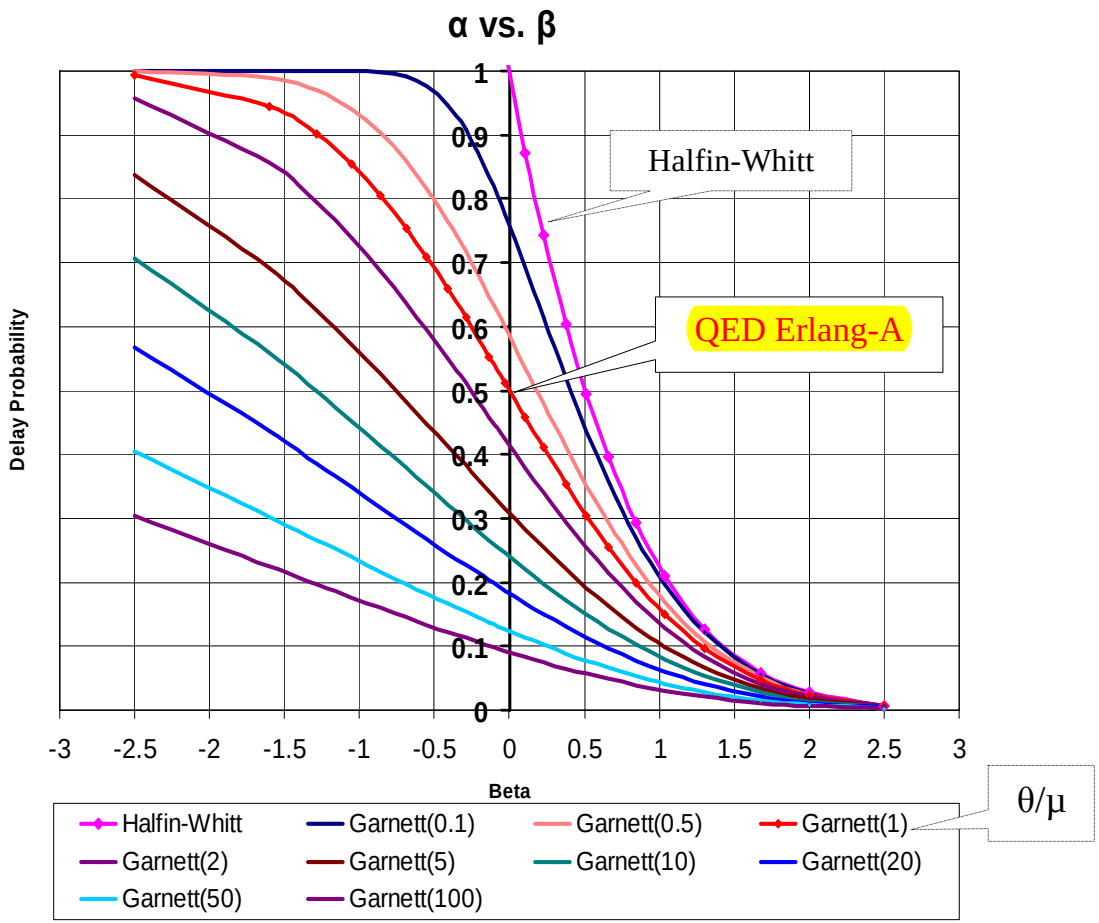
HW/GMR Delay Functions

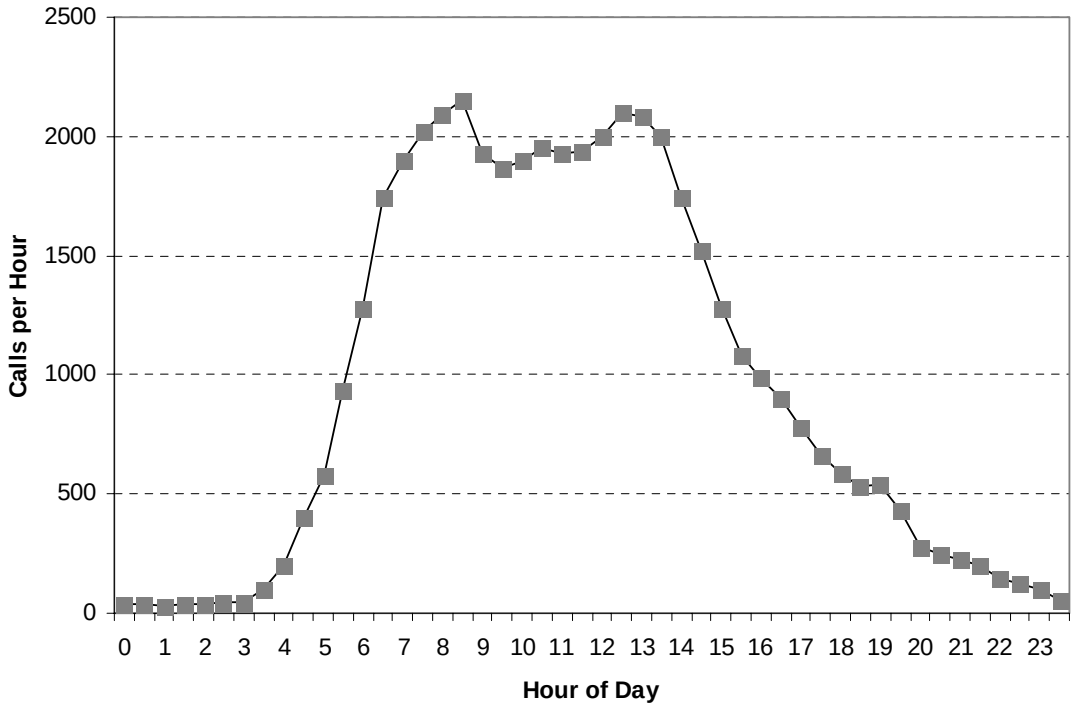


Example: "Real" Call Center

(The "Right Answer" for the "Wrong Reasons")

Time-Varying (two-hump) arrival functions common

(Adapted from Green L., Kolesar P., Soares J. for benchmarking.)



Assume: Service and abandonment times are both

Exponential, with mean 0.1 (6 min.)

Time-Varying Arrivals

Model $M_t / M / N_t$ + M

Parameters $\lambda(t)$ μ **?** θ

? $N_t = R_t + \beta\sqrt{R_t}$

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Average # in $M_t / M / \infty$

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Average # in $M_t / M / \infty$

Gives rise to TIME-STABLE PEFORMANCE

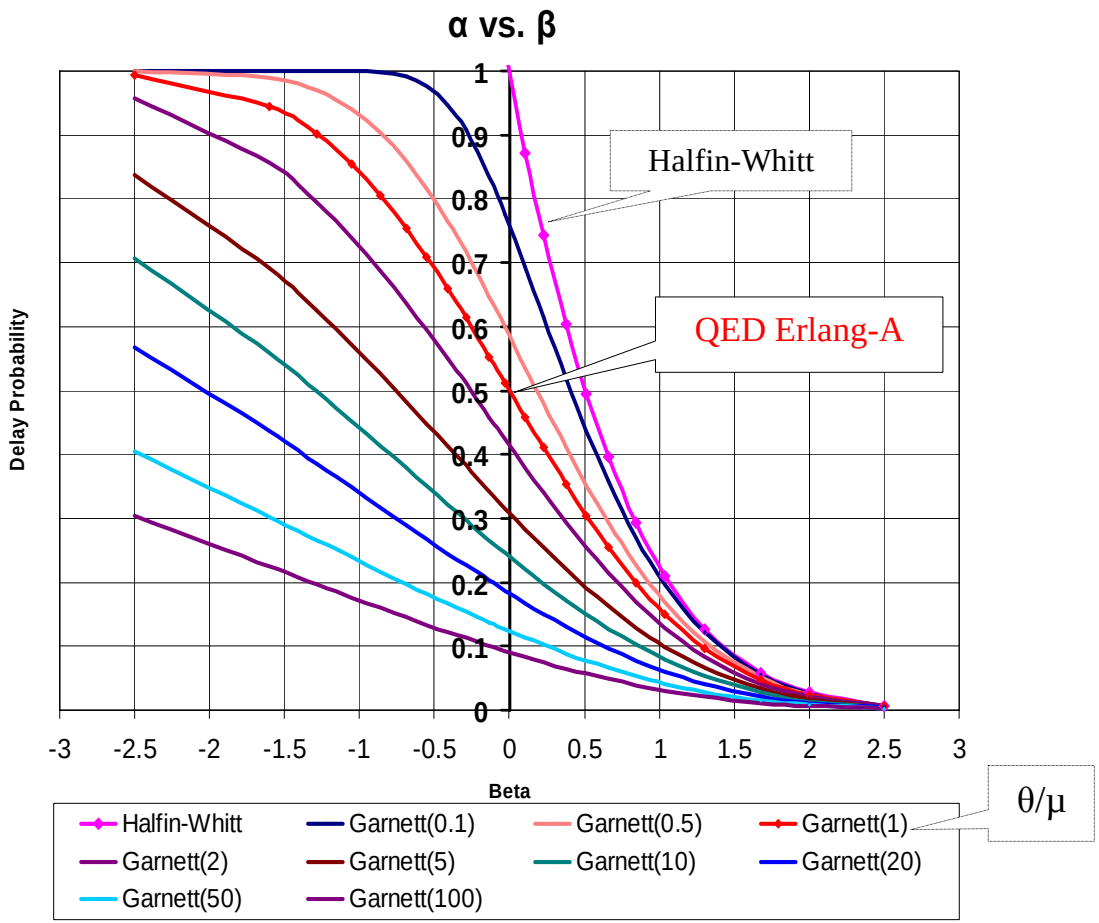
(Why? Think $M_t / M / N_t$ + M with $\mu = \theta$;

And if $\mu \neq \theta$, or generally:

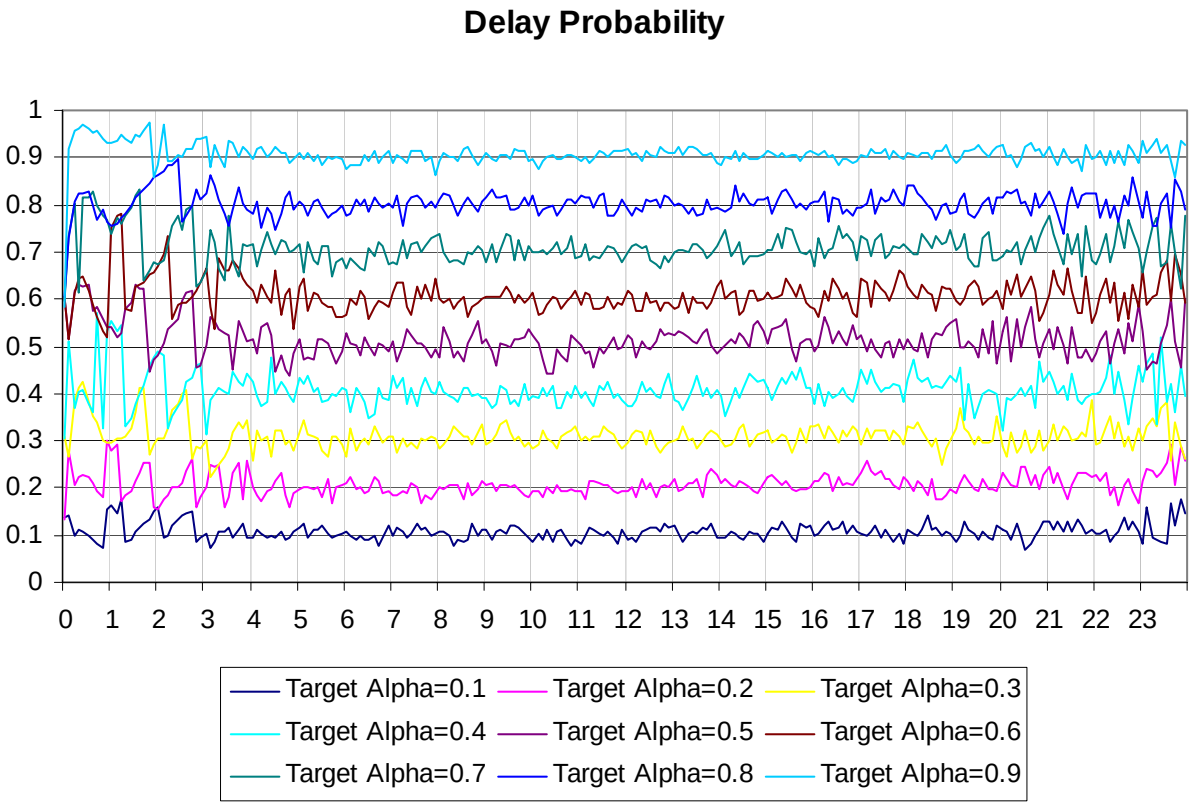
use the Iterative Simulation-Based Staffing Algorithm

in Feldman, M., Massey and Whitt, 2005.)

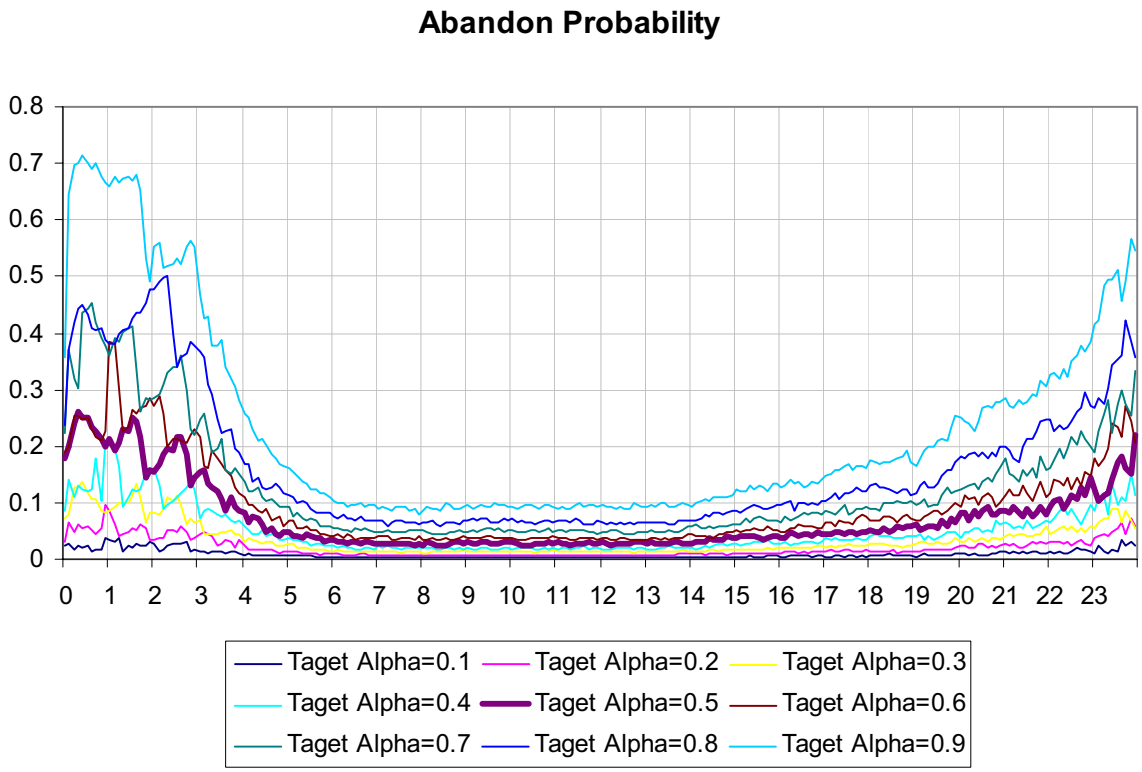
HW/GMR Delay Functions

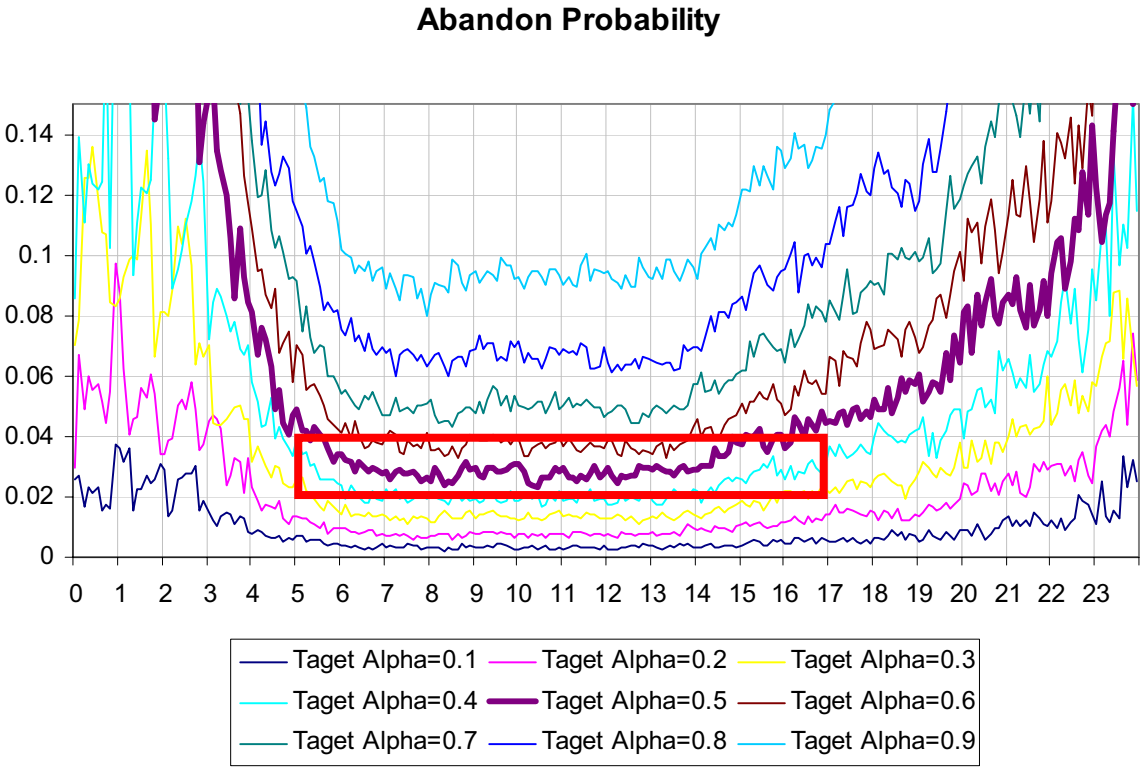


Delay Probability α



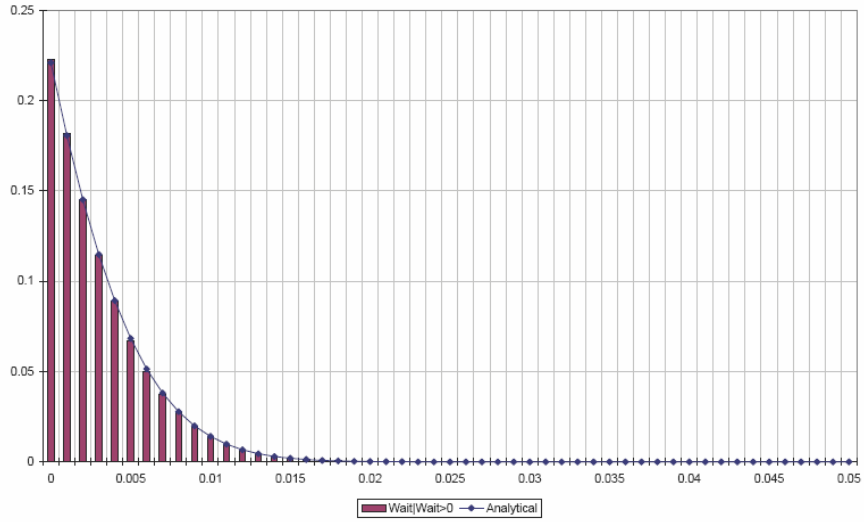
Abandon Probability

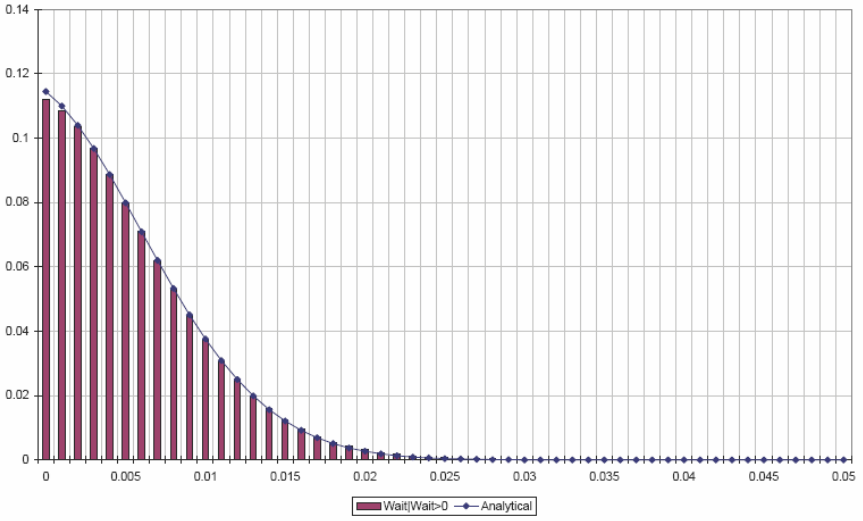


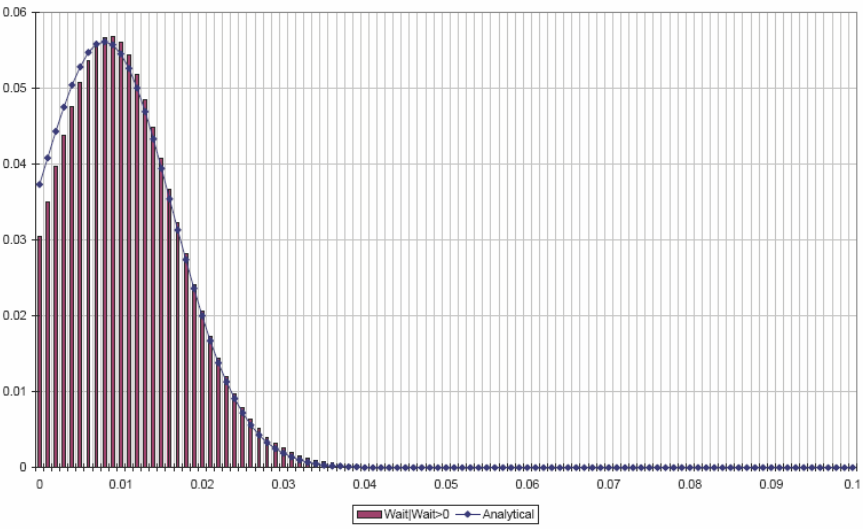


Real Call Center: Empirical waiting time, given positive wait

(1) α=0.1 (*QD*) **(2)** α=0.5 (*QED*) **(3)** α=0.9 (*ED*)







The "Right Answer" (for the "Wrong Reasons")

Prevalent Practice $N_t = \lceil \lambda(t) \cdot E(S) \rceil$ (PSA)

"Right Answer" $N_t \approx R_t + \beta \cdot \sqrt{R_t}$ (MOL)

$$R_t = E\lambda(t - S) \cdot E(S)$$

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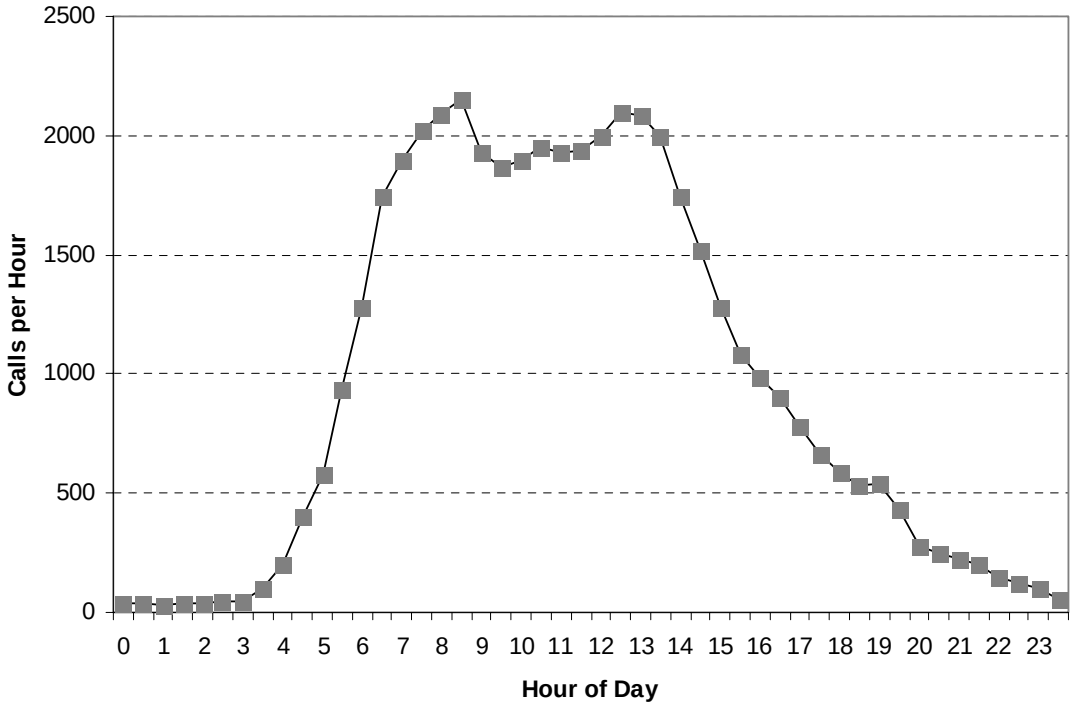
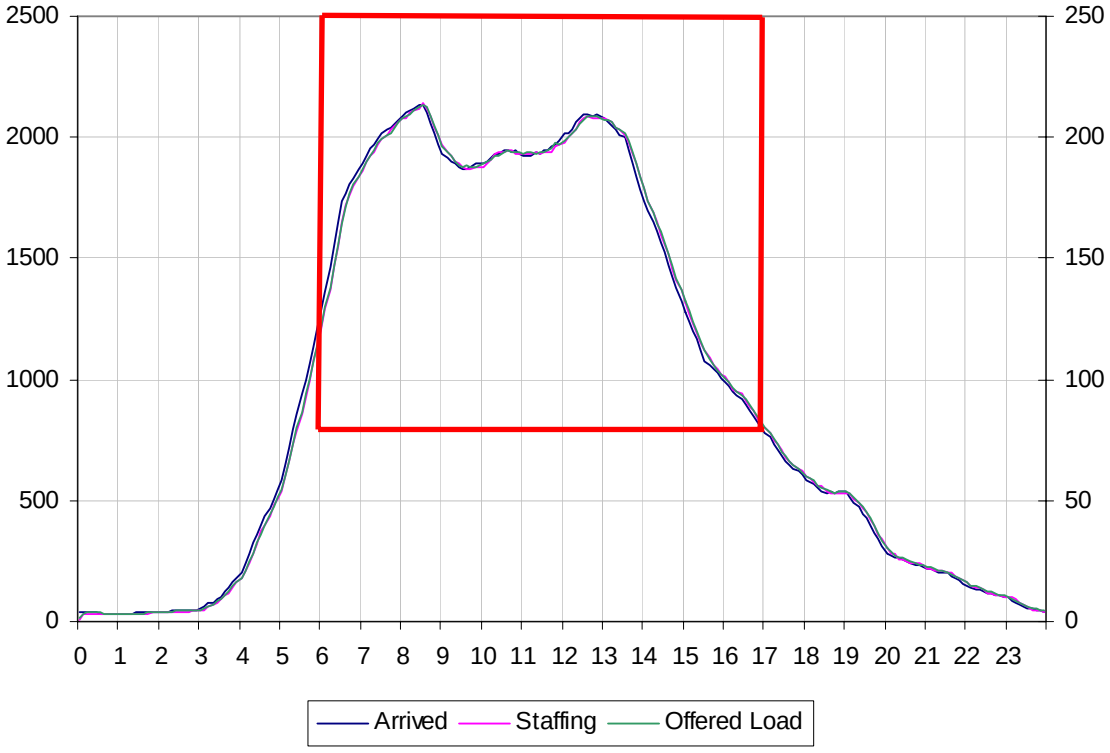
Practice ≈ "Right" $\beta \approx 0$ (QED)

and $\lambda(t) \approx$ stable over service-durations

Practice Improved $N_t = \lceil \lambda[t - E(S)] \cdot E(S) \rceil$



QED Staffing (β=0 iff α=0.5)



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When Optimal ? for moderately-patient customers:

1. Satisfization ⇔ At least 50% to be serve immediately
2. Optimization ⇔ Customer-Time = 2 x Agent-Salary

Time-Varying Arrivals: $\sqrt{\cdot}$ Safety-Staffing

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$\mu = \theta$: $L_t \overset{d}{=} \text{Poisson}(R_t) \overset{d}{\approx} N(R_t, R_t)$, since $M_t / M / \infty$

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Given $L_t \approx R_t + Z\sqrt{R_t}$, $Z \overset{d}{=} N(0,1)$

choose $N_t = R_t + \beta\sqrt{R_t}$

$\Rightarrow$ $\alpha = P(W_t > 0) \underset{\text{PASTA}}{\approx} P(L_t \geq N_t) = P(Z \geq \beta) = 1 - \phi(\beta)$

$\Rightarrow$ $\beta = \phi^{-1}(1-\alpha)$ time-stable $\alpha \equiv P(W_t > 0)$?

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($\mu \neq \theta$, or generally : Iterative Simulation-Based Algorithm)