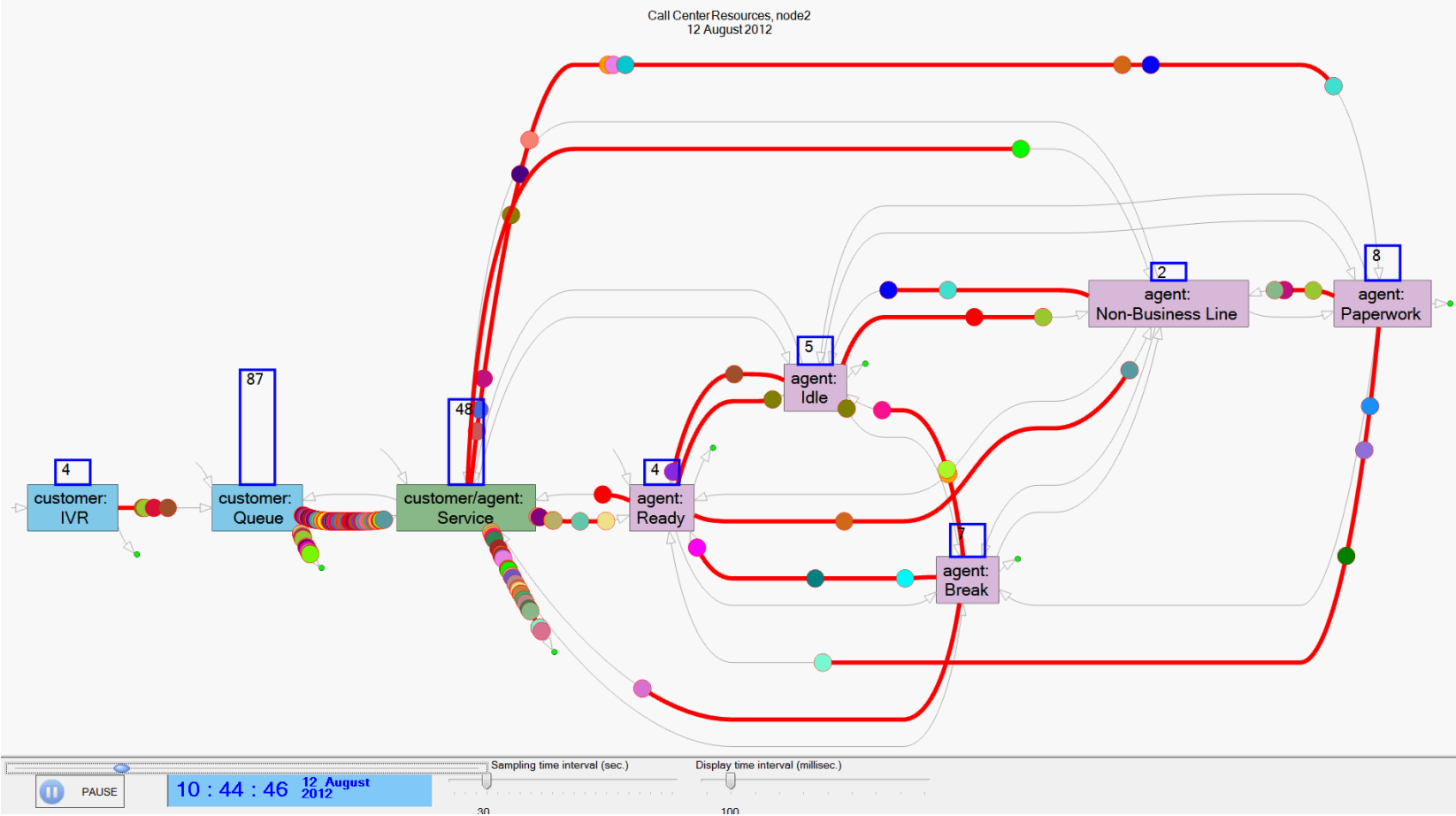
Symmetric Viewpoint



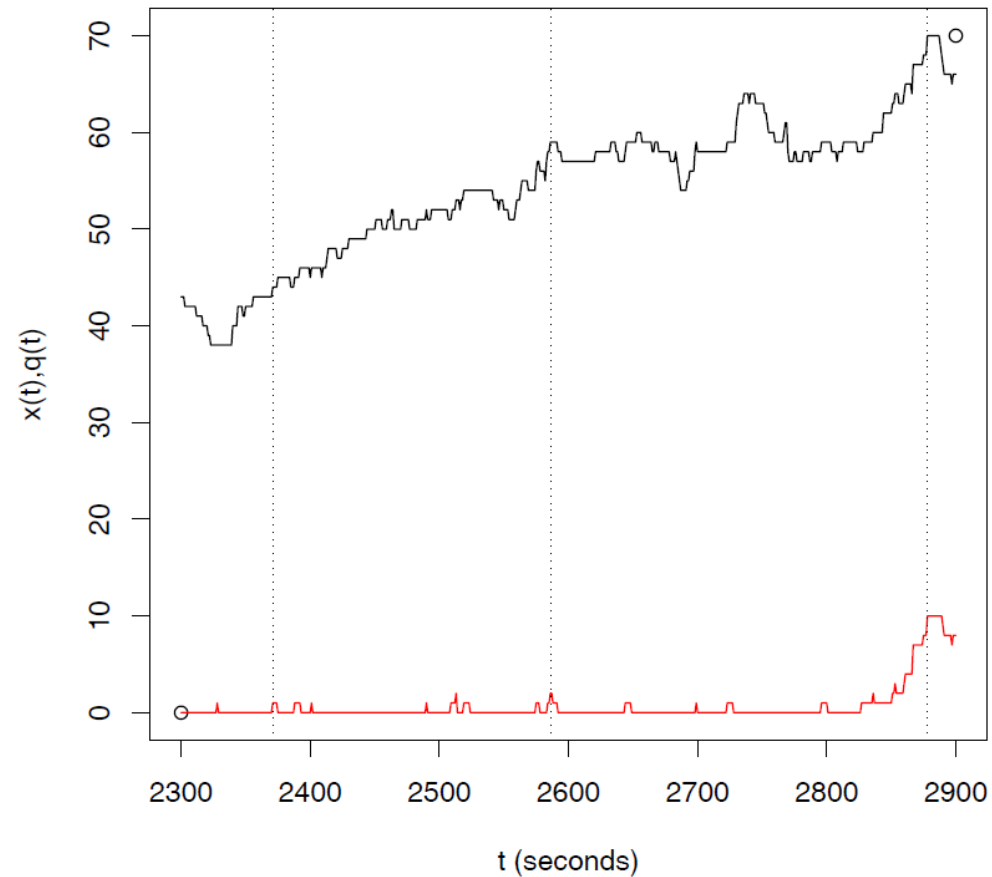
Telephone Services: Customers and Servers – Erlang-S, or **Resource-Driven Activity Networks**



Telephone Services: Customers and Servers - Erlang-S, or Resource-Driven Activity Networks



Number of Servers: Present (=Constant) vs. Available (Random)

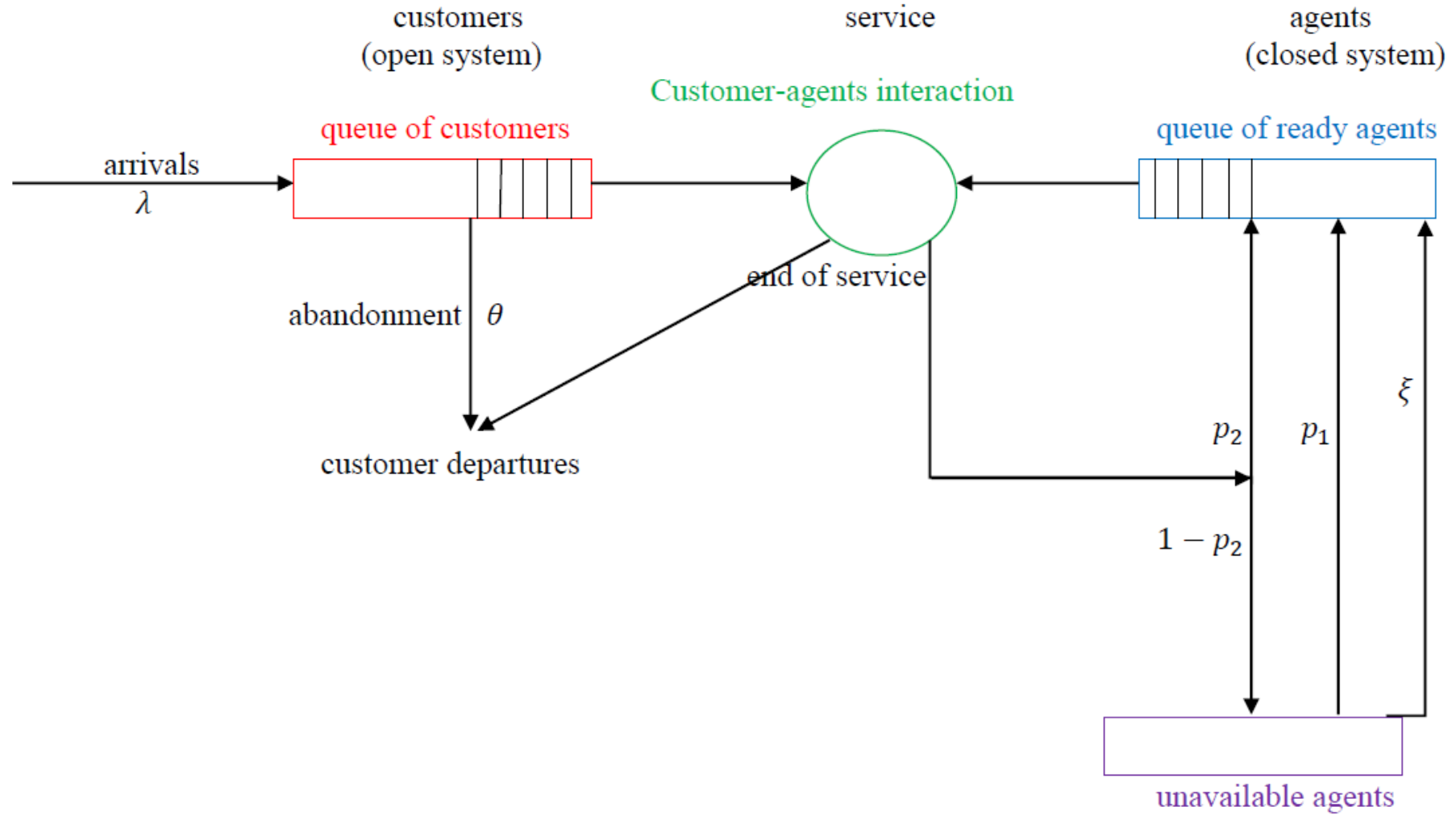


$x(t)$, $q(t)$ in real data (U.S. Bank; telesales; 12/1/2002)

$n(t) = (43, 57, 60)$ @ $t = (10:36:54, 10:43:07, 10:47:23)$:
17 agents became available within 11 minutes

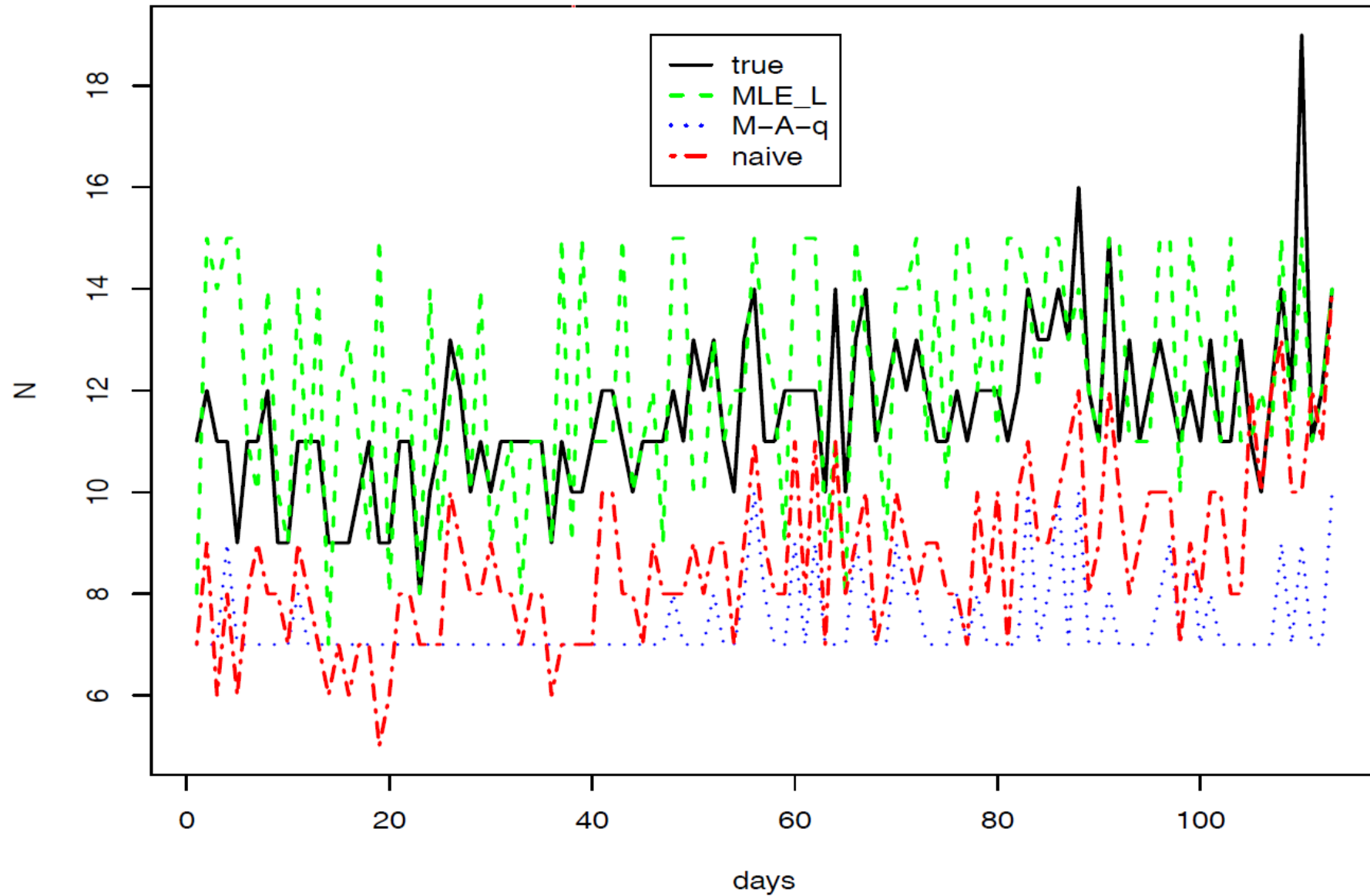
Erlang-S

w/ David Azriel & Paul Feigin

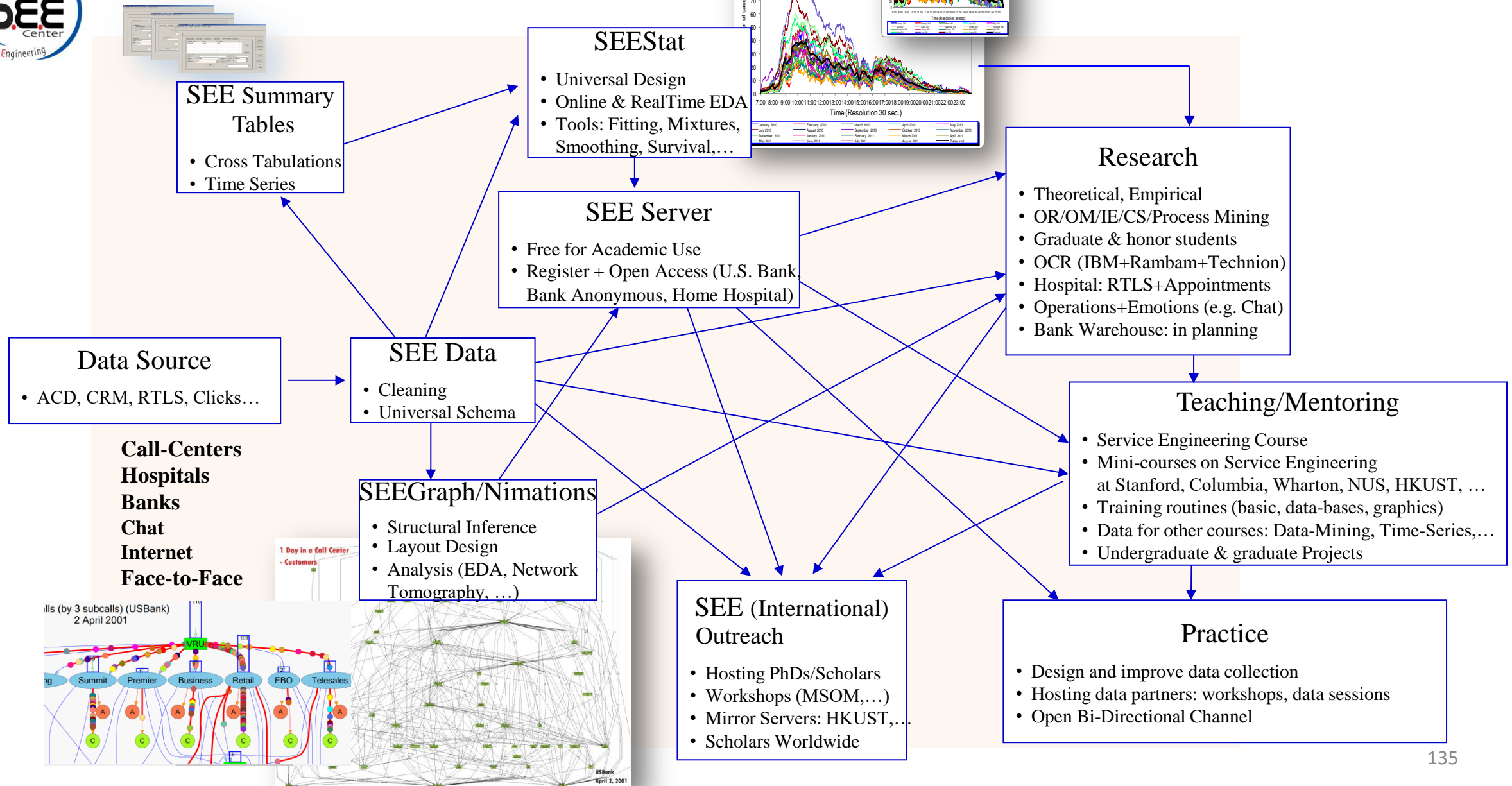
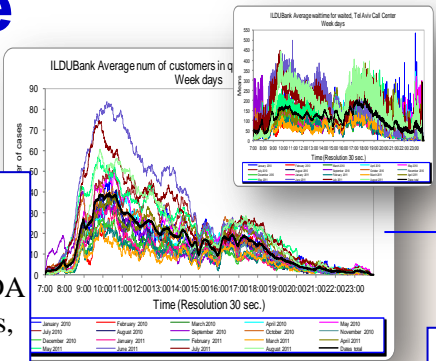


Estimating the Number of Present Servers

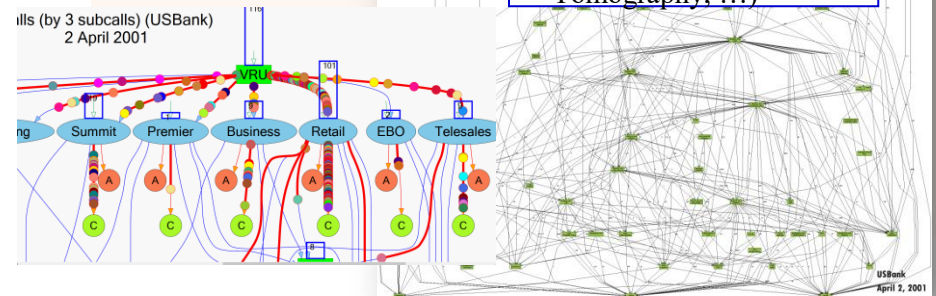
2/1/2005 – 27/6/2005



SEELab: Research, Teaching, Practice



Call-Centers
Hospitals
Banks
Chat
Internet
Face-to-Face



Data Cleaning: MCE with RFID Support

Data-base				Company report		comment
Asset id	order	Entry date	Exit date	Entry date	Exit date	
4	1	1:14:07 PM		1:14:00 PM		
6	1	12:02:02 PM	12:33:10 PM	12:02:00 PM	12:33:00 PM	
8	1	11:37:15 AM	12:40:17 PM	11:37:00 AM		exit is missing
10	1	12:23:32 PM	12:38:23 PM	12:23:00 PM		
12	1	12:12:47 PM	12:35:33 PM		12:35:00 PM	entry is missing
15	1	1:07:15 PM		1:07:00 PM		
16	1	11:18:19 AM	11:31:04 AM	11:18:00 AM	11:31:00 AM	
17	1	1:03:31 PM		1:03:00 PM		
18	1	1:07:54 PM		1:07:00 PM		
19	1	12:01:58 PM		12:01:00 PM		
20	1	11:37:21 AM	12:57:02 PM	11:37:00 AM	12:57:00 PM	
21	1	12:01:16 PM	12:37:16 PM	12:01:00 PM		
22	1	12:04:31 PM	12:20:40 PM			first customer is missing
22	2	12:27:37 PM		12:27:00 PM		
25	1	12:27:35 PM	1:07:28 PM	12:27:00 PM	1:07:00 PM	
27	1	12:06:53 PM		12:06:00 PM		
28	1	11:21:34 AM	11:41:06 AM	11:41:00 AM	11:53:00 AM	exit time instead of entry time
29	1	12:21:06 PM	12:54:29 PM	12:21:00 PM	12:54:00 PM	
31	1	11:40:54 AM	12:30:16 PM	11:40:00 AM	12:30:00 PM	
31	2	12:37:57 PM	12:54:51 PM	12:37:00 PM	12:54:00 PM	
32	1	11:27:11 AM	12:15:17 PM	11:27:00 AM	12:15:00 PM	
33	1	12:05:50 PM	12:13:12 PM	12:05:00 PM	12:15:00 PM	wrong exit time
35	1	11:31:48 AM	11:40:50 AM	11:31:00 AM	11:40:00 AM	
36	1	12:06:23 PM	12:29:30 PM	12:06:00 PM	12:29:00 PM	
37	1	11:31:50 AM	11:48:18 AM	11:31:00 AM	11:48:00 AM	
37	2	12:59:21 PM		12:59:00 PM		

- Imagine **“Cleaning” 60,000+ customers per day** (call centers) !
- **“Psychology”** of Data Trust and Transfer (e.g. 2 years till transfer)

Event-Logs in a Call Center (Bank Anonymous)

A Data Sample (Excel worksheet)

vru+line	call_id	customer_id	priority	type	date	vru_entry	vru_exit	vru_time	q_start	q_exit	q_time	outcome	ser_start	ser_exit	ser_time	server
AA0101	44749	27644400	2	PS	990901	11:45:33	11:45:39	6	11:45:39	11:46:58	79	AGENT	11:46:57	11:51:00	243	DORIT
AA0101	44750	12887816	1	PS	990905	14:49:00	14:49:06	6	14:49:06	14:53:00	234	AGENT	14:52:59	14:54:29	90	ROTH
AA0101	44967	58660291	2	PS	990905	14:58:42	14:58:48	6	14:58:48	15:02:31	223	AGENT	15:02:31	15:04:10	99	ROTH
AA0101	44968	0	0	NW	990905	15:10:17	15:10:26	9	15:10:26	15:13:19	173	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44969	63193346	2	PS	990905	15:22:07	15:22:13	6	15:22:13	15:23:21	68	AGENT	15:23:20	15:25:25	125	STEREN
AA0101	44970	0	0	NW	990905	15:31:33	15:31:47	14	00:00:00	00:00:00	0	AGENT	15:31:45	15:34:16	151	STEREN
AA0101	44971	41630443	2	PS	990905	15:37:29	15:37:34	5	15:37:34	15:38:20	46	AGENT	15:38:18	15:40:56	158	TOVA
AA0101	44972	64185333	2	PS	990905	15:44:32	15:44:37	5	15:44:37	15:47:57	200	AGENT	15:47:56	15:49:02	66	TOVA
AA0101	44973	3.06E+08	1	PS	990905	15:53:05	15:53:11	6	15:53:11	15:56:39	208	AGENT	15:56:38	15:56:47	9	MORIAH
AA0101	44974	74780917	2	NE	990905	15:59:34	15:59:40	6	15:59:40	16:02:33	173	AGENT	16:02:33	16:26:04	1411	ELI
AA0101	44975	55920755	2	PS	990905	16:07:46	16:07:51	5	16:07:51	16:08:01	10	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44976	0	0	NW	990905	16:11:38	16:11:48	10	16:11:48	16:11:50	2	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44977	33689787	2	PS	990905	16:14:27	16:14:33	6	16:14:33	16:14:54	21	HANG	00:00:00	00:00:00	0	NO_SERVER
AA0101	44978	23817067	2	PS	990905	16:19:11	16:19:17	6	16:19:17	16:19:39	22	AGENT	16:19:38	16:21:57	139	TOVA
AA0101	44764	0	0	PS	990901	15:03:26	15:03:36	10	00:00:00	00:00:00	0	AGENT	15:03:35	15:06:36	181	ZOHARI
AA0101	44765	25219700	2	PS	990901	15:14:46	15:14:51	5	15:14:51	15:15:10	19	AGENT	15:15:09	15:17:00	111	SHARON
AA0101	44766	0	0	PS	990901	15:25:48	15:26:00	12	00:00:00	00:00:00	0	AGENT	15:25:59	15:28:15	136	ANAT
AA0101	44767	58859752	2	PS	990901	15:34:57	15:35:03	6	15:35:03	15:35:14	11	AGENT	15:35:13	15:35:15	2	MORIAH
AA0101	44768	0	0	PS	990901	15:46:30	15:46:39	9	00:00:00	00:00:00	0	AGENT	15:46:38	15:51:51	313	ANAT
AA0101	44769	78191137	2	PS	990901	15:56:03	15:56:09	6	15:56:09	15:56:28	19	AGENT	15:56:28	15:59:02	154	MORIAH
AA0101	44770	0	0	PS	990901	16:14:31	16:14:46	15	00:00:00	00:00:00	0	AGENT	16:14:44	16:16:02	78	BENSION
AA0101	44771	0	0	PS	990901	16:38:59	16:39:12	13	00:00:00	00:00:00	0	AGENT	16:39:11	16:43:35	264	VICKY
AA0101	44772	0	0	PS	990901	16:51:40	16:51:50	10	00:00:00	00:00:00	0	AGENT	16:51:49	16:53:52	123	ANAT
AA0101	44773	0	0	PS	990901	17:02:19	17:02:28	9	00:00:00	00:00:00	0	AGENT	17:02:28	17:07:42	314	VICKY
AA0101	44774	32387482	1	PS	990901	17:18:18	17:18:24	6	17:18:24	17:19:01	37	AGENT	17:19:00	17:19:35	35	VICKY
AA0101	44775	0	0	PS	990901	17:38:53	17:39:05	12	00:00:00	00:00:00	0	AGENT	17:39:04	17:40:43	99	TOVA

- Unsynchronized transition times, consistently

SEELab: Environment for graphical EDA

Operational histories (customers, servers) at the **individual-transaction level**, e.g.

1. ***Bank Anonymous Call-Center**: 1 year, 350K calls by 15 agents - in 2000, **which paved the way to:**
2. ***U.S. Bank Call-Center** : 2.5 years, 220M calls, 40M by 1000 agents
- 3-4. Israeli Cellular Company: 2.5 years, 110M calls, 25M calls by 750 agents; **ILBank (IVR, SBR)**: 2 years
5. Back to Bank Anonymous: Call-Center and more - **from January 2010, daily-deposit at a SEESafe**

6. Service Engineering **internet site**: click-stream data (2 years)

7. ***Home (Rambam) Hospital** : 4 years, 1000 beds, inter-ward patient flow

8. **Emergency Departments (ED) patient flow**:
 - **5 EDs in Israel**: 1-2 years, **late David Sinreich**, ED arrivals & LOS
 - ED in Seoul: 2 months, **K. Song-Hee & W. Cha**, pilot
 - ED in Singapore: 2 years, pilot

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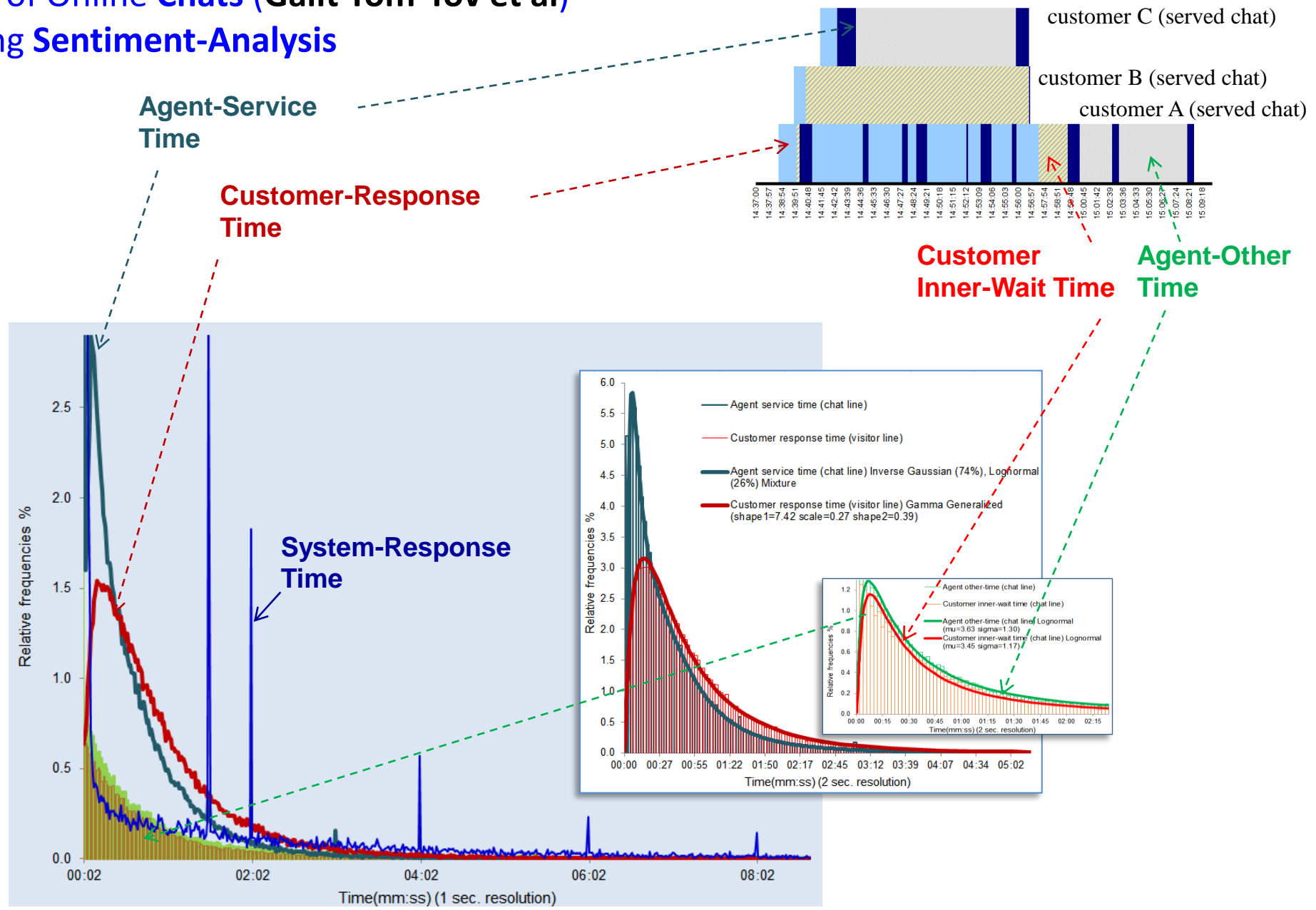
9. **U.S. Ambulatory Hospital RTLS (Real-Time Location System)**: Since November 2013
 - **250K events/day (1GB/week)**: **1000 patients, 350 staff (1500 tagged entities)**, every **3 sec's**
 - Infrastructure: **900 readers (sensors) over ceilings of 7 (now 8) clinical floors**
 - Both actual and planned (**appointment book** of resources: staff, patients, rooms)

- 10-13: **Chat Services (Europe); ILBank Warehouse; Smart-City Simulator (Haifa, Boston,...); Justice System**

***Open & Free for (reproducible) research and teaching**

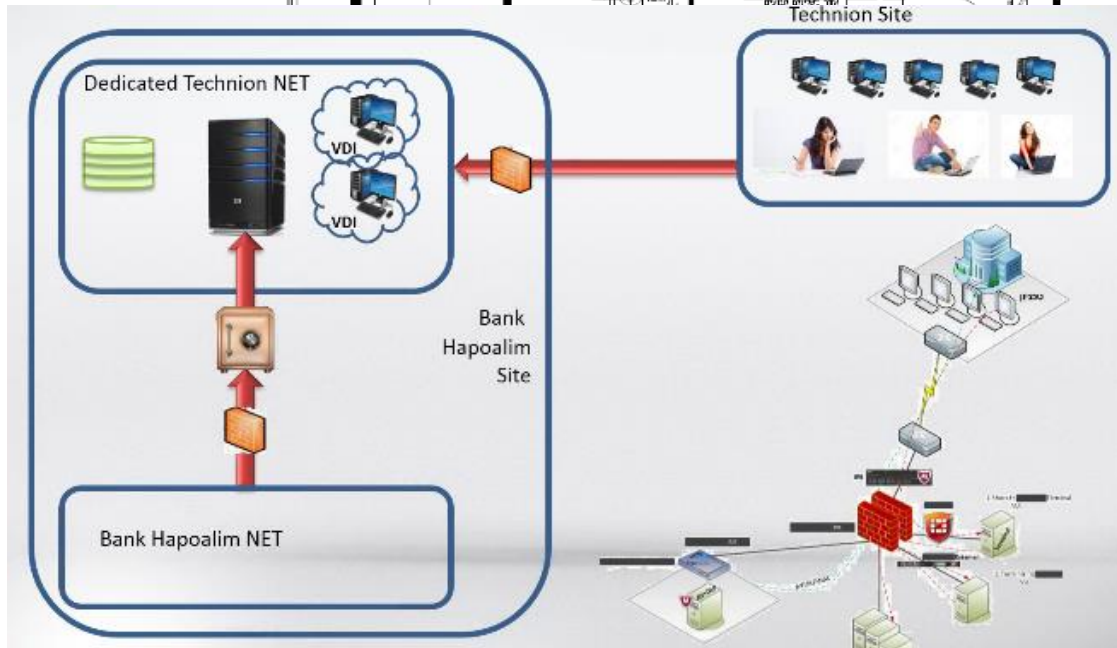
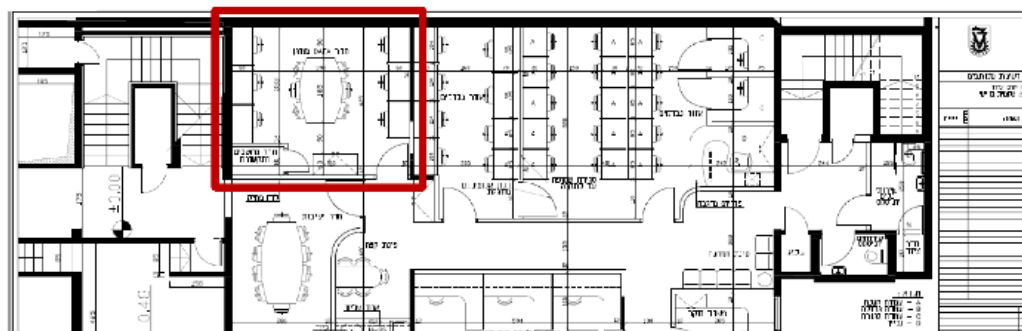
Anatomy of Online Chats (Galit Yom-Tov et al)

Supporting Sentiment-Analysis



Data-Room (Secured): Accessing 80% of Bank Warehouse

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- T-PADS מעבדה: איך, מי, איפה, ומתי
- מיקוד ראשוני Data Science & Engineering
- אפשרות הרחבה ל Blockchain, IoT, וכו'
- X5 חוזה 5
- - בעלות משותפת
- פרסומים משותפים בהסכמה
- דרך פרויקטים, לאימות המחקר POC
- סטטוס: 1ח, ועדת היגוי - 6ח, יום אירוע מרוכז- 1ש

Smart-City "Simulator" (40K sq ft "playground", 10K offices)

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Emergency services

Traffic lights

Lightening

Smart parking

Water, sewage

Energy systems

600 מ"ר משרדים + 2.5 דונם של "מגרש משחקים" + 2 דונם הרחבה עתידית



מתהווה: שת"פ עם מערכת המשפט

שיתוף פעולה יאפשר תמיכה מחקרית במטרות הבאות (דוגמאות בלבד):

- ❖ פיתוח מפה תהליכית המתארת הליכי טיפול בתיקי בתי המשפט, תוך איתור צווארי בקבוק תפעוליים ורעיונות לקיצור משכי הליכים.
- ❖ ניתוח מדיניות חלוקת התיקים והתיעדוף הקיימים, במטרה לקצר תורי תיקים ושיפור חלוקתם (תורת התורים, תורת המשחקים, ...).
- ❖ בחינה וחקר (עידון) המדד "משקלות תיקים" (עומס תפעולי=Offered-Load), ויותר (למשל רגשי, דוגמת 2 מחלקות יולדות ברמב"ם, לאיזון עומסים "הוגן").
- ❖ תכנון "יומנים" של שופט, עו"ד, תובע ונתבע (**Appointment Scheduling**)

The Skeptics

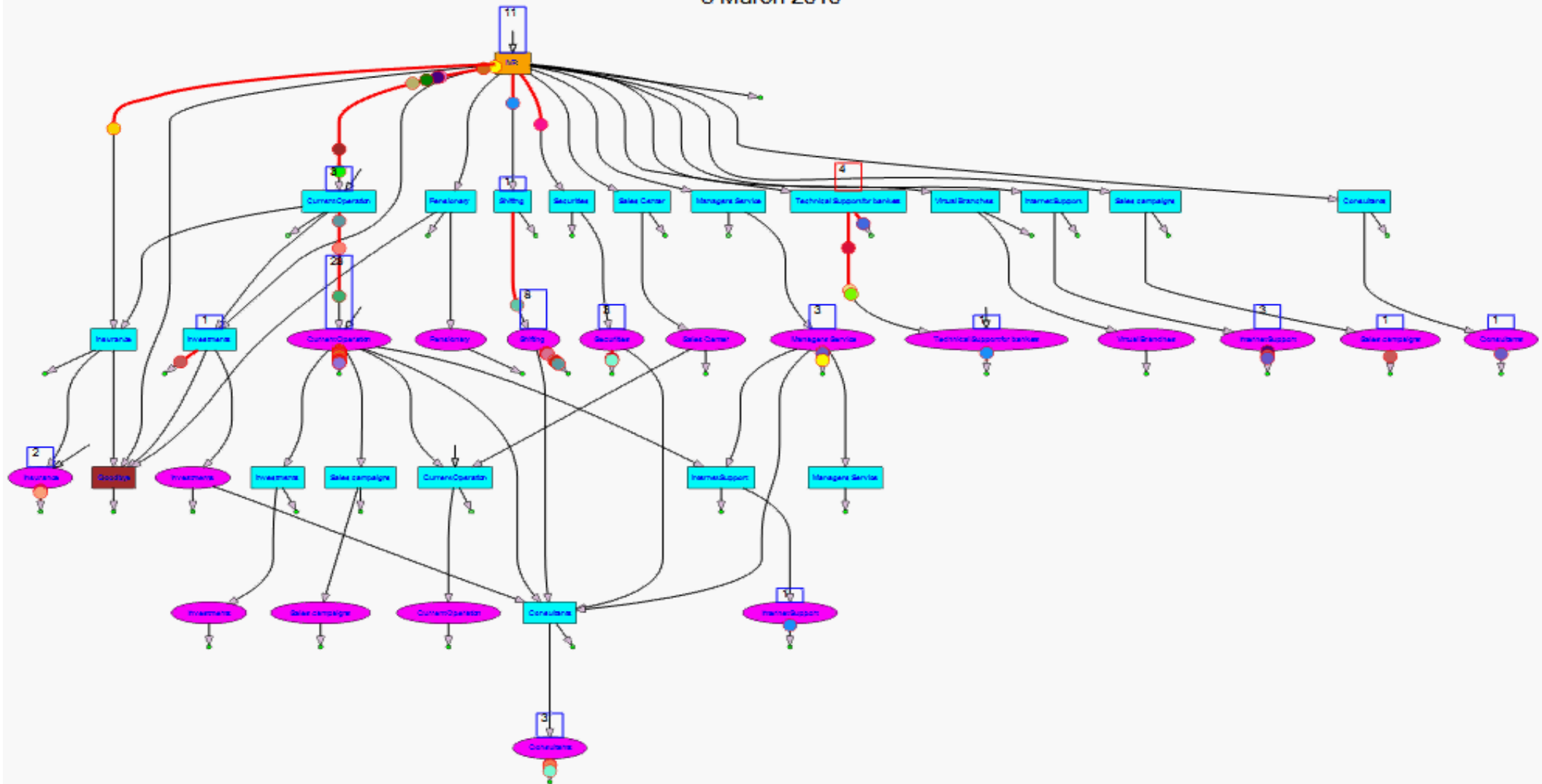
ERC: on using data to motivate theory (as in say Physics, Biology,...)
How can breakthrough mathematics come out of so much data?

NSF: on funding data-collection and maintenance in OR research grants
Finding an interested industry-partner w/ data \Rightarrow problem solved

ISF: on measuring judicial workload (JW)
For the most part, ... the applicant proposes to quantify the unquantifiable and solve the unsolvable, namely JW.
At least since the eighteenth century, there are continuous ...

Still: Nurse-workload in maternity wards: operational+emotional(+cognitive)

Customers flow (ILDUBank)
8 March 2010



PAUSE

10 : 24 : 32 8 March 2010

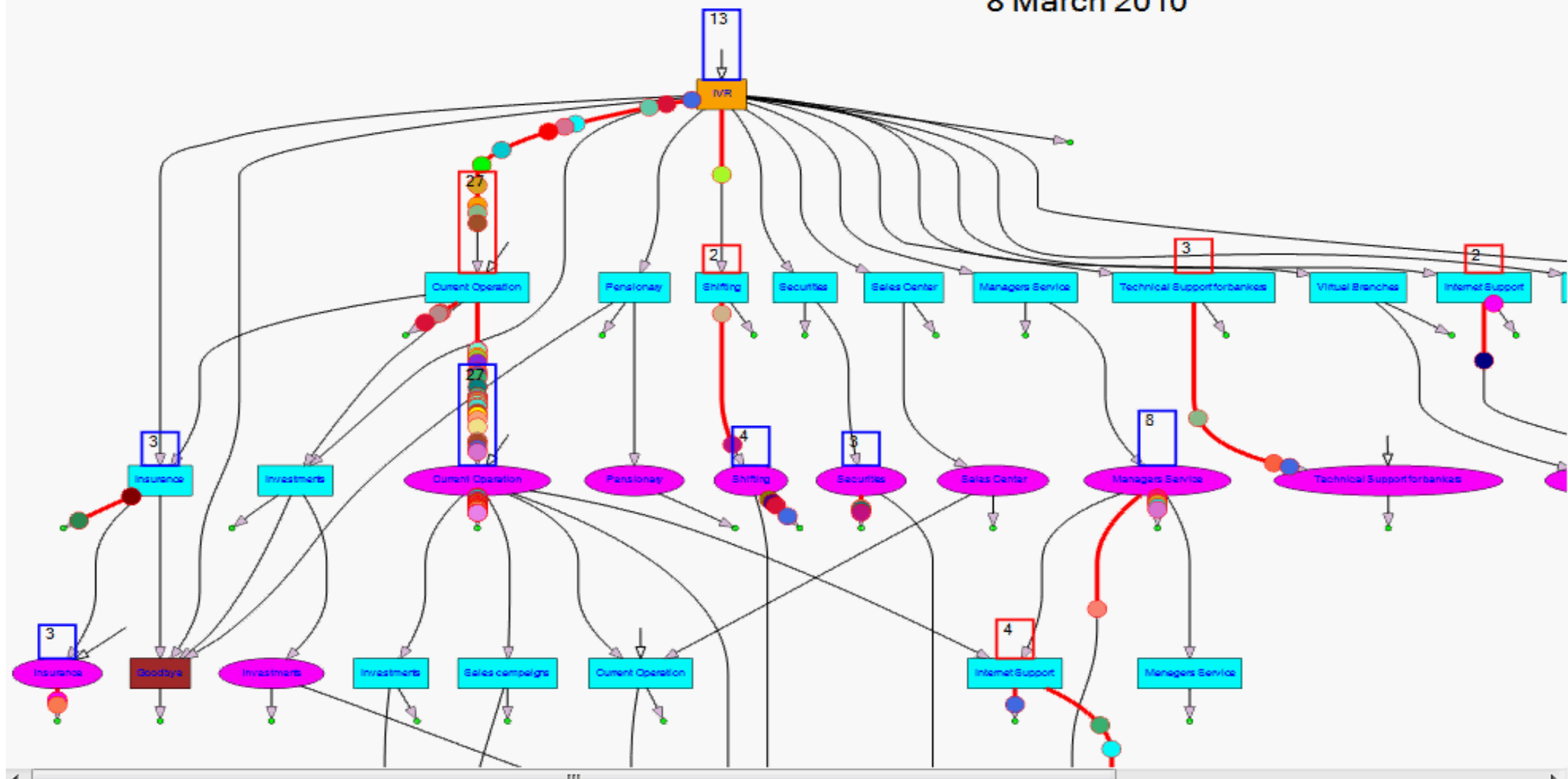
Sampling time interval (sec.)

60

Display time interval (millisec.)

100

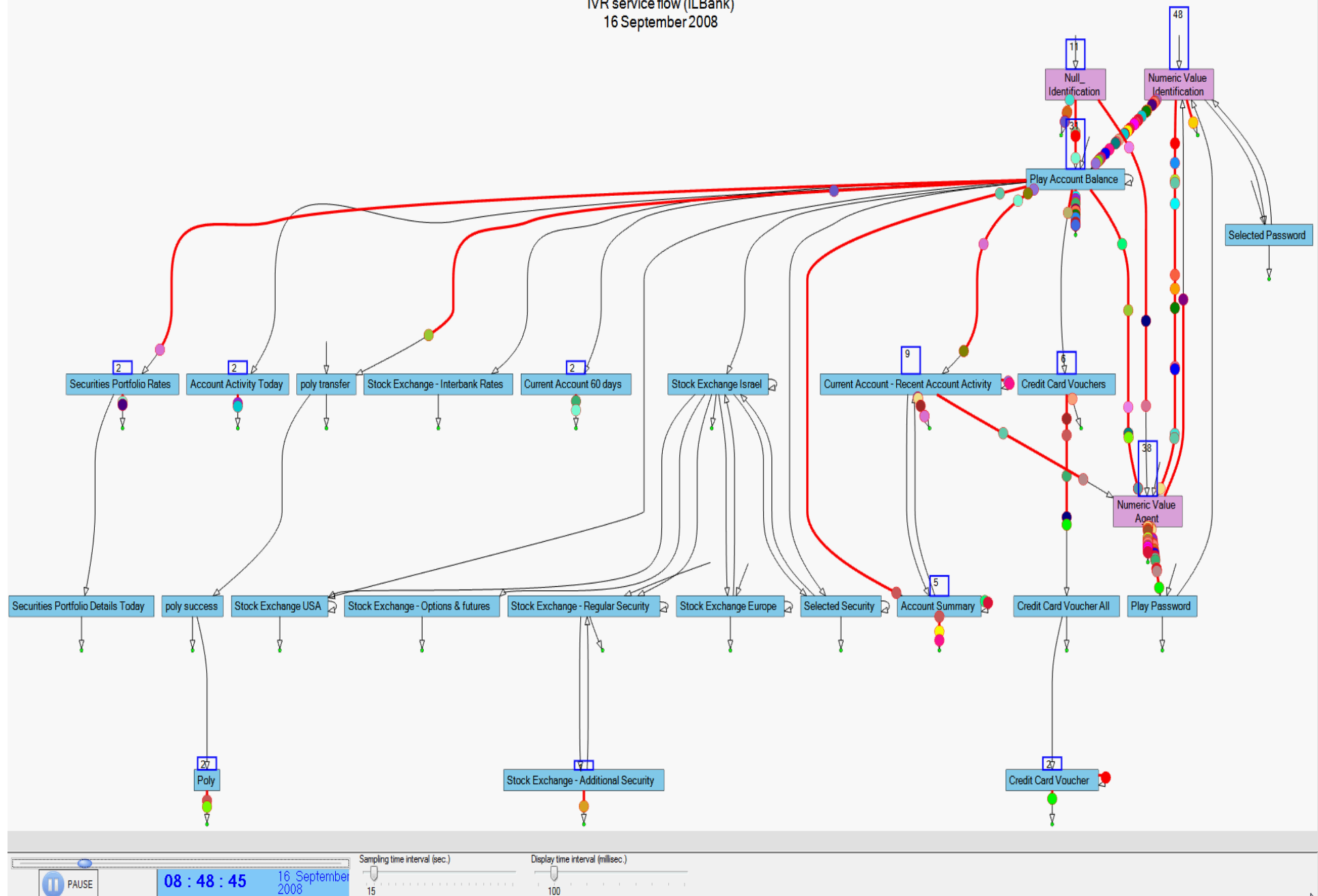
Customers flow (ILDUBank) 8 March 2010

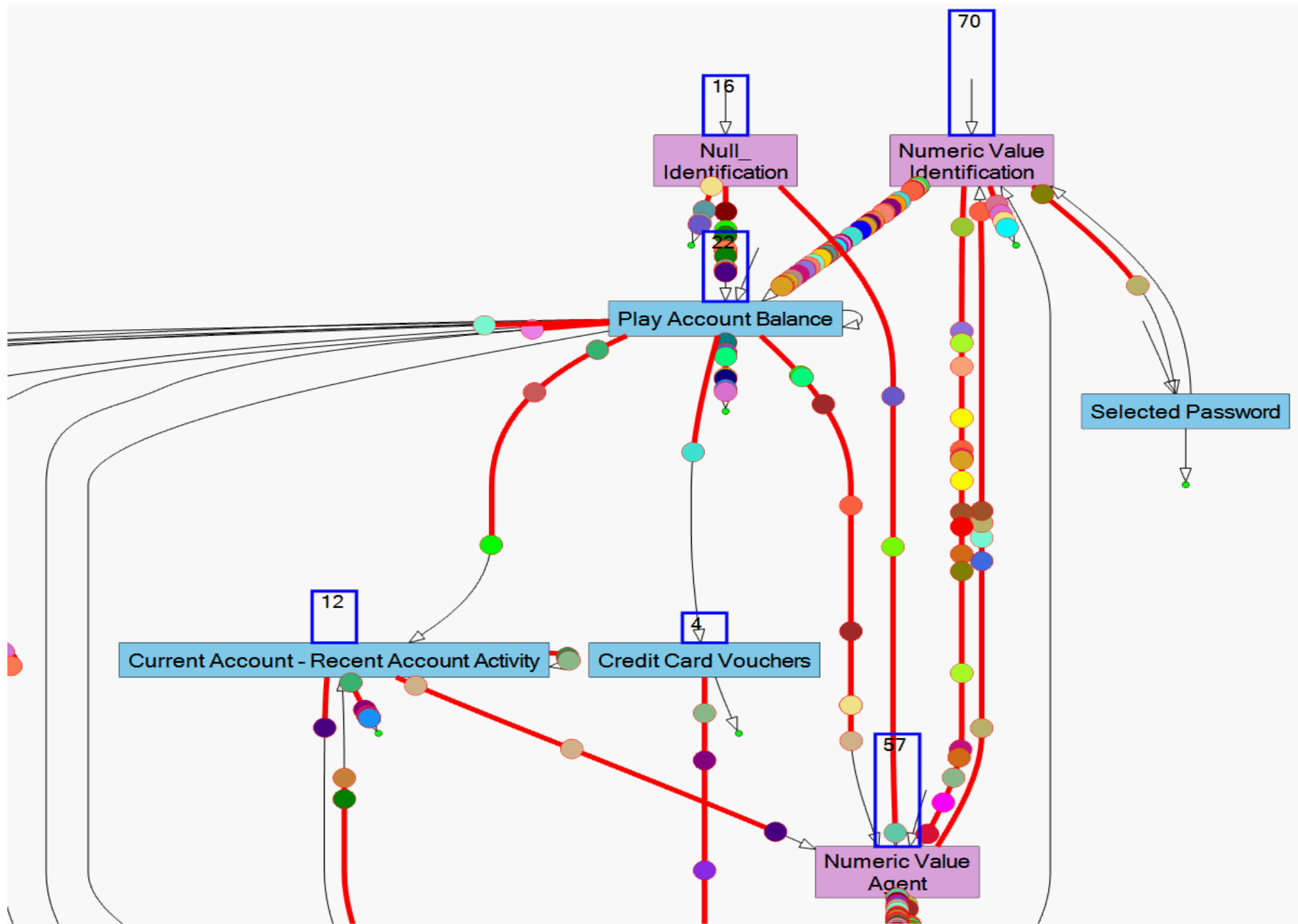


PAUSE 09 : 48 : 40 8 March 2010

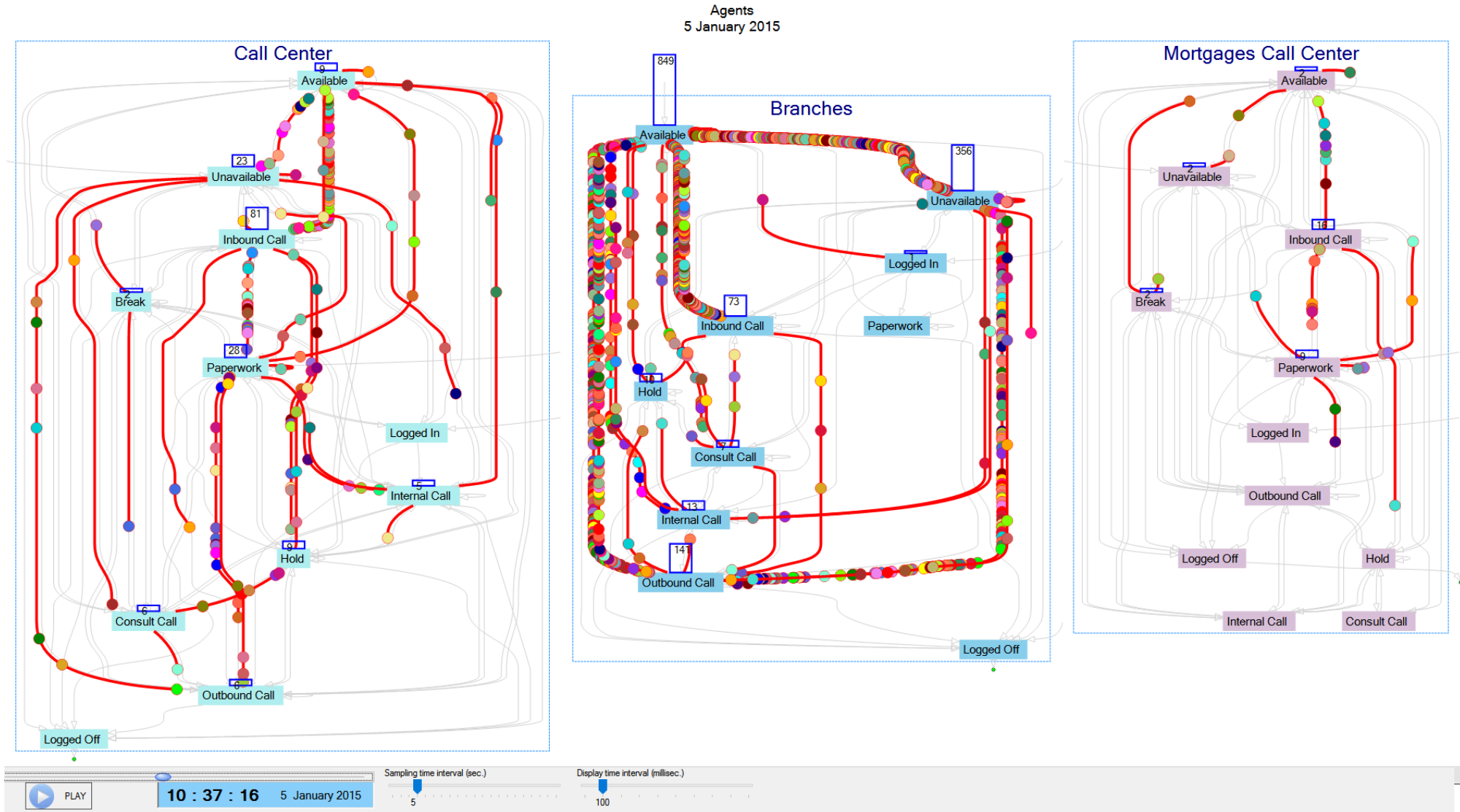
Sampling time interval (sec.) 60

Display time interval (millisec.) 100

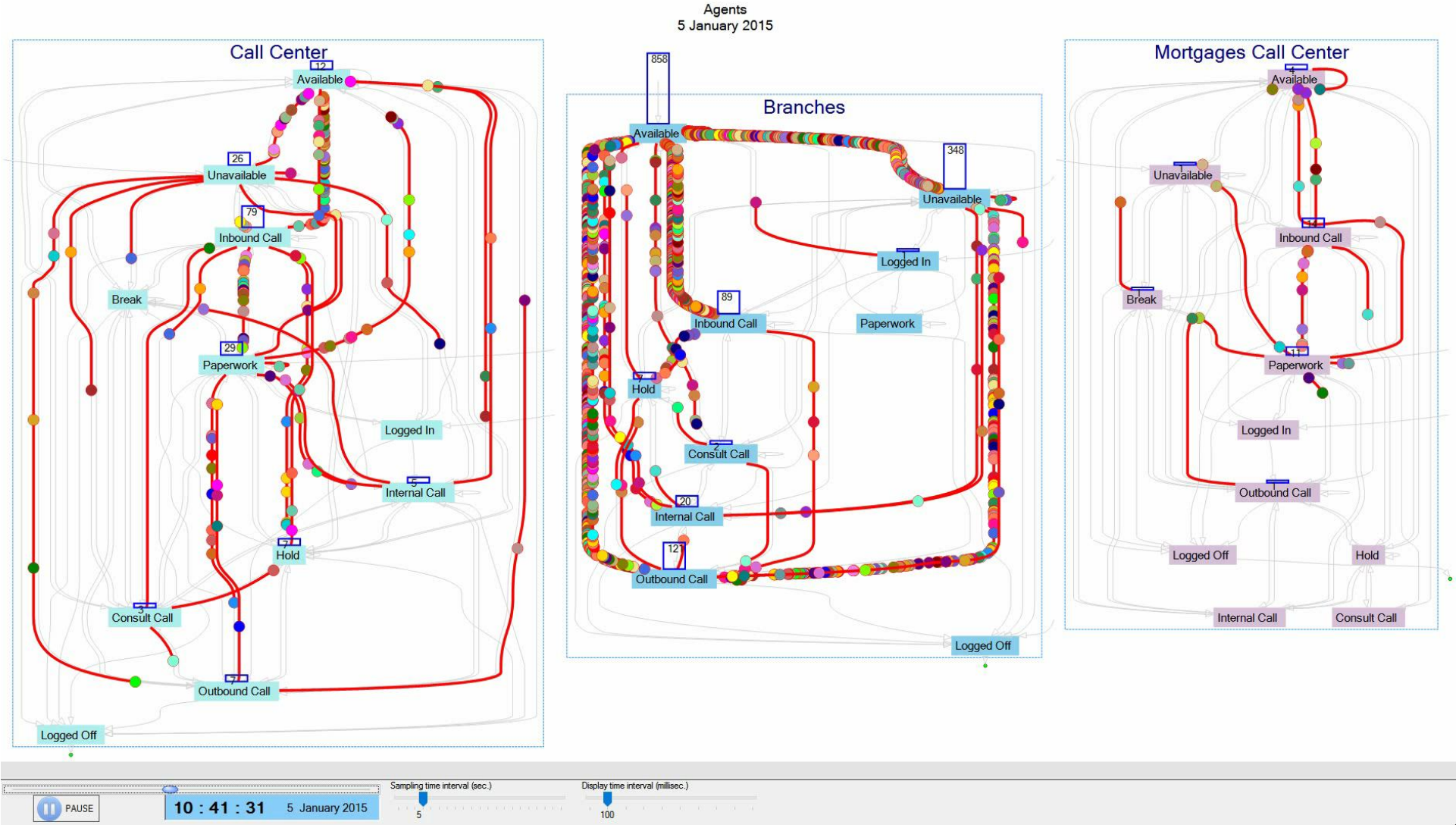




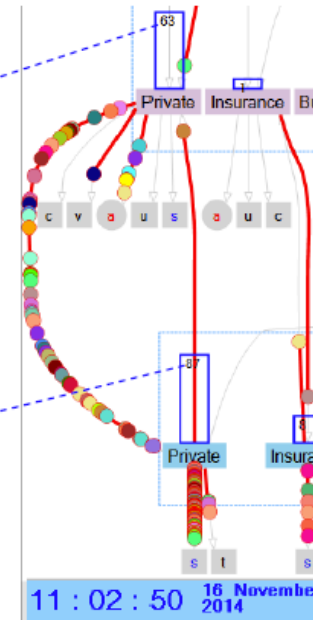
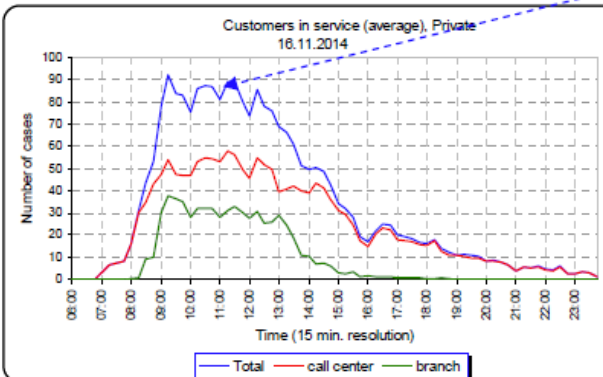
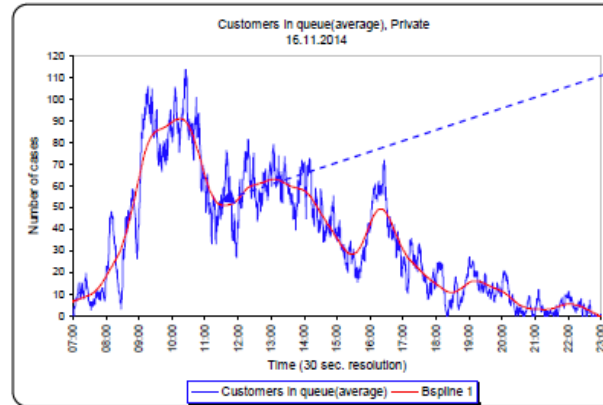
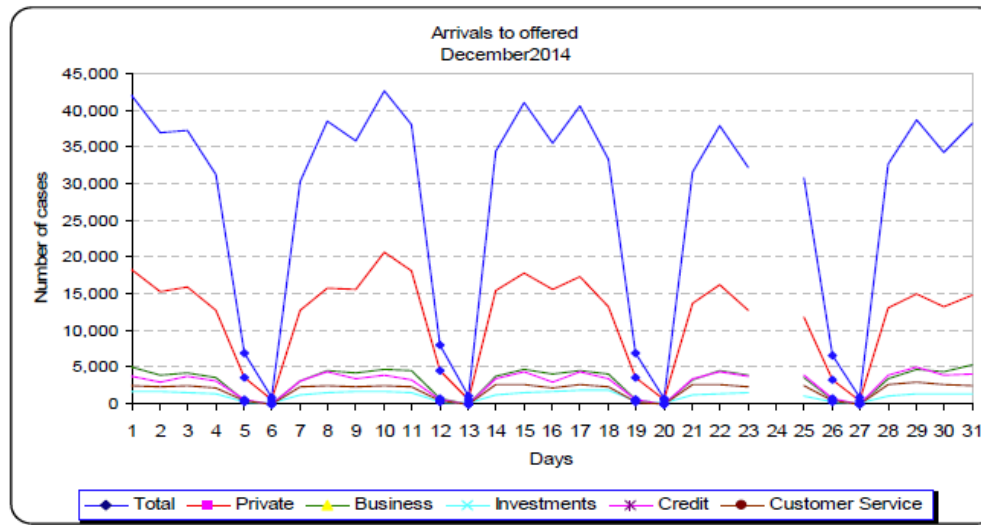
Agents (2000): Branches (1700) and Call Centers - Commercial (270), Mortgages (50)



Agents: Branches, Commercial and Mortgages Call Centers



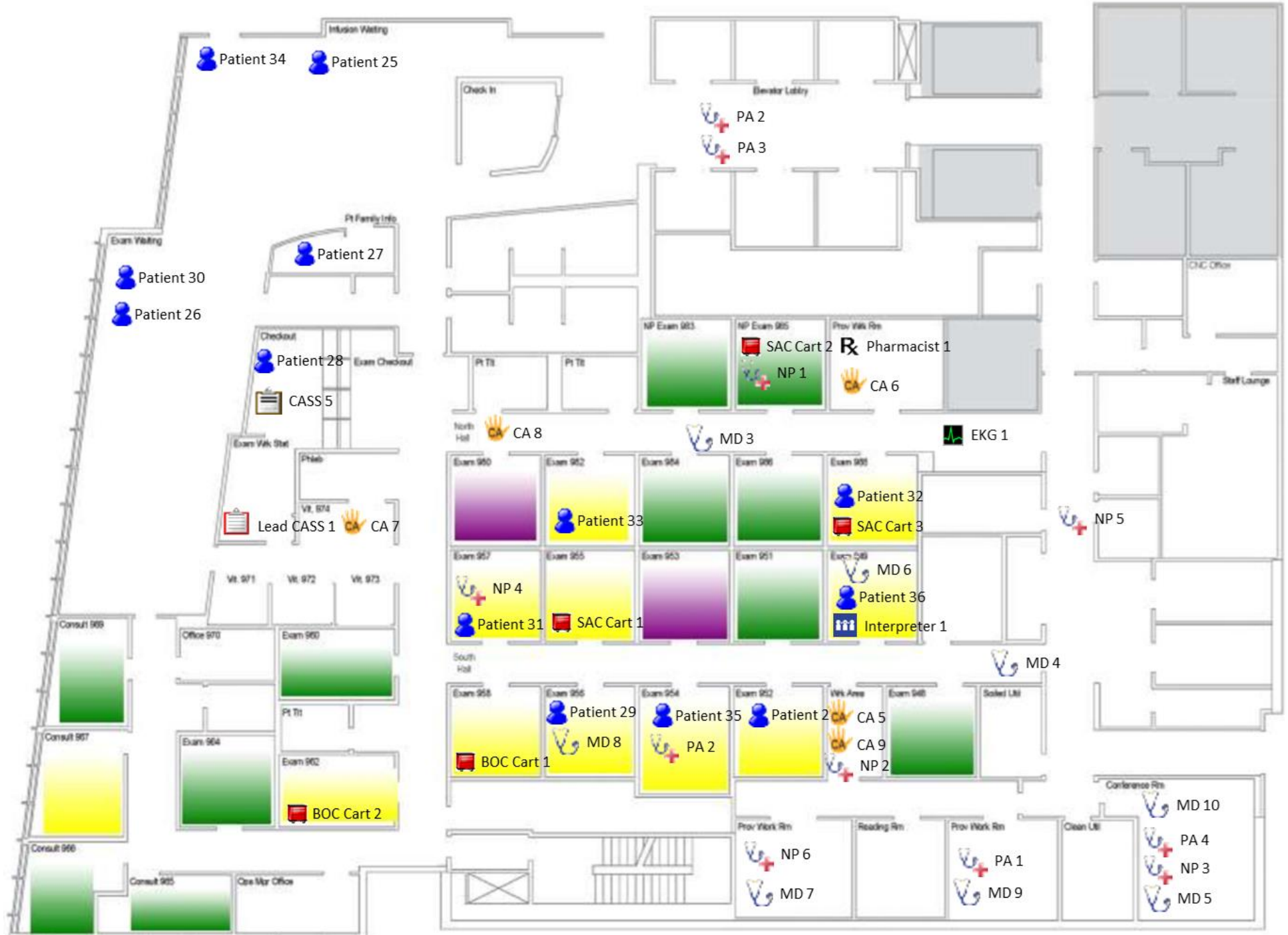
ILDU Banking:
branches and call centers
Service Types



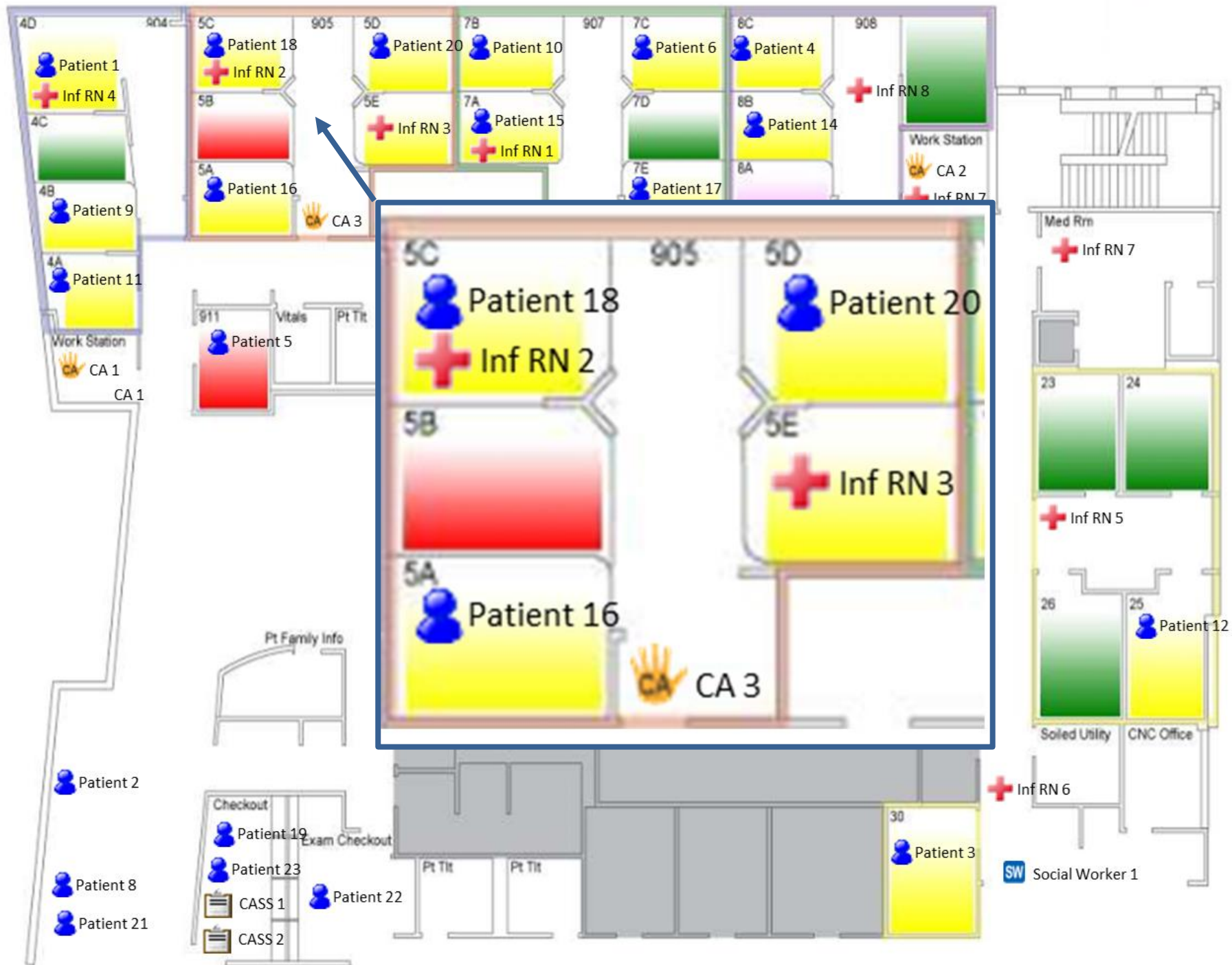


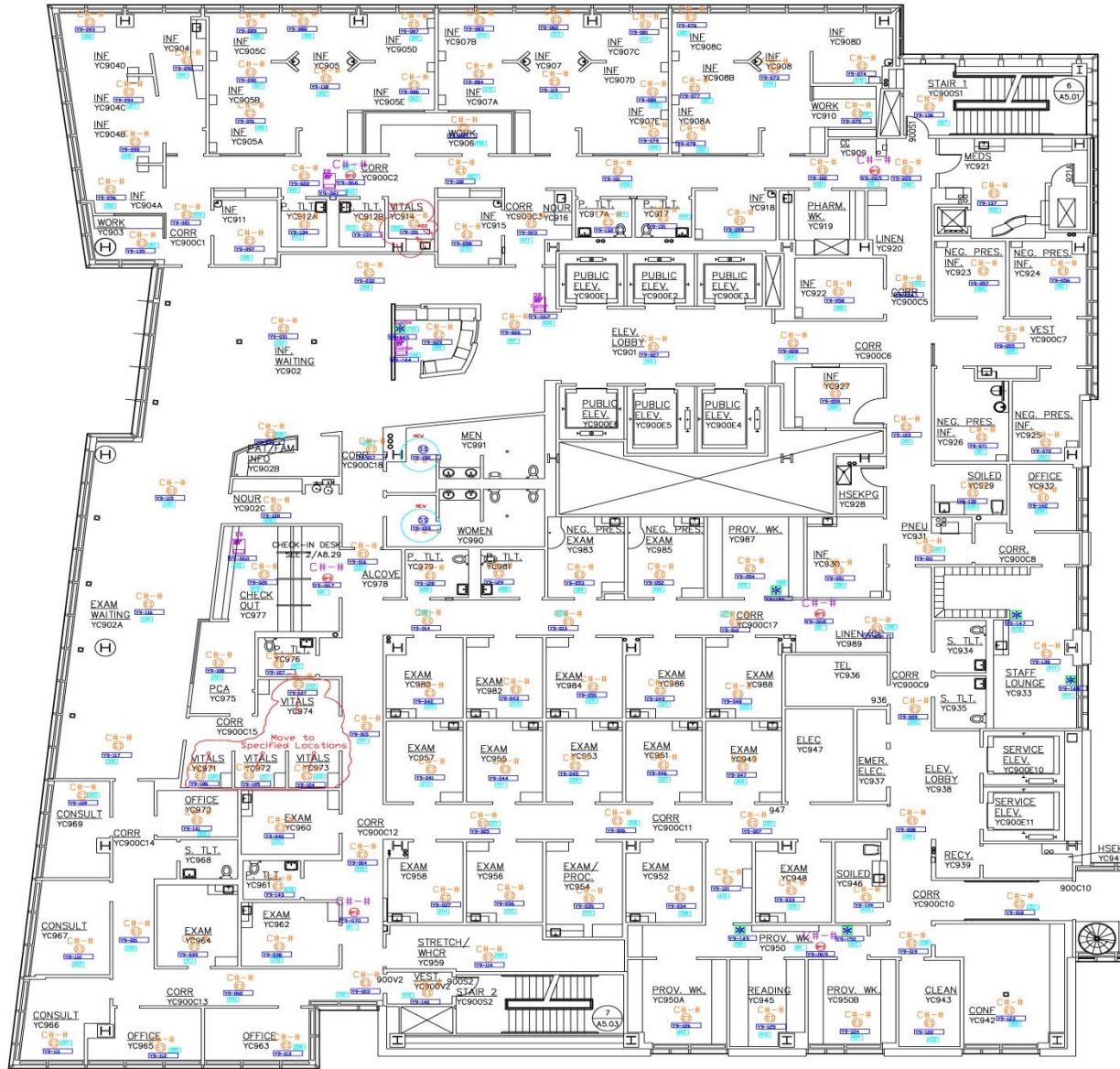
Closing the Data-Gap: from Call-Centers to Hospitals, now Banks

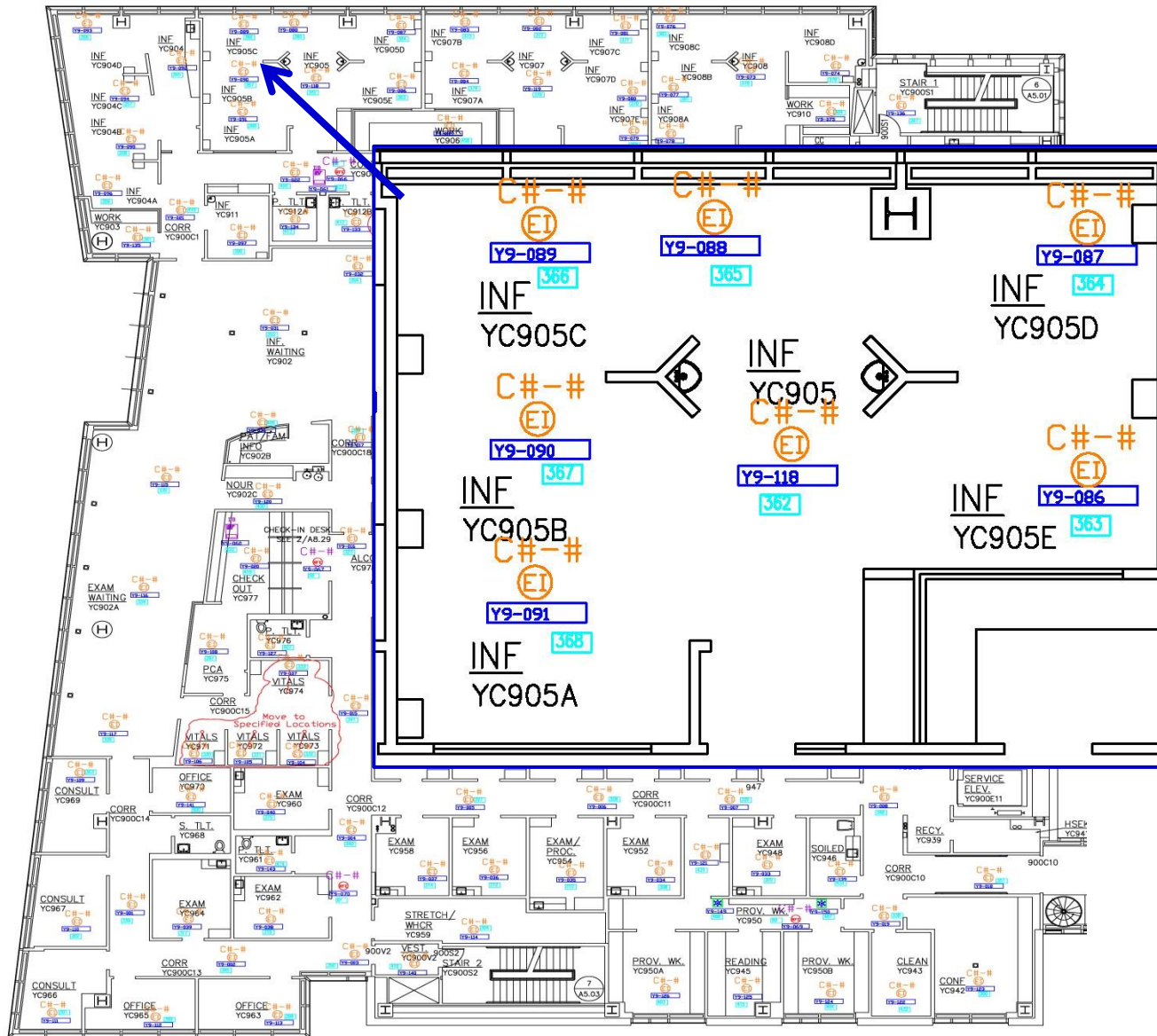
- **Large call center:**
 - 1000s of agents
 - Hundreds of thousands of calls per day
 - Data: operational, psychological, financial – **automatic** collection
- **Large hospital:**
 - 1000+ Beds
 - 1000s of patients & nurses, hundreds of doctors
 - Data: operational, clinical, financial – mostly **inaccessible (to academia)**
- **Large Bank:** “Enjoys” characteristics of both of the above











Applications in DFCI

Control: rooms status, physicians location, long wait times

Planning: number infusion chairs, load-balancing among floors

Management: evidence-based

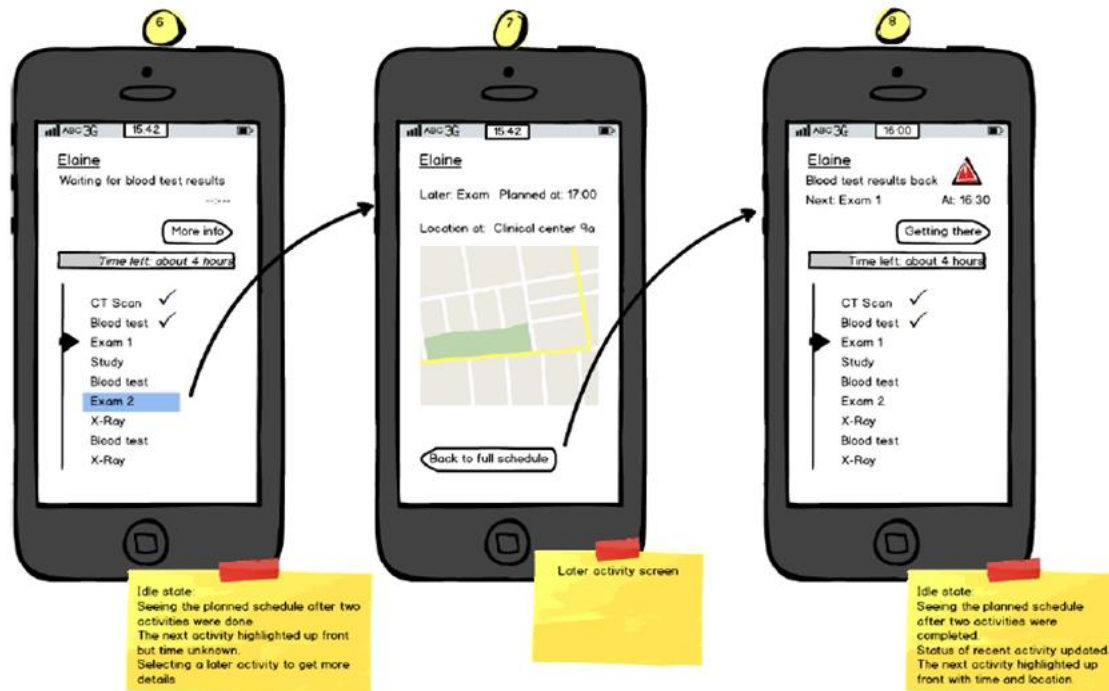
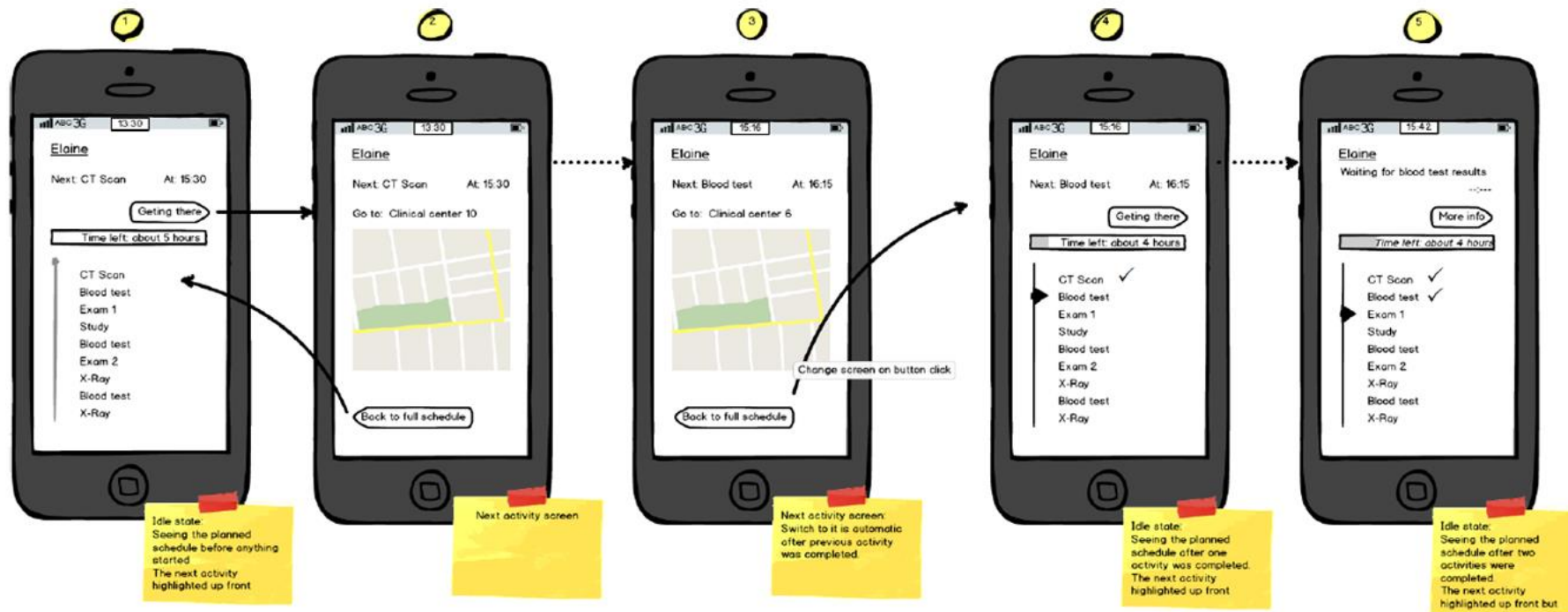
Motivating improvement: room for physician vs. room for patient

BUT how about

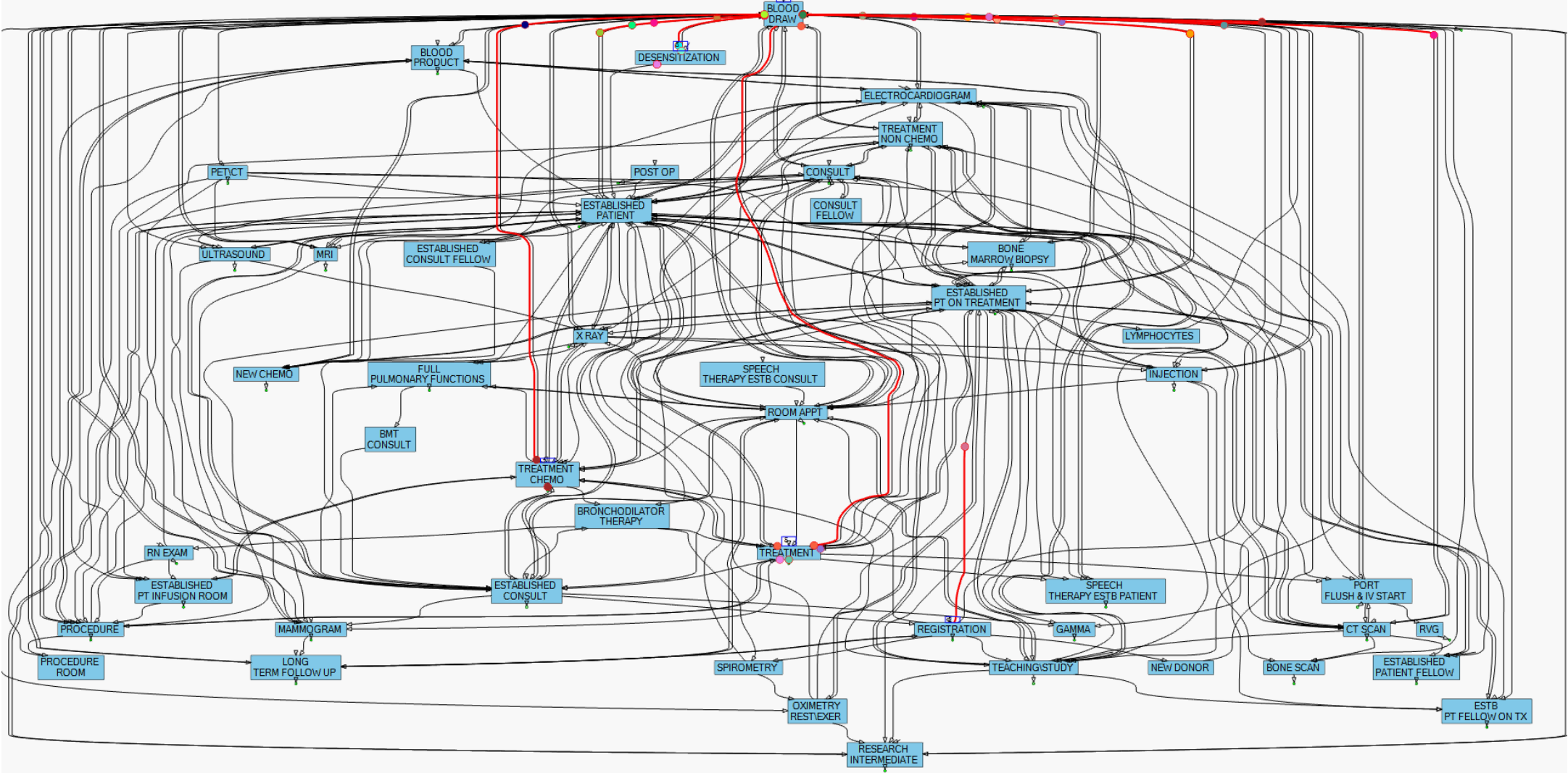
Time & Motion Studies of Resources (IEM 21st century), or

Mining Social Network(s), or

...



Patients appointments (DayHospital)
30 September 2014



time interval (sec.) Display time interval (millisec.)

07 : 04 : 30 30 September 2014

PAUSE 30 100

Appointment-Driven Services (Networks)

e.g. Healthcare, Courts, Projects, MSEs, ..., even Banks now

- Ample research (1000's papers, books) since the 50's; **theory = 1-server**
- Significant in heavy-traffic hence begs for heavy-traffic theory, yet
- Aware of 1 example: **Atar, Armony, Honnappa** (2017): Single server, via ["Asymptotically optimal appointment schedules with customer no-shows"](#) (submitted)
- Ongoing: **with Momcilovic, Trichakis, DFCI partners; Huang**
["Data-Driven Appointment-Scheduling Under Uncertainty: ... Infusion..."](#) (submitted)
 - QED Appointments (ongoing)
 - Ultimately: DFCU = Network in heavy-traffic (start AAH, then mix with QED)

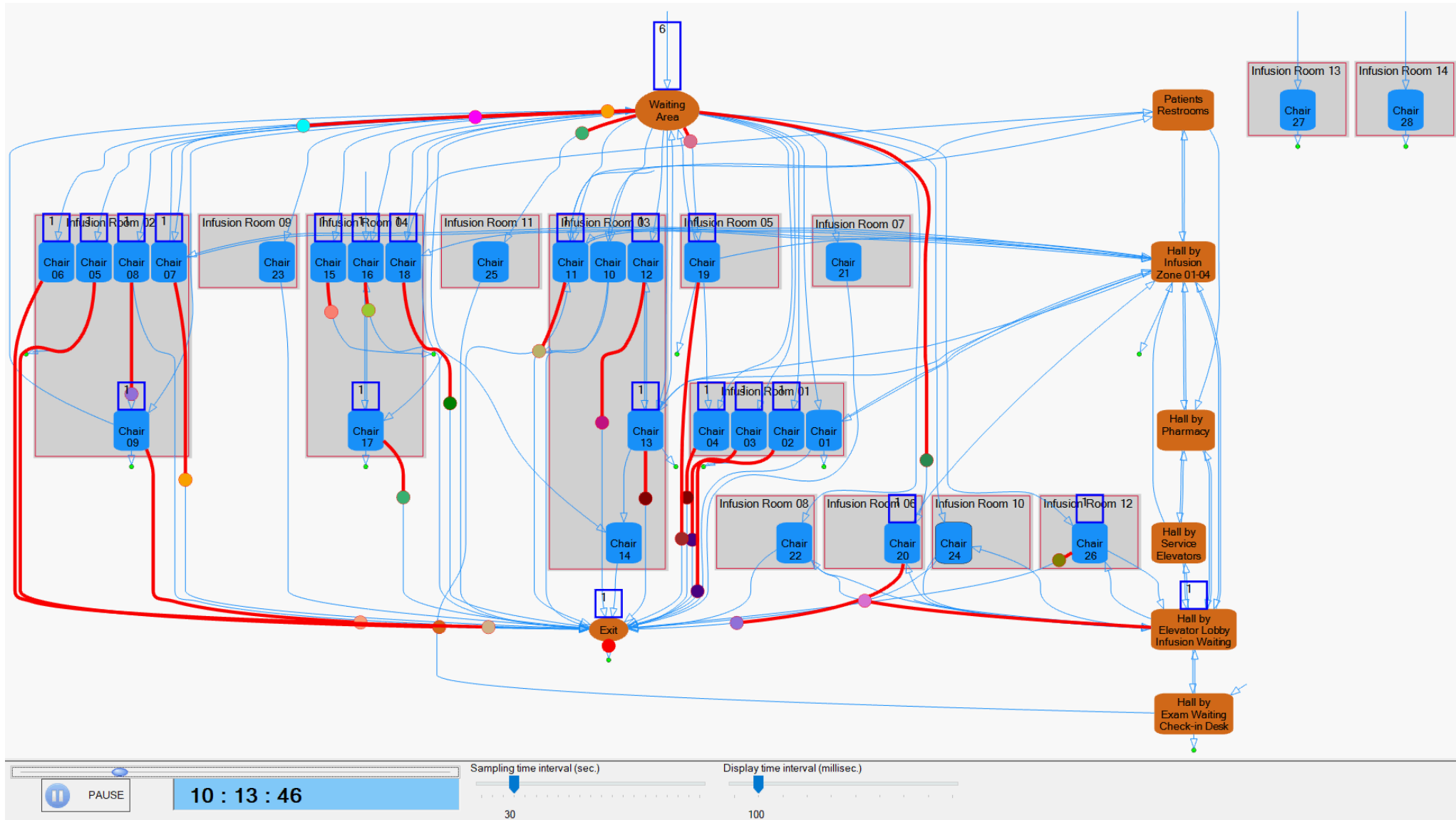
Appointments dominate healthcare systems. Such systems are intrinsically stochastic, yet appointment systems (too) often view them deterministically:

DFCI Sample Infusion Schedule on Sep 3, 2014

time	patient_id	duration (min)	link_flag	floor_id	disease_center
15:00	01	60	unlinked	9	breast onc.
12:30	02	120	unlinked	9	breast onc.
10:30	03	180	linked	9	genitourinary onc.
12:30	04	60	linked	9	breast onc.
12:00	05	120	linked	9	genitourinary onc.
07:00	06	60	unlinked	9	breast onc.
⋮	⋮	⋮	⋮	⋮	⋮

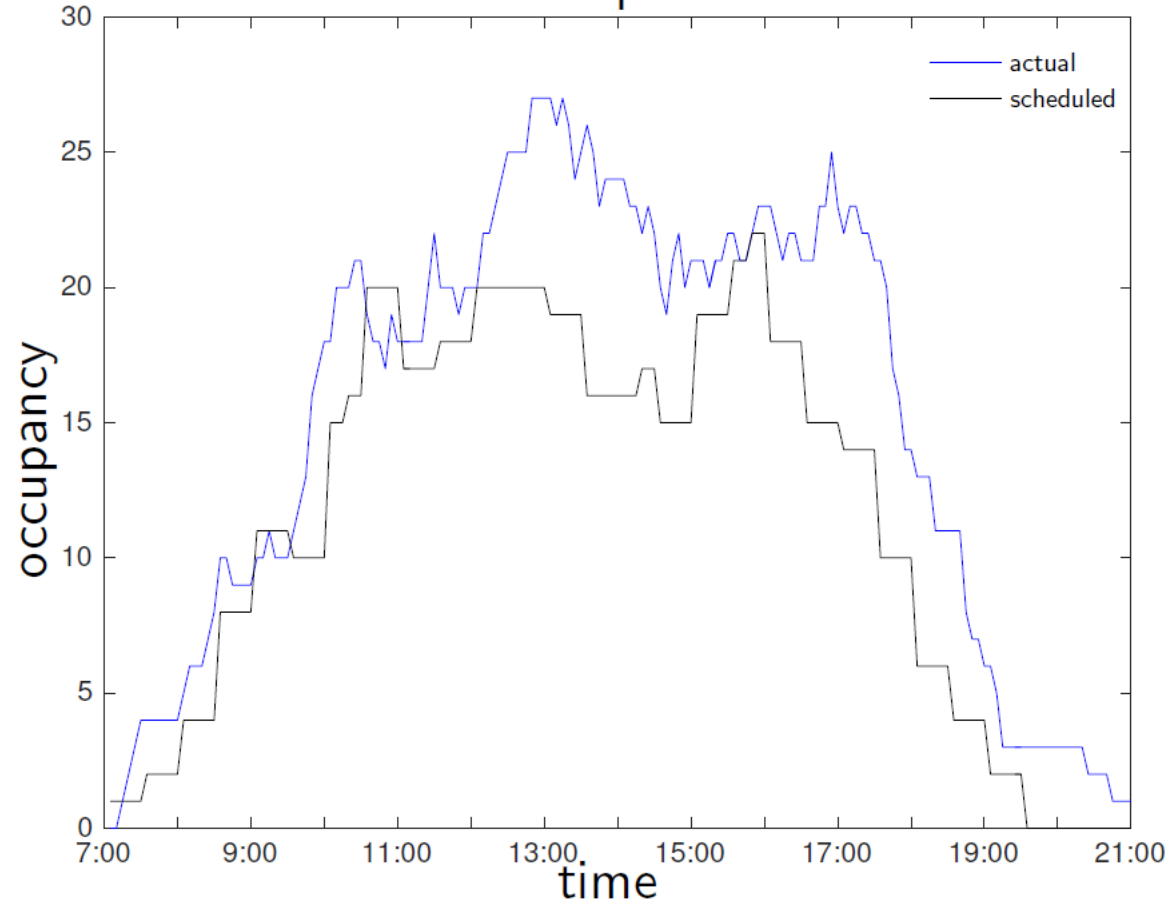
w/ P. Momcilovic & N. Trichakis: Develop methodologies for appointment scheduling, in **multi-server** environments, that take into account **stochastic** punctuality and service-durations

Patients Flow: Nth Floor, Infusion Unit w/ P. Momcilovic & N. Trichakis



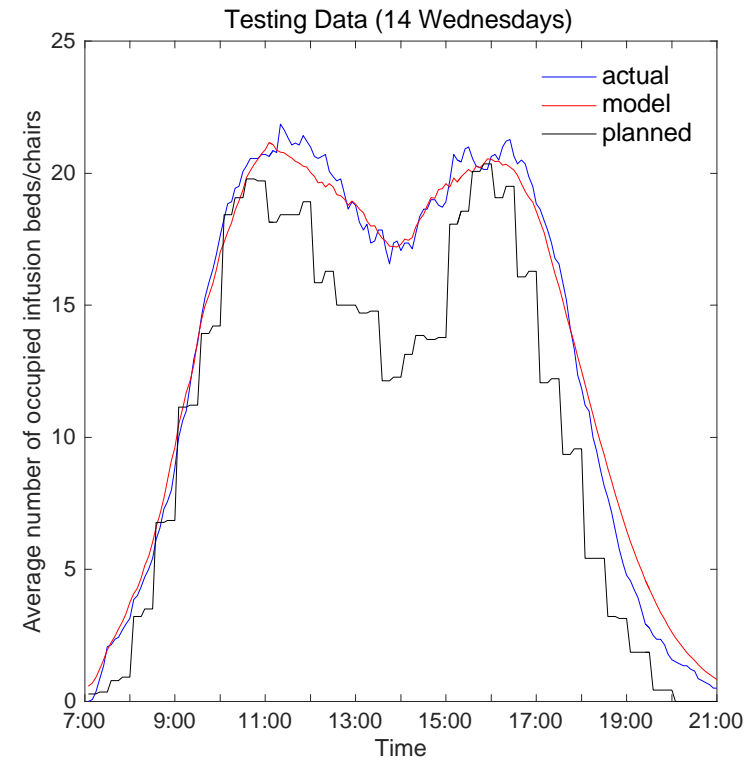
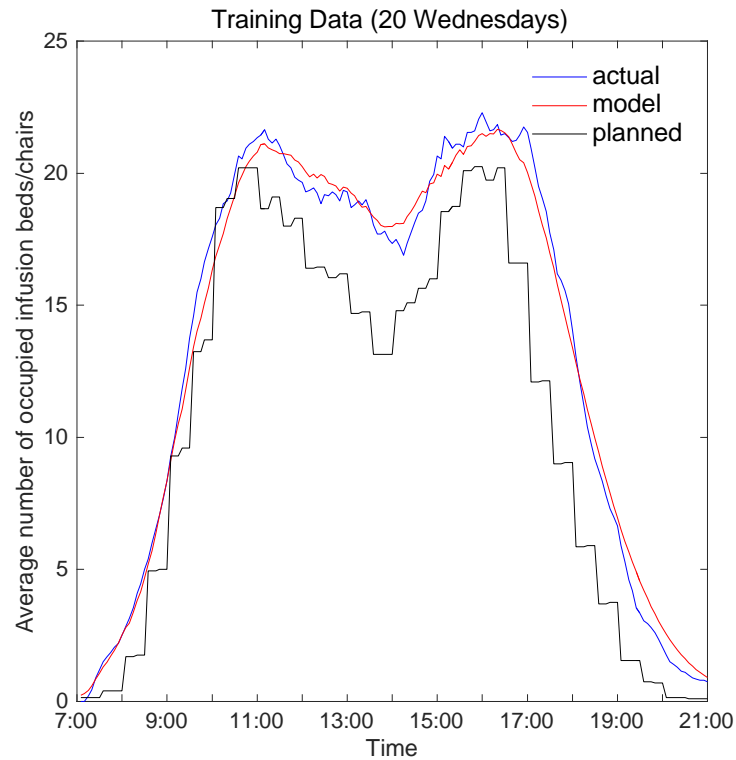
Scheduled vs Actual

03-Sep-14



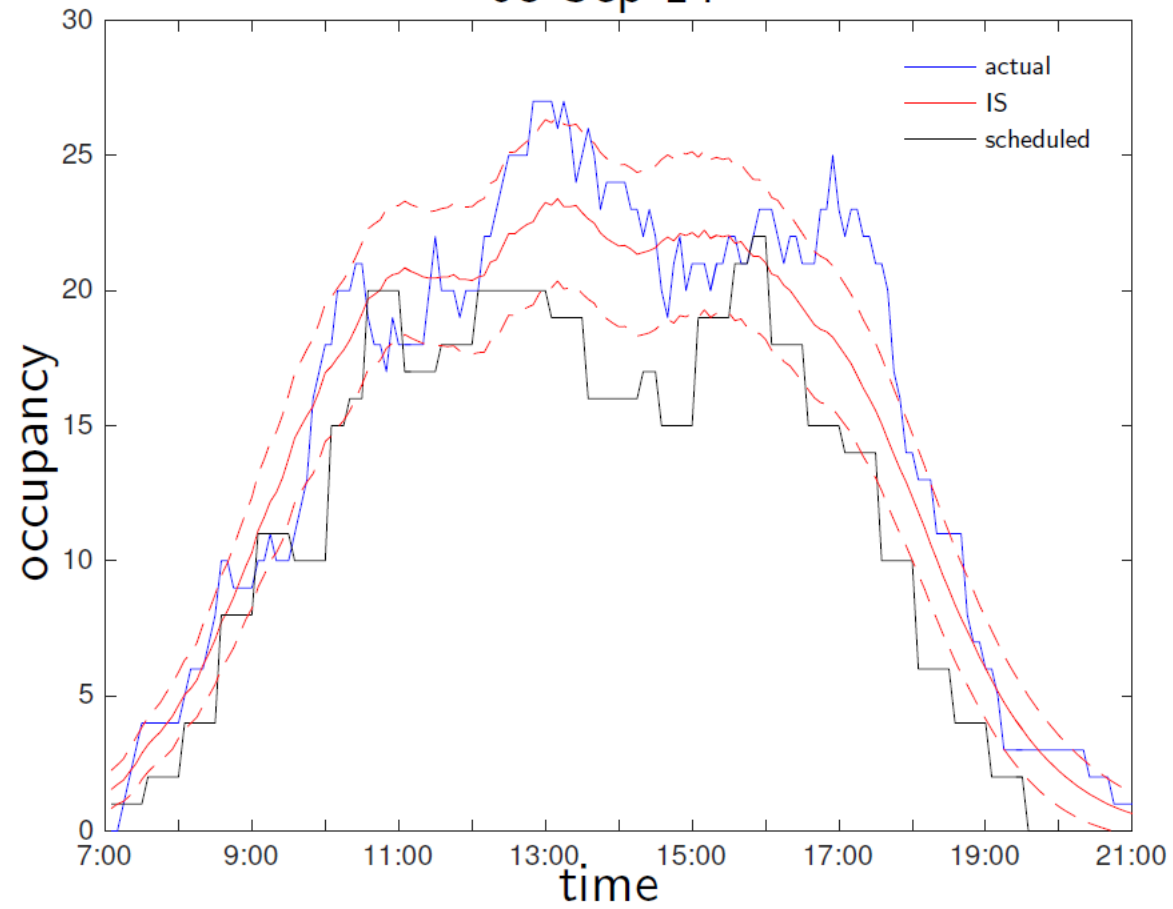
Approach & Validation

Appointment scheduling in **multi-server** environments: accounting for **stochastic** punctuality and service-durations, via infinite-server models (offered-load)



Validation with DFCI Data

03-Sep-14



Infinite-Server (IS) Approximation

- i th customer arrives $a_i + P_i$, leaves $a_i + P_i + D_i$ ($c_t = \infty$)
- $Z_i(t) := 1_{\{a_i + P_i \leq t < a_i + P_i + D_i\}}$ “presence” indicator
- $Z(t) = \sum_{i=1}^n Z_i(t)$

$$\mathbb{E}Z(t) = \sum_{i=1}^n \mathbb{E}\tilde{F}_i(t - a_i - P_i) =: \sum_{i=1}^n \Omega_i(t)$$

$$\text{Var}(Z(t)) = \sum_{i=1}^n \Omega_i(t) (1 - \Omega_i(t))$$

where $\tilde{F}_i(x) := 1_{\{x \geq 0\}}(1 - F_i(x))$

IS Solution Approach

- recall: $Z = \{Z(t)\}$ occupancy process
- sample path cost

$$Q(Z) := \int_{-\infty}^{\infty} r(Z(t) - c_t) dt + \tilde{\gamma} \int_T^{\infty} Z(t) dt,$$

$r(\cdot)$ cost function, *e.g.*, $(\cdot)^+$

- CLT approximation $\tilde{Z}(t) := \mathbb{E}Z(t) + \xi(t) \sqrt{\text{Var}(Z(t))}$
($\xi(t)$ std normal)

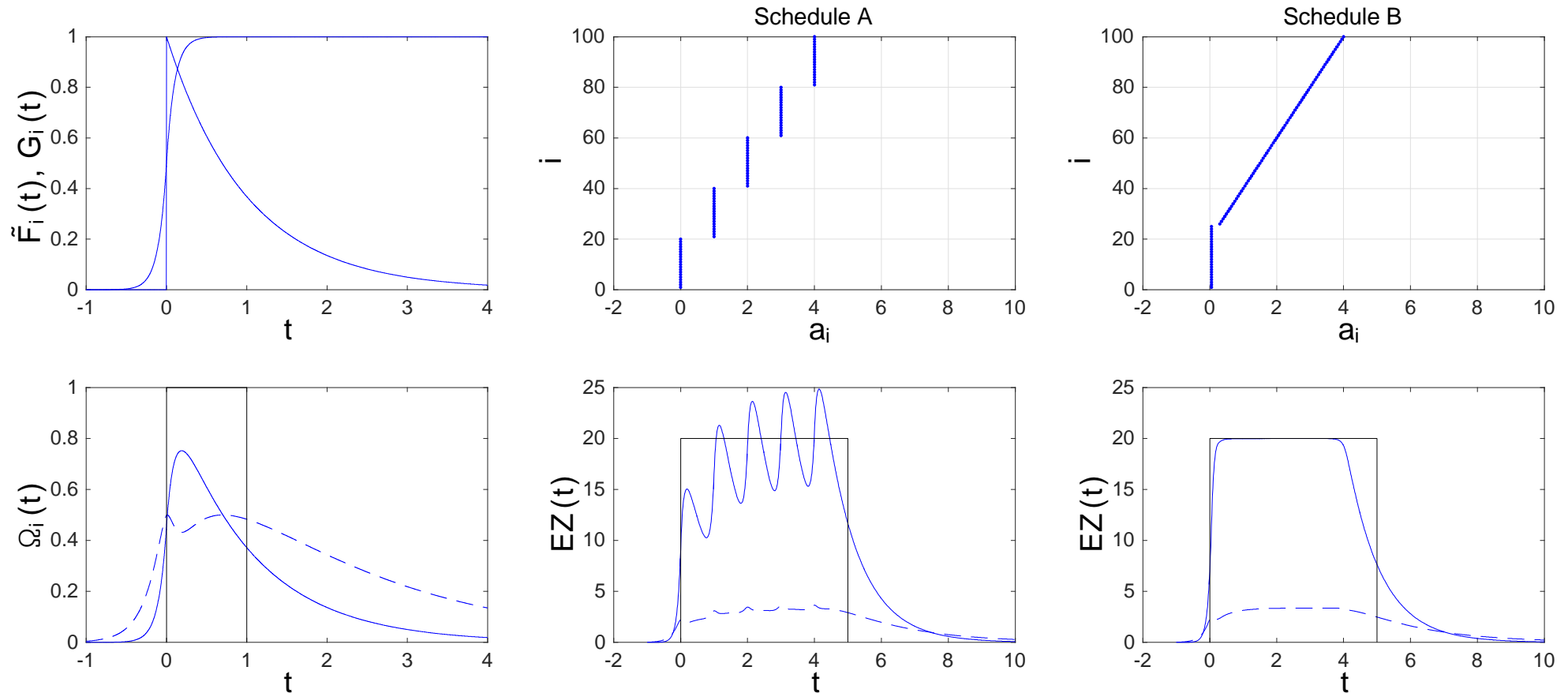
$$\min_{0 \leq a \leq T} \mathbb{E}Q(\tilde{Z})$$

Comparison with DFCI Practice

- infusion service on floors 8, 9
 - $c_t \approx 25$, $n \approx 90$
 - compare IS with “means-based” scheduling
 - consider 85% and 95% utilization levels
- Reduce costs (waiting, overtime) by approximately **30%**
- Hopefully a pilot soon

Appointment Scheduling Matters: Means-Based (Prevalent) vs. (Reasonable/almost Optimal) Alternative

- 100 customers, 20 servers: $\{a_i\}_i$ appointment times
- **Exponential** service times with mean=1; **Laplace** distribution of punctuality
- Compare 2 schedules:



Observations on “Academia-Industry Partnerships” (not MSR or the Original-Bell-Labs)

- Intriguingly: RTLS successful at a **leading research** hospital?
- Intriguingly: initiative of **Physicians** at DFCI (as opposed to managers)?
- **Conjectures?**

Some Observations on “Academia-Industry Partnerships” (not MSR or the Original-Bell-Labs)

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- **Conjectures?**

Partnership with “Strong” partners, who appreciate Research/Evidence-Based-Management, and it is naturally based on **Knowledge** partnering with **Data**

Data enables **SYMMETRIC Partnerships**, caters to goals of both partners (even IP)

The Technion SEE Center / Laboratory
Data-Based Service Science / Engineering

